Lawrence J. Christiano and Terry J. Fitzgerald

Introduction and summary

Good fiscal and monetary policy requires a clear understanding of the workings of the economy, especially what drives the business cycle—the periodic ups and downs in economic activity. Since at least the late 1800s, a full swing from the start of an economic expansion to a recession and back to the start of another expansion has generally taken between two and eight years. Every citizen is keenly aware of the state of the economy, whether it is in prosperity or recession.

Everyone is so conscious of the business cycle because most sectors of the economy move up and down together.¹ This phenomenon, referred to as *comovement*, is a central part of the official definition of the business cycle. The definition is set by the National Bureau of Economic Research (NBER), which decides when recessions begin and end. Under the NBER's definition,

"... a recession is a [persistent] period of decline in total output, income, employment, and trade, usually lasting from six months to a year, and *marked by widespread contractions in many sectors of the economy*."²

Even though comovement is a defining characteristic of the business cycle, in recent decades macroeconomists have tended to focus on understanding the persistence in the ups and downs of aggregate economic activity. They have generally been less concerned with understanding the synchronized nature of this pattern across sectors. In part, the omission reflects the conceptual difficulties inherent in thinking about an economy with many sectors.³ Standard models of business cycles assume there is only one good being produced and so they consider only one economic sector. These models do not encourage thinking about the comovement of economic activity across many sectors. Since these models were first introduced, in the late 1970s and early 1980s, the state of macroeconomics has advanced rapidly. The conceptual and computational barriers to thinking about multiple sectors are quickly falling away. As a result, recent years have witnessed a renewed interest in understanding comovement.

We have two objectives in this article. The first is to document business cycle comovement. We examine data on hours worked in a cross section of economic sectors. We examine the business cycle components of these data and show that the degree of comovement is substantial. Our second objective is to analyze explanations for this comovement. We find that none is completely satisfactory. Still, this is a growing area of research, and we are seeing some progress.

Identifying comovement

To study comovement across sectors over the business cycle, we need the following two things: a measure of the level of economic activity in the various sectors of the economy; and a precise definition of what we mean by the business cycle component of the data. Below, we address these two issues. After that, we present our results, characterizing the degree of comovement in the data.

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The data

We measure economic activity in a given sector by the number of hours worked in that sector. Table 1 lists the sectors we consider. The hours worked measure that covers the most sectors is *total private hours worked.*⁴ This measure covers all sectors of the economy, except government and agriculture. It is broken into hours worked in goods-producing industries and in service-producing industries. Goods-producing industries are further broken into mining, manufacturing, and construction. Similarly, service-producing industries are broken into five subsectors. The subsectors of manufacturing, durable goods and nondurable goods, are broken into yet smaller sectors. The data in the third column give an indication of the relative magnitude of each subsector. In particular, any given row reports the average number of people employed in that sector, divided by the average number of people employed in the sectoral aggregate to which that sector belongs. Thus, for example, 58 percent of manufacturing employment is in the durable goods sector and 42 percent is in the nondurable

	TABLE 1				
	Properties of the business cycle components of hours worked				
Variable number	Hours worked variable	Relative magnitude	Relative volatility	Business cycle comovement	
1	Total private	1.00	1.00	.00	
2	Goods-producing industries	.33	3.91	.99	
3	Mining	.03	5.46	.38	
4	Construction	.17	6.75	.88	
5	Manufacturing	.80	3.92	.97	
6	Durable goods	.58	6.90	.97	
7	Lumber and wood products	.06	10.18	.89	
8	Furniture and fixtures	.04	8.14	.94	
9	Stone, clay, and glass products	.05	4.98	.95	
10	Primary metal industries	.09	9.89	.86	
11	Fabricated metal products	.13	7.21	.96	
12	Machinery, except electrical	.19	11.10	.93	
13	Electrical and electronic equipment	.15	8.75	.88	
14	Transportation equipment	.17	7.83	.89	
15	Instruments and related products	.08	5.03	.76	
16	Miscellaneous manufacturing	.04	3.23	.90	
17	Nondurable goods	.42	1.39	.91	
18	Food and kindred products	.21	.16	.50	
19	Tobacco manufactures	.01	1.83	.08	
20	Textile mill products	.11	3.92	.76	
21	Apparel and other textile products	.15	2.64	.85	
22	Paper and allied products	.09	1.97	.85	
23	Printing and publishing	.16	.91	.90	
24	Chemicals and allied products	.13	1.01	.80	
25	Petroleum and coal products	.02	2.02	.16	
26	Rubber and misc. plastics products	.09	7.82	.89	
27	Leather and leather products	.03	2.71	.64	
28	Service-producing industries	.67	.25	.93	
29	Transportation and public utilities	.10	.87	.95	
30	Wholesale trade	.10	.65	.87	
31	Retail trade	.31	.36	.87	
32	Finance, insurance, and real estate	.10	.35	.48	
33	Services	.38	.19	.49	

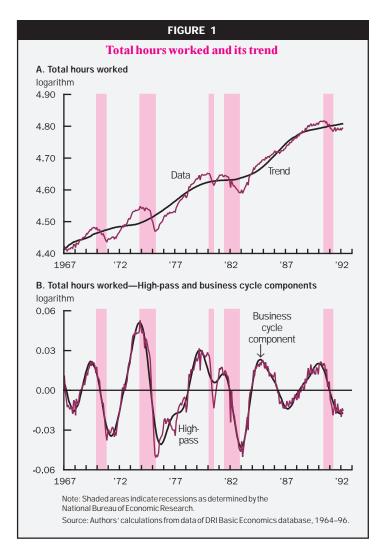
Notes: The column labeled "Relative magnitude" reports an indication of the relative magnitude of each sector. Any given row reports the average number of people employed in that sector divided by the average number of people employed in the sectoral aggregate to which that sector belongs, for example, 58 percent of manufacturing employment is in the durable goods sector and 42 percent is in the nondurable goods sector. The column labeled "Relative volatility" reports the variance of the business cycle component of the logarithm of hours worked in the indicated row variable divided by the variance of the business cycle component of the logarithm of total private hours worked. The column labeled "Business cycle comovement" is calculated using the process described in note 6 of the article. Source: Authors' calculations from data of DRI Basic Economics database, 1964–96.

goods sector. Also, the largest goods-producing industry, by far, is manufacturing, which has 80 percent of all goods-producing employees.

Next, we try to characterize how much business cycle comovement there is across the economic sectors we consider. That is, if we limit ourselves to the business cycle range of fluctuations in the data—fluctuations that last between two and eight years—to what extent do the data move up and down together?⁵

Business cycle component of the data

A detailed discussion of our notion of the business cycle component of the data is in technical appendix 1. Figure 1 illustrates the basic idea behind what we do. The choppy line in panel A of figure 1 displays total private hours worked. The reported data are the logarithm of the raw data. The advantage of using the logarithm of the data in this way is that the resulting movements correspond to percent changes in the



underlying raw data. The deviations between the actual data and the trend line in panel A of figure 1 are graphed in panel B. Those deviations contain the rapidly varying, erratic component, inherited from the choppy portion of the data that is evident in panel A. The smooth curve in panel B is our measure of the business cycle component of the total private hours worked data. Specifically, that measure excludes both the trend part of the data and the rapidly varying, erratic component. It includes only the component of the data that contains fluctuations in the range of two to eight years. According to our approach, the economy is in recession when our business cycle measure is negative and in prosperity when it is positive.

Figure 1 also compares our measure of the business cycle with the one produced by the NBER. The start of each shaded area indicates the date when, according to the NBER, the economy reached a business cycle peak. The end of each shaded area indicates

> a business cycle trough. Note how total private hours worked fall from peak to trough and then generally grow from trough to peak. An obvious difference in the two business cycle measures is that ours is a continuous variable, while the NBER's takes the form of peak and trough dates. As a result, our measure not only indicates when a recession occurs, but also the intensity of the recession. Apart from these differences, however, the two measures appear reasonably consistent. For example, note that near the trough of every NBER recession, our measure of the business cycle is always negative. But the two measures do not always agree. According to our measure, the economy was in recession in 1967 and in 1987, while the NBER did not declare a recession during those periods. In part, this is because there must be several months' negative employment growth before the NBER declares a recession. However, our procedure only requires a temporary slowdown.

> Figure 1 provides informal evidence in support of the facts we wish to document. As noted in the introduction, the NBER must see a broad-based decline before declaring a recession. Thus, the NBER dates in figure 1 indicate periods when many economic sectors showed weakness. Since these dates roughly coincide with periods of weakness in total

private hours worked, this is consistent with the view that most sectors move up and down together, at least in the two- to eight-year frequency range. We stress, however, that the NBER's dating procedures are informal. Our objective in this section is to provide a formal, quantitative assessment of the degree of comovement among economic sectors.

We computed a business cycle component for each of the 33 series listed in table 1. As we anticipated, we find that the business cycle components in most of the series move together closely. This is true, despite a striking lack of uniformity in other respects. For example, note how different the trends in figure 2 are. The first two columns report data for the goodsproducing industries and its major components. The second two columns report the analogous data for the service-producing industries. Generally, trend employment is down in the goods-producing industries and up in the service-producing industries. The levels of volatility in the business cycle components of the various series are also very different. The fourth column of table 1 reports the variance of the business cycle component of a variable, divided by the variance of aggregate hours worked. The relative variance of hours worked in goods-producing industries is typically quite high, substantially above 2, and it is quite low for service-producing industries. That goodsproducing industries are volatile relative to the serviceproducing industries is well known.

Measuring business cycle comovement

Despite the very substantial differences in the trends of the data series shown in figure 2, their movements over the business cycle are quite similar. Figure 3 illustrates the business cycle components of the same variables used in figure 2. In each case, we computed the business cycle component using exactly the same method underlying the calculations in panel B of figure 1. Each graph contains the business cycle component of the variable indicated and the business cycle component for total private hours. This was taken directly from panel B of figure 1.

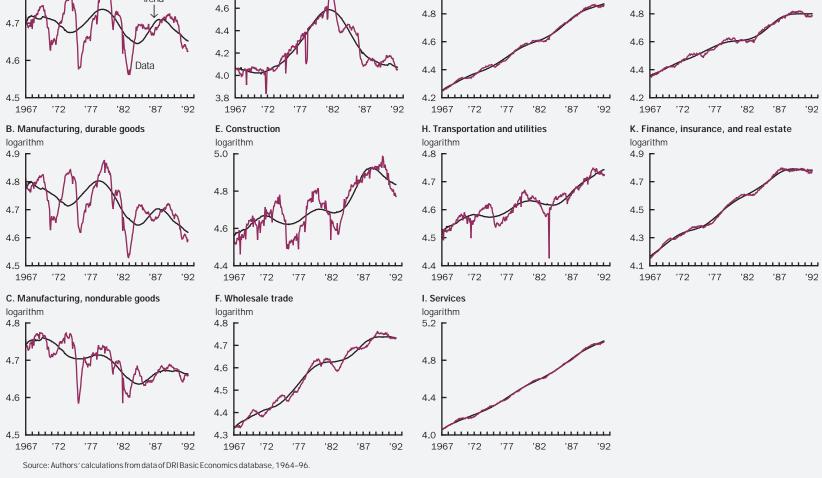
In most of the series in figure 3, the data move up and down closely with the business cycle component of total hours worked. There are some exceptions. For example, the business cycle movements in mining bear little resemblance to the business cycle movements in total hours worked. At the same time, mining represents a very small part of the private economy and employs only 3 percent of workers in the goodsproducing industry. Another exception is the finance, insurance, and real estate (FIRE) industry, whose business cycle component exhibits reasonably high comovement with aggregate employment until the 1980s, after which this relationship breaks down.

To measure the degree of business cycle comovement between a given series and total hours worked, we use a statistic that is like the square of the correlation between the business cycle components in the two variables. Our statistic measures the fraction of the variance in the series that can be accounted for by the total hours worked data.⁶ If this number is, say, 98 percent, this means that 98 percent of the business cycle variance in the variable can be accounted for by the business cycle in aggregate hours worked. These results are reported in the fifth column of table 1. As expected, the results indicate that this measure of comovement is relatively low, in the sense of being below 0.50, for the mining, FIRE, and services sectors. Overall, however, the degree of comovement by this measure is high.

Going one step further in the level of disaggregation, we can get an idea about the comovement in the components of durable and nondurable manufacturing. Figure 4, panel A displays the business cycle movements in the components of durable manufacturing sectors. Panel B does the same for nondurable manufacturing. In each case, the data series graphed at the top of the figure is the business cycle component of total hours worked. The series are presented so as to allow one to focus exclusively on the degree of comovement between them. Thus, we added a constant to each series to spread them out across the figure and divided each series by its sample standard deviation, so that the standard deviation of the reported data is unity in each case.⁷ The number to the right of each line in the figure identifies the data series. Figure 4 also displays the NBER peak and trough dates as a convenient benchmark.

Figure 4, panel A shows that the comovement among sectors in durable manufacturing is very high. With only one minor exception, the variables move closely with each other and with aggregate employment. The exception is that instruments and related products, series 15, does not move closely with the other variables during 1987, when the other business cycle components are signaling a recession. However, overall the degree of comovement is strikingly high. Figure 4, panel B shows that the business cycle comovement in the nondurable goods manufacturing industries is lower than in the durable goods sector. Two variables that do not comove closely with the others at business cycle frequencies are tobacco manufactures, series 19, and petroleum and coal products, series 25. Both these variables are rising in the first and last NBER recession periods in our data. The

FIGURE 2 Hours worked in various sectors: Data and trends A. Goods-producing industries D. Mining G. Service-producing industries logarithm logarithm logarithm 5.0 r 4.8 Trend 4.6 4.8 4.4 4.6

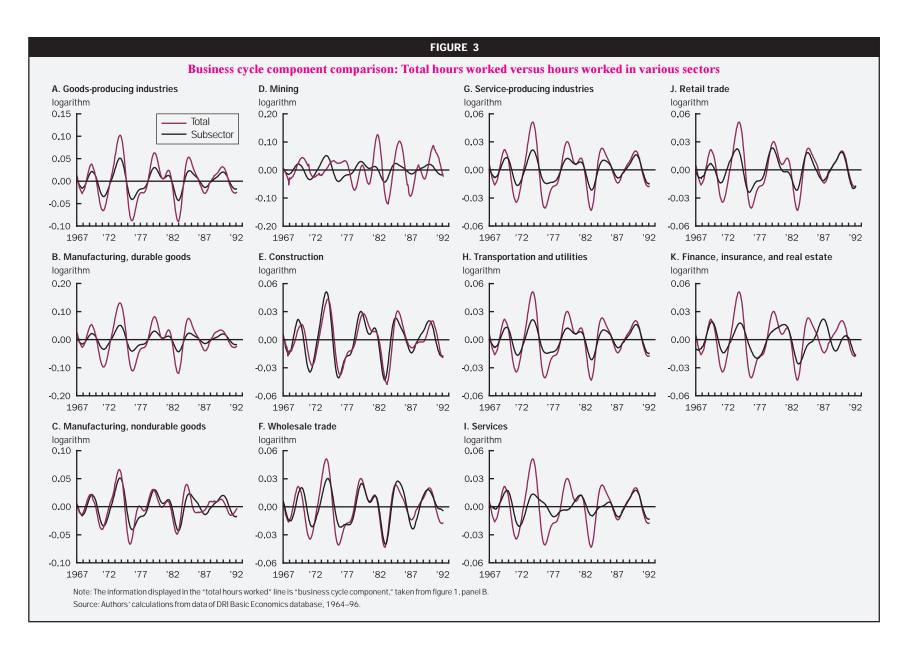


J. Retail trade

logarithm

5.0 r

4.8



comovement statistic for these variables reported in table 1 is very low, 0.08 for tobacco manufactures and 0.16 for petroleum and coal products. The other variables in nondurable manufacturing display stronger comovement, with comovement statistics of 0.50 or higher.

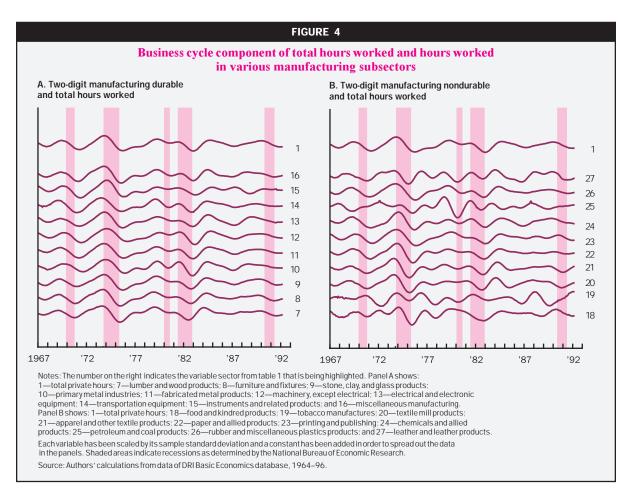
Up to now, the statistics we have used to characterize comovement emphasize association with aggregate hours worked. This nicely complements the visual evidence in the graphs. However, there is a pitfall to relying exclusively on statistics like this to characterize comovement. A simple example illustrates the point. Suppose there is a variable, y_p which is the sum of two other variables, y_{it} and y_{2} :

$$y_t = y_{1t} + y_{2t}$$

Suppose further that y_{1t} and y_{2t} are uncorrelated. No one would say there is comovement between these variables. Still, each variable is strongly correlated with the aggregate. To see this, take the simple case where the variance of y_{1t} and y_{2t} is σ^2 . Then, the correlation between y_{it} and y_t is 0.71, for i = 1, 2, despite the absence of comovement between the variables.⁸ This example exaggerates the point somewhat, since results are less severe when there are more than two sectors.⁹ Still, this pitfall is of some concern.

With this in mind, we consider the correlation between the business cycle components of all the variables. A difficulty with this is that there are many such correlations. For example, with three variables, there are three possible correlations, with four there are six, with five there are ten, and with *n* there are n(n-1)/2. So, with n = 33, there are 528 possible correlations. It is a challenge to organize and present this many correlations in a coherent way. We present the mean and histogram of the correlations for different subsectors in figure 5. The histograms display, on the vertical axis, the fraction of correlations lying within a given interval, whose midpoint is indicated on the horizontal axis.¹⁰

Figure 5, panel A displays the correlations for the finest levels of aggregation for which we have data. This means hours worked in mining, construction, the 20 components of manufacturing, and the five components of the service-producing industries. Thus, we



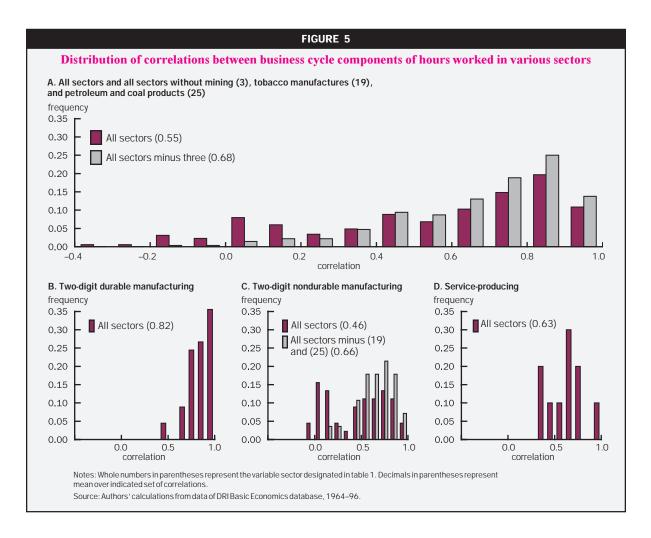
have 27 data series, with 351 correlations between them. Figure 5, panel A indicates that the mean of these correlations is 0.55. When we eliminate the three data series that we already know do not display strong business cycle comovement, the mean rises to 0.68. The histogram shows that there is a substantial fraction of high correlations in these data. We infer that the data are consistent with the impression from the preceding statistics that there is considerable evidence of comovement. Figure 5, panel B presents the results for the manufacturing durable sector. Consistent with our previous findings, the degree of comovement in this sector is very high, with a mean correlation of 0.82. Figure 5, panels C and D show the results for the nondurable manufacturing sector and the serviceproducing sectors, respectively. Again, the results are consistent with the notion that there is less comovement in these sectors than in manufacturing durables. Still, the degree of comovement is substantial, with mean correlations in excess of 0.6 if we consider all sectors except tobacco and petroleum and coal products.

In conclusion, we find that there is substantial business cycle comovement in the data. Only two relatively small sectors—tobacco manufactures and petroleum and coal products—exhibit little tendency to move up and down with general business conditions over the business cycle.

Explaining business cycle comovement

What is it that at times pulls most sectors of the economy up, and at other times pushes them down again? This is one of the central questions in business cycle analysis. Although economists have developed a number of possible explanations, the phenomenon remains a puzzle.

In a classic article devoted to this puzzle, Robert E. Lucas, Jr., conjectures that the resolution must lie in some sort of shock that hits all sectors of the economy, a so-called aggregate shock (Lucas, 1981). Many economists today would probably agree with this conjecture. That is why, in practice, the search for the ultimate cause of business cycles often focuses



on identifying an aggregate shock. However, research conducted since Lucas published his article suggests that identifying the *cause* of business cycles may not be so simple.

First, even if we do manage to identify a shock that clearly affects the whole economy, it does not necessarily follow that shock is responsible for the business cycle. A shock might well be experienced by all sectors of the economy, but they need not all *respond* in the same way. The business cycle shock, if indeed there is only one, seems to lead to a synchronized response across sectors. Second, we now know that the search for a single aggregate shock may itself be off base. Following the work of Long and Plosser (1983), we know that, at least in theory, disturbances to individual industries, even if they are uncorrelated across industries, could result in comovement.

Currently, there is no consensus among economists as to what causes business cycles and, in particular, their key feature, comovement. At the same time, researchers are exploring a large range of possibilities. Next, we provide a selective overview of this research.

A natural starting point is what is perhaps the most thoroughly developed theory of business cycles, the *real business cycle* theory associated with Kydland and Prescott (1982), Long and Plosser (1983), and Prescott (1986).¹¹ We focus specifically on the *stan-dard real business cycle model*, developed in Hansen (1985) and analyzed in Prescott (1986). Although that model posits an aggregate shock, it is inconsistent with business cycle comovement. We then explore two sets of modifications to this model. The first can be viewed as natural extensions of the model. The second depart more significantly from the model's assumptions.

Standard real business cycle theory¹²

A key component of real business cycle theory is a production technology. This is a relationship that specifies how much output a firm can obtain from a given amount of capital and labor resources. This technology is subject to shocks. Sometimes a good shock occurs and more output can be produced for a given level of inputs. In this case, we say the technology has been shifted up. A good technology shock might reflect the implementation of a more efficient way to organize the work force, the acquisition of more efficient manufacturing equipment, or perhaps the discovery of a way to alter the firm's product so that it better meets customers' needs. At other times, a bad technology shock can shift a production technology down. A bad shock might reflect bad weather, a labor dispute, an accident in the workplace, a machine breakdown, or a government policy that encourages an inefficient way of organizing production. According to real business cycle theory, business cycle expansions reflect that shocks affecting firms are mostly on the positive side, while recessions reflect periods when most firms' shocks are on the negative side. Standard formulations abstract from the differences between firms and simply assume they all have the same production technology and are affected by the same shock. Thus, real business cycle theory proposes that the aggregate shock to which Lucas refers is a productivity shock.¹³

The standard real business cycle model not only assumes that all firms are affected by the same productivity shock, but also that there is just one type of good produced (and, therefore, one industry sector) in the economy. At least at first glance, this model does not seem useful for examining business cycle comovement among many sectors. However, it has recently been pointed out that this impression is misleading.¹⁴ In fact, one *can* use the model to examine business cycle comovement. When we do so, we find that its implications are strongly counterfactual. The standard real business cycle model is at variance with the observation of business cycle comovement, despite the fact that it views the economy as being driven by a single aggregate shock. Understanding why it is incompatible with comovement is useful for gaining insight into the various lines of inquiry researchers have pursued.

The standard real business cycle model imagines that households interact with firms in competitive markets, in which they supply labor and physical capital and demand goods for consumption and to add to their stock of capital. Although there is only one type of production technology in this model, we can reinterpret the model to suggest that one type of firm produces goods for consumption (the consumption goods industry) and another type produces new investment goods for maintaining or adding to the stock of capital (the investment goods industry).

When a positive productivity shock hits, so that the real business cycle model shifts into a boom, the output of both consumption and investment goods industries increases. However, there is a relatively larger increase in the output of investment goods. This reflects a combination of two features of the model. First, a positive technology shock increases the expected return to investment, raising the opportunity cost of applying resources to the consumption sector. Second, the model assumes that households prefer not to increase consumption substantially during booms but to smooth consumption increases over a

longer time horizon. The increase in the demand for investment goods relative to consumption goods that occurs in a boom implies, in the standard model, that capital and labor resources are shifted out of the production of consumption goods and into the production of investment goods. The model does predict a small rise in consumption in a boom. However, this rise is driven by the favorable technology shock, which is not fully offset by the flow of productive resources out of that sector. Thus, the model implies that hours worked in the consumption sector are countercyclical, in contrast with our empirical findings in the previous section. This is a feature of the model, despite its implication that total hours worked rise in a boom. That is, the additional hours of work all flow into the investment good sector. The standard real business cycle model also implies that investment in capital for use in the consumption sector is countercyclical. This too, is counterfactual, according to the results reported in Huffman and Wynne (1998).

So, this model is strongly at variance with comovement. Why is this so? The result may seem especially surprising to those who expect an aggregate shock to all sectors of the economy to produce comovement.

Intuitively, there are two related ways to understand the model's implication that inputs are allocated away from the sector that produces consumption goods during a boom. One is that the model overstates the value of leisure at that time. This inflates the cost of allocating labor resources to the consumption sector then. The other is that the model understates the value of the output of the sector producing consumption goods in a boom. This undercuts the incentive to allocate resources to that sector then.¹⁵

Natural extensions of the standard theory

Among the various extensions to the model that economists have pursued,¹⁶ we focus on approaches that stress 1) factors that prevent the rise in the marginal utility of leisure in a boom and 2) factors that prevent the decline in the value of the output of the consumption sector in a boom. As in the discussion above, the work we survey here assumes two market sectors.¹⁷

Value of leisure

One factor that can slow the decline of the marginal utility of leisure when the economy moves into a boom was explored in Benhabib, Rogerson, and Wright (1991) and Einarsson and Marquis (1997). Each of these papers points out that if there is a third use of time, in addition to leisure and time spent working in the market, and if that use of time declines during a boom, the marginal utility of leisure need not increase as market effort increases.¹⁸

Benhabib, Rogerson, and Wright (1991) suggest that the third use of time may be working in the home. For example, the amount of leisure time enjoyed by a homemaker may not change significantly if the homemaking job is exchanged for a market job. Considerations like this lead Benhabib, Rogerson, and Wright to argue that work time can be reallocated from the home to the market during a boom without substantially raising the marginal utility of leisure.¹⁹

Einarsson and Marquis (1997) suggest that the third use of time may be time spent accumulating human capital, such as going to school. In principle, this is an appealing idea, since it is known that time spent in educational pursuits goes down in business cycle expansions. Their work is primarily theoretical, however. A crucial issue one would have to address in pursuing this explanation at the empirical level is whether the time spent on education is sufficiently countercyclical, in a quantitative sense, to have a substantial effect in a suitably modified real business cycle model. In assessing this, one would have to confront a substantial measurement problem. In particular, time spent in educational institutions is only part of the time spent in education. Some of that time is applied in the workplace, by diverting workers from direct production. Our understanding is that there do not exist reliable measures of this use of time.

Value of the output of the consumption sector

Several papers attempt to get at comovement by reducing the decline in the value of output in the consumption sector during booms. For example, Baxter (1996) adapts the standard real business cycle model by assuming that the consumption of market goods and the services of home durables are good substitutes. An example of two goods that substitute is a movie watched in a theater (a market good) and a movie watched on a home television set (a home durable good).²⁰

Under Baxter's substitutability assumption, the appropriate measure of household consumption is not just market consumption, but consumption of market goods *plus* the service flow on the stock of home durables. If home durables consumption is sufficiently large, then a given jump in the consumption of market goods leads to a smaller percent drop in the marginal utility of consumption. In the extreme case where the stock of home durables is extremely large and accounts for essentially all of consumption, then a rise in market consumption would produce essentially *no* drop in the marginal utility of consumption.²¹ Although

Baxter shows that this mechanism does indeed produce comovement in employment across consumption and investment sectors in her model, there is a sense in which the comovement is not strong enough. That is because investment in the capital used in the two sectors is essentially uncorrelated. As noted above, the data suggest that investment across sectors comoves as well, in addition to output and employment.

One can also view the home production approach of Benhabib, Rogerson, and Wright (1991) as a strategy to generate comovement by reducing the decline in the value of output in the consumption sector during booms. In a boom, as labor is allocated away from home-produced goods toward the production of market goods, the marginal utility of the market good does not fall much because the market and home goods are assumed to be highly substitutable.²² This allows the value of output in the consumption sector to rise sufficiently so that employment in that sector is procyclical. A shortcoming of the analysis, emphasized by Benhabib, Rogerson, and Wright (1991), is that the high substitutability between home and market goods needed for comovement of labor inputs hurts on another dimension. It has the effect that purchases of durables are countercyclical over the cycle.23

Christiano and Fisher (1998) take another approach. Following Boldrin, Christiano, and Fisher (1995), they modify a standard real business cycle model in two ways. First, they specify that it takes time before labor can shift between economic sectors in response to a shock. The reasons for this are not modeled explicitly, but the assumption is motivated with an informal reference to such factors as the search and training costs which inhibit real world labor mobility between industry sectors. This assumption alone is not sufficient to guarantee comovement, however. Without further changes, their model predicts that resources would be reallocated out of the consumption sector and into the investment sector as soon as labor becomes fully mobile, which they specify to occur in three months' time. As a result, this version of the model is still inconsistent with the evidence on business cycle comovement. Christiano and Fisher therefore introduce a second modification, by changing the specification of household preferences over consumption. They assume that households have a tendency to become accustomed to the level of consumption they have enjoyed in the recent past. This property of preferences is known as habit persistence. A household with habit persistence preferences whose consumption has recently increased

is particularly unhappy if later it must return to its previous level of consumption.²⁴ Habit persistence preferences have the implication that when consumption rises in a boom, the marginal value of continuing to maintain consumption at a high level is increased. Christiano and Fisher show that habit persistence and limitations on the intersectoral mobility of labor are sufficient to produce comovement in hours worked and investment. To our knowledge, this is the only quantitative model in the comovement literature with this property.

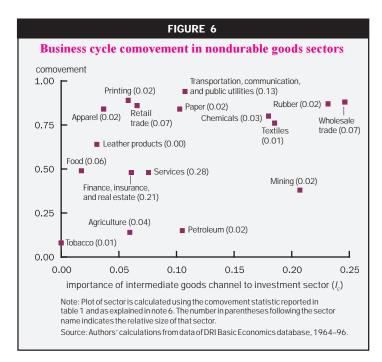
The credibility of this result depends on the credibility of the underlying assumptions. The assumption that there are limitations on the speed with which productive resources can be transferred across sectors seems uncontroversial, though the model certainly takes an extreme stance. What does call for a defense is the assumption of habit persistence preferences. One defense is that these preferences help to account for observations that otherwise seem puzzling. For example, Boldrin, Christiano, and Fisher (1995) show that, consistent with results in Constantinides (1990), habit persistence and limited intersectoral mobility can account for the magnitude of the observed average premium in the return on equity over risk-free securities. The solution to this premium has eluded many researchers.²⁵ In addition, Christiano and Fisher (1998) show that habit persistence can help account for the so-called inverted leading indicator property of interest rates, that high interest rates tend to forecast bad economic times. King and Watson (1996) document that standard models have difficulty accounting for this observation.²⁶

A third approach toward understanding comovement was recently pursued by Hornstein and Praschnik (1997). They observe that some of the output of the sector that produces consumption goods (the nondurable goods sector) is also used as intermediate goods in the production of investment goods. For example, both households and investment-good producing firms make use of the services of the transportation sector. Hornstein and Praschnik (1997) modify a real business cycle model to accommodate this feature of the economy. The modification has the effect of increasing the value of output in the consumption sector in a boom. This increased value reflects the increased need for the output of the consumption good sector during a boom for use in the investment good sector.²⁷ We refer to this demand channel going from investment sector to the nondurable goods sector as the intermediate goods channel.

There are two shortcomings of the Hornstein and Praschnik (1997) analysis. First, the model is not

consistent with the observed comovement in investment across sectors. That is, the intersectoral linkages in the model are not strong enough to produce full comovement. Second, data on subsectors of the nondurable goods sector cast doubt on the notion that the intermediate good channel is the only reason there is comovement. We studied the subsectors of the nondurable goods sector and found that there is considerable variation in the fraction of total output sent to the investment goods industry. But, as documented in the previous section, most of these sectors nevertheless display strong business cycle comovement. Figure 6 is a scatter plot of the subsectors' degree of comovement, drawn (with one exception) from the fifth column of numbers in table 1, against the strength of each sector's intersectoral linkage with the investment sector, I_{a} . The variable I_{a} is the fraction of the gross output of a sector which is allocated to intermediate goods destined directly or indirectly for the production of final investment goods.²⁸ (Technical appendix 3 has details of how we computed this.) The numbers in parentheses in figure 6 indicate the relative magnitude of the gross output of the sector (gross output of the sector in 1987, divided by the sum of the gross outputs across all 17 sectors). Hornstein and Praschnik's concept of the nondurable goods sector is broader than the one in table 1. They also include agriculture; retail trade; wholesale trade; transportation, communication, and utilities; services; FIRE; and mining.²⁹

Figure 6 shows that employment in most (13 of



17) nondurable good sectors is substantially procyclical (that is, the comovement statistic is 0.45 or higher), even though the strength of the intermediate good channel (the magnitude of I_{i}) varies from almost zero in the case of food to nearly 0.25 in the case of wholesale trade. Interestingly, although the comovement in mining is moderately weak in our data set, it is one of the sectors in which the intermediate goods channel is the strongest. Conversely, the comovement in apparel is strong, although this sector's intermediate goods channel is almost nonexistent. Based on these results, we suspect that the intermediate goods channel to the investment sector plays at best only a small role in accounting for comovement of employment in nondurable goods.³⁰ To further explore the Hornstein and Praschnik hypothesis, one would have to construct a version of their model with a disaggregated nondurable goods sector and see if it is consistent with comovement, in the sense of being able to reproduce patterns like those in figure 6^{31}

Alternative approaches

Here, we summarize three other approaches that may ultimately lead to a satisfactory explanation of business cycle comovement—*strategic complementarities, information externalities,* and *efficiency wage theory.* The first two approaches emphasize the importance in business decisions of expectations about the future. They draw attention to the possibility that general shifts in expectations may trigger business cycle fluctuations. If so, these shifts in ex-

> pectations may well constitute the aggregate shock to which Lucas (1981) refers. The third approach looks at efficiency wage theory. Although promising, the ability of these three theories to quantitatively account for the comovement aspect of business cycles is yet to be fully explored.

Strategic complementarities

Suppose there are two people, A and B. Each has to decide on a level of work effort: high or low. Suppose that the net gain to A of exerting a high level of effort is greater if B exerts a high level of effort and that B is in a similar position. The situation is depicted in table 2.

Table 2 has four entries, one for each possible combination of work effort. In each entry, the first number indicates the net gain to A, and the second number indicates the gain to B. Suppose A exerts high effort. Then, if B is putting

	TABLE 2				
Example of strategic complementarity					
		Person B			
		High effort	Low effort		
Person A	High effort	(5, 5)	(–5,0)		
	Low effort	(0, -5)	(2, 2)		

out high effort too, A receives 5. But, if B exerts low effort, then A receives -5. The situation is the same for B. Table 2 captures the idea that the gain to either person from exerting high effort is high only when the other person exerts high effort. A situation like this is said to be characterized by strategic complementarity. What do we expect to happen? If the two people could sit down and reach an agreement, they would clearly both choose to exert high effort. But what if they have difficulty coordinating in this way? There are now two possibilities. One is that each expects the other to exert low effort, in which case each finds it optimal to exert low effort. This would put the two people in the bottom right box, with a low payoff going to each. They would stay there until they found a way to communicate and reach an agreement or until something happened to alter their expectation about the other's plans. Another possibility is that each expects the other to exert high effort, in which case it is in the private interest of each person to exert high effort. This situation could persist for a while, again, unless something happens to shift one person's expectations about what the other one will do.

What does this have to do with business cycles and comovement? Possibly a lot. There are aspects of business decisions that exhibit strategic complementarity. For example, suppose a firm is considering reopening a plant or starting a large capital investment project. Suppose the project involves a substantial outlay of funds, not just to hire more workers but also to purchase materials and supplies from other firms. The higher the sales the firm expects in the future, the more inclined it will be to shift to a high level of activity in this way. However, much of a firm's sales come from other firms. And those sales are greater if other firms are themselves operating at a high level of activity, for example, reopening plants or undertaking new capital investment projects. So, firm A has a greater incentive to shift to a high level of activity if it believes firm B plans to operate at a high level of activity.

What do we expect in this situation? Coordination in this setup is much more difficult than in the two person example. There are millions of firms in the economy and, even if it were technically feasible for some firms to coordinate, the antitrust laws represent another barrier. In light of these considerations, we might well expect to find results similar to those in the two person example. Thus, if firms were pessimistic about prospects for future sales, they would choose to be inactive and their pessimistic expectations would be fulfilled.³² Optimistic expectations would be self-fulfilling in the same way. It is clear that in this setting, expectations have the potential to act as an aggregate shock driving the business cycle. Of course, that does not guarantee that they can necessarily account for comovement.³³ This is an important topic of research.³⁴

Information externalities

Another potential source of comovement is the way information about the state of the economy is transmitted to individual firms. Forecasts of the future strength of the economy are a factor in individual firms' current investment decisions. If a firm observes a series of construction projects being initiated by other firms, it may infer that those other firms have information that bodes well for the general economic outlook. When the firm combines this inference with its own information about the economic outlook, it may decide to invest too. Other firms may follow for similar reasons.

These considerations are logically distinct from the strategic complementarities discussed above. There, a firm is interested in the actions of other firms because these actions have a direct impact on the firm's profitability. Here, a firm is interested in the actions of other firms because of the associated information externality. The externality refers to the fact that a firm's action may reveal information it has on something of interest to other firms, such as the state of the economy. It is a *positive* externality, unlike the more familiar examples of externalities which tend to be negative.³⁵

We present an example, taken from Banerjee (1992), to illustrate the sort of things that can happen when there are information externalities. When firms look to what other firms are doing for guidance in deciding what they should do, this can lead to what Banerjee (1992, p. 798) calls *herd behavior*, a situation with "everyone doing what everyone else is doing, even when their private information suggests doing something quite different." It hardly needs to be stated that herd behavior sounds like comovement.

Here is the example. Suppose there are 100 people trying to decide between two restaurants, A and B. Each person knows very little about the two

restaurants, but thinks the odds favor A slightly. In addition, each person receives a signal about the relative quality of the two restaurants. For example, one person may read a review of the two restaurants in a travel guide. The review is several years old, however, and the signal may not be accurate. The signals received by each of the 100 persons are equally reliable. Everyone knows this, but they do not know what signal the others received. Now, suppose that 99 people get a signal that suggests B is better than A, while one person gets a signal that A is better than B. If all information were known to everyone, they would recognize that the preponderance of the evidence favors restaurant B and all 100 people would go to B. However, what actually happens is that the 99 people whose signal indicates B is better ignore their signal and flock to restaurant A, following the one person who received the signal that A is better.

This result is not due, as one might suppose, to an assumption that agents are irrational. On the contrary, the example assumes agents are completely rational. The result reflects that not all people make their decisions at the same time. Some have to be first, and as a result, the information they have has disproportionate impact, since almost everyone else is watching them. This timing assumption does not seem unrealistic. In practice, the exact timing of firms' decisions is not completely under their control.³⁶

The example adopts an extreme version of the assumption that the timing of a decision is out of the agents' control, specifying that someone must choose first, then someone else must choose second after observing the choice of the first, and so on. The person choosing first happens to be the one who receives the signal that A is better than B. Since person 1's suspicion that A is better is apparently confirmed by the signal, this person rationally chooses A. The second person's signal suggests that B is better. However, person 2 knows that person 1's signal must have favored A. Since the two signals are equally reliable, they cancel in the mind of person 2. Since person 2 originally thought restaurant A was better, the rational thing for person 2 to do is to go to restaurant A. Person 3 is in precisely the same position as 2, because person 3 knows that, given person 1 went to A, person 2 would have gone to A no matter what signal she received. That is, person 3's observation that person 2 went to A provides no information at all about the relative quality of the two restaurants. Being in the same position as 2, person 3 also chooses A regardless of the signal received. In this way, all 99 people after the first ignore their own signal and go

to restaurant A. Although there is a lot of information in the economy about the relative quality of the restaurants, one person acts on a small piece of it, and everyone else follows.

This example and others like it hold out some hope that a fully developed business cycle model incorporating information externalities might exhibit the synchronization of behavior across economic sectors that we observe over the business cycle. However, the above example only illustrates how information externalities can lead rational people to ignore information and synchronize on bad decisions. Synchronization of actions would have occurred anyway, even if there had been no information problem and all signals had been known to everyone. Another concern with this example is how heavily dependent it is upon details of the environment. For example, the outcome is very different if two people are required to choose a restaurant first. In this case, the 99 people who received the signal that B is better than A go to B.^{37,38} Despite these considerations, we believe the growing literature on information externalities may eventually provide at least part of the explanation for business cycle comovement.39

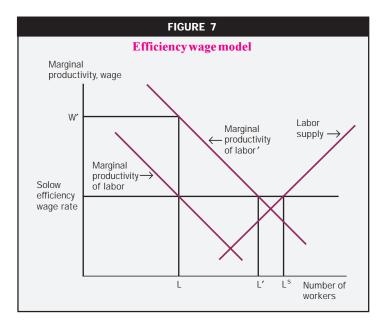
Efficiency wage theory

A third strategy for understanding comovement is to make use of efficiency wage theory.

Efficiency wage theory: A sketch

Under this view of labor markets, the amount of effort a worker makes (the worker's efficiency) depends on the wage that the worker is paid. Development economists hypothesized that in economies at a very early stage of development, a higher wage leads to greater worker efficiency because it facilitates improvements in diet and health. Efficiency wage theory holds that a higher wage also results in greater worker efficiency in a modern, developed economy, but for different reasons. Because employers cannot perfectly monitor the amount and quality of work effort expended by their employees, there is a temptation for workers to shirk. Efficiency wage theory says that a high wage rate is an effective way to combat this temptation. The higher the wage, the more a worker has to lose if caught and fired for poor job performance.

The simplest version of this idea was articulated by Robert Solow,⁴⁰ who theorized that the firm selects a wage rate, the *efficiency wage*, which maximizes worker effort per dollar paid. The firm is not willing to pay more because the resulting increase in worker effort would not be enough to warrant the extra cost. The firm is also not willing to pay less, because the



resulting fall in effort would exceed the fall in cost.⁴¹ In the Solow model, the amount of effort expended per hour by a worker is a function only of the current wage and, for example, does not depend on the general state of business conditions. As a result, the efficiency wage rate does not vary over the business cycle under Solow's efficiency wage theory.

The firm also has to decide how many workers to hire. It hires workers up to the point at which the marginal productivity of the last worker is just equal to the efficiency wage.42 The downward sloping marginal productivity of labor curve in figure 7 shows how the marginal productivity of labor is lower at higher levels of employment. At the level of employment, L, the marginal productivity of the last worker hired is equal to the efficiency wage. Since the efficiency wage in the Solow model is a constant, it follows that employment over the business cycle is determined by the requirement that the marginal product of labor does not change. The downward sloping curve in figure 7. marginal productivity of labor', shows the marginal productivity curve after it has been shifted up by a positive technology shock.43 If the firm kept employment fixed at L when technology shifted up, marginal productivity would rise to W', a point far above the efficiency wage. By increasing L to L', the firm keeps marginal productivity unchanged despite the shift up in technology.44

A notable feature of efficiency wage theory is that labor supply plays no role in the determination of the wage rate. The theory assumes that there are more workers willing to work than the firm is willing to hire at the efficiency wage. Still, unemployed workers cannot bid the wage down below the level of the efficiency wage. Firms are not interested in hiring workers at such a low wage because they fear it would not provide workers with enough incentive to work hard. The quantity of unemployed people is the number who are willing to work at the efficiency wage, minus the number that firms want to hire. Note how the upward sloping labor supply curve in figure 7 is shifted to the right. At the efficiency wage, L^s, workers would like to work, but only L are hired, so unemployment is L^s -L. At the higher level of technology, unemployment falls to $L^s - L'$.

Efficiency wage theory and business cycle comovement

How might efficiency wage theory help account for business cycle comovement? Suppose the business cycle is driven by an aggregate, real-businesscycle-type technology shock. As we explained earlier, in the standard real business cycle model such a shock does not lead to comovement in employment. In that model, a positive shock leads to a transfer of resources—labor and capital—away from the firms producing consumption goods and toward the firms producing investment goods. Now suppose the labor market part of the real business cycle model is replaced by efficiency wage theory, which implies that firms vary the number of workers they employ to ensure that the marginal product of labor remains constant and equal to the efficiency wage rate. So, when a positive technology shock shifts up the marginal productivity of labor, employment must increase to maintain equality between the marginal product of labor and the efficiency wage.45

We indicated earlier that a positive real-businesscycle-type shock pushes up the production functions and the marginal labor productivity curve of all firms. According to efficiency wage theory, this results in an increase in employment by all firms, as they seek to maintain equality between marginal labor productivity and the unchanging efficiency wage rate. This is the intuition underlying the idea that efficiency wage theory may help explain business cycle comovement.^{46,47}

Have we now established that efficiency wages are sufficient to account for comovement? Absolutely not. When we examine efficiency wage theory more closely, we discover that it need not necessarily work

as just outlined. The relationship between how hard a worker works and the wage rate (the worker's effort function) is a function of the household's attitude toward risk, the resources it has available if the worker is caught shirking and fired, the probability of being caught conditional on shirking, and the precise consequences of being fired for shirking. The household effort function used in the analysis must integrate all these factors in a logically coherent way. In addition, it must be consistent with other household decisions, such as how to split income between consumption and saving. To build confidence in the idea that efficiency wage theory helps account for comovement, we must integrate all these aspects of the household into a coherent framework which also includes firms and their decisions to see if it works.

To understand why it might not work, recall the Solow model's assumption that worker effort is a function only of the wage rate. That is what led to our conclusion that the efficiency wage is a fixed number, unrelated to the state of the business cycle. But the logic of the efficiency wage argument suggests that the Solow assumption may not be consistent with rational behavior by households. According to efficiency wage theory, what motivates hard work is the fear of losing a high-wage job. Of course, the cost of that loss is not a function of the wage rate alone. It is also a function of the amount of time the worker can expect to be out of a job after being fired. This suggests that the horizontal line in figure 7 shifts up in a boom, when the duration of unemployment is low.48 However, if it shifts up high enough, the comovement result could disappear. This highlights the importance of integrating efficiency wage theory into a logically coherent model, before we conclude that it provides a solid foundation for understanding business cycle comovement.

Important steps have been taken in this direction, for example, Shapiro and Stiglitz (1984) and Danthine and Donaldson (1995).⁴⁹ Recent work by Gomme (1998) and Alexopoulos (1998) makes a significant further contribution toward understanding the implications of efficiency wage theory for business cycles. However, this work does not focus on the implications for business cycle comovement. We argue that doing so is a good idea.⁵⁰

Conclusion

A key feature of the data is that, in a frequency range of two to eight years, output, employment and investment across a broad range of sectors move up and down together. We have documented this phenomenon—business cycle comovement—as it pertains to employment. Our survey of possible explanations for it is by no means exhaustive. Many other approaches—those based on sticky prices and wages, countercylical markups, and credit market frictions—also deserve consideration.⁵¹ Still, we have covered a wide range of models, from straightforward modifications to standard business cycle theory to theories that suggest analogies between businesspeople and herds of animals.

Many of the approaches we have surveyed are in early stages of development, while some have been developed to the point where their implications have been quantified and compared with the data. Among these, only one has been shown to be consistent with the observed strong comovement in output, employment, and investment across sectors of the economy the model presented in Christiano and Fisher (1998). This model incorporates a specification of household preferences, habit persistence, that is not currently standard in the macroeconomics literature. We believe that the success this model has in generating comovement warrants giving this specification of preferences further consideration.

Because comovement is such a central feature of business cycles and because we do not have a generally agreed upon theory of comovement, we conclude that the business cycle is still a puzzle.

TECHNICAL APPENDIX 1 Extracting the business cycle component of a time series

In casual discussions of economic time series, we often think of the data as being the sum of components that have different frequencies of oscillation: the business cycle component lasting two to eight years, components lasting shorter periods, etc. The theory of the spectral analysis of time series makes this intuition rigorous. It clarifies how one can think of data as being composed of components that fluctuate at different frequencies. The method we use to extract the business cycle component of economic time series builds on this theory. For this reason, we begin with a brief section which attempts to convey the basic intuition of spectral analysis. The second section uses this intuition to describe and motivate our method for extracting the business cycle component of a time series.

Decomposing a time series into frequency components

At the core of the spectral analysis of time series is the view that the data can be thought of as the sum of periodic functions. The purpose of this section is to explain this. We begin by reviewing the basic periodic function used in spectral analysis, which is composed of a sine and a cosine function.

Consider the following cosine function of time, t:

$$\cos(t\omega), t=0, 1, 2, ...$$

A graph of this, with $\cos(t\omega)$ on the vertical axis and *t* on the horizontal axis, exhibits the oscillations between 1 and -1 familiar from high school trigonometry. Recall too, that the period of the cosine function is 2π . That is, $\cos(y) = \cos(y + 2\pi h)$, for h = 1, 2, Thus, after the argument of the cosine function increases by 2π , the function repeats itself in a periodic fashion.

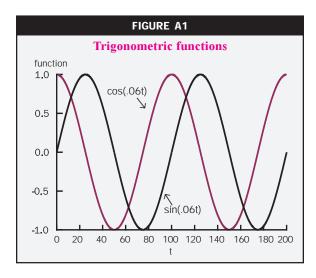
What is of interest here is the period of oscillation of $\cos(t\omega)$, expressed in units of time. This is the amount by which *t* must increase so that $t\omega$ increases by 2π . Thus, suppose t_1 is a given point in time. We want to know what is the later point in time, $t_2 > t_1$, when the cosine function begins to repeat itself. This is just t_2 such that $t_2\omega - t_1\omega = 2\pi$, or, $t_2 - t_1 = 2\pi/\omega$. Thus, the period of oscillation of $\cos(t\omega)$, in units of time, is $2\pi/\omega$. The parameter ω is referred to as the frequency of oscillation.

The function, $\sin(t\omega)$, behaves similarly. It fluctuates between 1 and -1, and has a period of oscillation of $2\pi/\omega$. Thus, the two functions have the same *amplitude* (magnitude of vertical variation) and period. However, the sine function has a different *phase* than $\cos(t\omega)$. For example, a graph of the two functions together shows that one looks like the other, apart from a horizontal shift. The phase difference between the two functions is a measure of the magnitude of this horizontal shift. Figure A1 displays sine and cosine functions for t = 0, 1, ..., 200. The period of oscillation is 100, so that the frequency is $\omega = 2\pi/100$. Thus, the figure displays the graphs of $\cos(t2\pi/100)$ and $\sin(t2\pi/100)$.

We can now describe the central periodic function in spectral analysis, namely the linear combination of a sine and a cosine function

1) $a\cos(t\omega) + b\sin(t\omega)$,

where *a* and *b* are parameters. This function obviously has a period, in units of time, equal to $2\pi/\omega$. But, its



amplitude and phase depend on the values of a and b. If a and b are both very small, the resulting function has very small amplitude and if a and b are both very large it has a large amplitude. Also, as the size of a is increased and the size of b is decreased, the phase of the function shifts, as more weight is allocated to the cosine and less to the sine.

It turns out that sums of periodic functions like equation 1 look very much like actual data. Thus, suppose we have a time series of data, x_i , t = 1, ..., T. To see that x_i can be expressed as a sum of periodic functions, suppose we specify T/2 (suppose *T* is even) such functions, each with a different frequency of oscillation ω . For concreteness, let $\omega_j = 2\pi j/T$, for j = 1, ..., T/2. To distinguish the parameters associated with each of these functions, we denote them by a_j and b_j for j = 1, ..., T/2. It should not be surprising that, in general, a time series, $x_1, ..., x_T$ can be written as the sum of these T/2 functions

2)
$$x_t = a_1 \cos(t\omega_1) + b_1 \sin(t\omega_1) + ...$$

 $+ a_{T/2} \cos(t\omega_{T/2}) + b_{T/2} \sin(t\omega_{T/2})$

for t = 1, ..., T. That is, we can always find values for the *T* parameters, $(a_j, b_j; j = 1, ..., T/2)$, so that the *T* equations, equation 2 for t = 1, ..., T, are satisfied. To see this, consider the following regression. Let the explanatory variables be:

$$X = \begin{bmatrix} \cos(\omega_1) & \sin(\omega_1) & \cdots & \cos(\omega_{T/2}) & \sin(\omega_{T/2}) \\ \cos(2\omega_1) & \sin(2\omega_1) & \cdots & \cos(2\omega_{T/2}) & \sin(2\omega_{T/2}) \\ \vdots & \vdots & \vdots & \vdots \\ \cos(T\omega_1) & \sin(T\omega_1) & \cdots & \cos(T\omega_{T/2}) & \sin(T\omega_{T/2}) \end{bmatrix}.$$

Let the $T \times 1$ vector of 'independent variables,' *Y*, and the $T \times 1$ vector of regression coefficients, β , be

$$Y = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_T \end{bmatrix}, \quad \beta = \begin{bmatrix} a_1 \\ b_1 \\ \vdots \\ a_{T/2} \\ b_{T/2} \end{bmatrix}$$

Then, the regression is

$$Y = X\beta + u$$

Note, however, that since the number of explanatory variables is *T*, the error term is exactly zero, and β is computed from $(X' X)^{-1} X' Y = X^{-1}Y^{-1}$ Thus a time series of length *T* can be expressed exactly as the sum of *T*/2 simple processes like equation 1, each having a different frequency of oscillation.

Unfortunately, taken literally, equation 2 is *not* a very sensible way to think of the data. With *T* observations, one has only to compute the a_i s and the b_i s and then the T + 1st observation can be predicted *exactly*. No one believes that there is *any* way to use *T* observations on any economic data series and predict the next observation exactly. Imagine, for example, that you could do this with the Dow Jones Industrial Average. If you could, then after one minute of reading this appendix, you would have the information needed to go out and become fabulously wealthy.

Of course, one could instead suppose that the data are a realization from an expression like equation 2, in which the number of periodic functions exceeds the number of observations by, say, 10. In this case, there is no longer the implication that one can perfectly predict next period's value of x_i . However, there is the implication that after 20 more periods, the data series will then become perfectly predictable. No one would think this is a sensible way to view economic data either. The theory of spectral analysis assumes, sensibly, that no matter how many observations on x_{i} one accumulates, the data never become perfectly predictable. That is, it in effect assumes that the number of periodic functions in equation 2 is infinitely large by comparison with the size of the available data set. When this is so, equation 2 is written in the form of an integral, as follows:2

3) $x_t = \int_0^{\pi} [a(\omega)\cos(\omega t) + b(\omega)\sin(\omega t)]d\omega$,

where $a(\omega)$ and $b(\omega)$ are functions of ω . In view of these observations, it is perhaps not surprising that

any covariance stationary time series process, x_p , can be expressed in the form of equation 3 (Koopmans, 1974).

Extracting the business cycle component

Equation 3 allows us to make precise the notion of extracting the business cycle component of x_i . That representation views the time series process, x_i , as the sum of components with periods of oscillation $2\pi/\omega$ for ω lying in the interval 0 to π . In monthly data, the business cycle corresponds to components with period greater than 24 months and less than 96 months. In terms of frequencies of oscillation, this corresponds to ω belonging in the interval $\underline{\omega} = 2\pi/96$ to $\overline{\omega} = 2\pi/24$. Thus, we seek the business cycle component of x_i , y_i , such that

4)
$$y_t = \int_{\omega}^{\omega} [a(\omega)\cos(\omega t) + b(\omega)\sin(\omega t)]d\omega$$

It is well known that y_t can be computed as a particular centered moving average of observations on the observed data, x_t

5)
$$y_t = B_0 x_t + B_1 (x_{t-1} + x_{t+1}) + B_2 (x_{t-2} + x_{t+2}) + \dots,$$

where

$$B_{j} = B_{-j} = \frac{\sin(j\omega) - \sin(j\omega)}{\pi j}, j = 1, 2, ...$$

6)
$$B_{0} = \frac{\overline{\omega} - \omega}{\pi}.$$

There is a major practical stumbling block to using equation 5 for extracting the business cycle component of x_t . It requires an infinite amount of data! Some sort of approximation is needed, if one is to estimate y_t given only the available data, $x_1, ..., x_t$.

An extensive analysis of this problem appears in Christiano and Fitzgerald (1998), which also provides a review of the related literature. We provide only the briefest review of that discussion here, just enough to enable us to describe exactly how we isolated the business cycle component of the data.

We denote our approximation of y_t by \hat{y}_t . Here, we focus on the approximations of the following form:

7)
$$\hat{y}_t = \hat{B}_0 x_t + \hat{B}_1 (x_{t-1} + x_{t+1}) + ... + \hat{B}_K (x_{t-K} + x_{t+K}).$$

That is, we approximate y_t by a finite ordered, centered, symmetric moving average. But, how should we choose the weights? The natural way is to choose

them so that \hat{y}_t is as close to y_t as possible, that is, so that they solve

8)
$$\min_{\hat{B}_{i}, i=0,1,...,K} E(y_{t} - \hat{y}_{t})^{2}.$$

The solution to this problem is a function of the details of the time series representation of x_t . For example, if we suppose that x_t is a random walk, that is, $x_t - x_{t-1}$ is a process that is uncorrelated over time, then the solution is:³

9)
$$\hat{B}_j = B_j, j = 0, ..., K-1$$

 $\hat{B}_K = -\left[\frac{1}{2}B_0 + B_1 + B_2 + ... + B_{K-1}\right]$

Suppose the data at hand are $x_1, ..., x_T$, so that the objects of interest are, $y_1, ..., y_T$. We computed \hat{y}_{36} , ..., \hat{y}_{T-36} as follows. For \hat{y}_{36} we applied equation 7 with K=35, for \hat{y}_{37} we applied equation 7 with K=36, and so on. For each \hat{y}_t that we computed, we used the largest value of K possible. Christiano and Fitzgerald (1998) argue that this procedure for estimating y_t works well in terms of optimizing equation 8, even if the truetime series representation of x_t is not a random walk. They show that an even better approach uses an asymmetric set of weights, so that the estimate of \hat{y}_t for each t uses all the available data on x_t .

$$a_j = \frac{2}{T} \sum_{t=1}^{T} \cos(\omega_j t) x_t,$$

$$b_j = \frac{2}{T} \sum_{t=1}^{T} \sin(\omega_j t) x_t.$$

TECHNICAL APPENDIX 2

Comovement and the elasticity of substitution

The standard real business cycle model assumes that the elasticity of substitution between capital and labor in production is unity. In the text, we discussed a result due to Benhabib, Rogerson, and Wright (1991): With this kind of production function and with utility functions consistent with balanced growth, comovement in employment is impossible (see note 16). Here, we show that comovement is a technical possibility when the elasticity of substitution is different from unity. However, we find that comovement does not Also,

$$a_{T/2} = \left[\sum_{t=1}^{T} \cos(\omega_{T/2}t) x_t\right] / T$$
$$b_{T/2} = \left[\sum_{t=1}^{T} x_t\right] / T.$$

²To gain further intuition into the relationship between equations 2 and 3, it is useful to recall the simplest definition of an integral, the Riemann integral. Thus, let f(y) be a function, with domain $y \le y \le \overline{y}$. Let $y_{e^j} j = 1, ..., M$ be a set of numbers that divide the domain into M equally spaced parts. That is, $y_1 = y + \Delta_M, y_2 = y_1 + \Delta_M, ..., y_M = y_{M-1} + \Delta_M$,

where $\Delta_M = (\overline{y} - \underline{y}) / M$. Note that $y_M = \overline{y}$. The integral of f over its domain is written,

$$\int_{y}^{\overline{y}} f(y) dy.$$

This is approximated by the sum of the areas of the $Mf(y_i)$ by Δ_M rectangles:

$$\sum_{j=1}^{M} f(y_j) \Delta_M.$$

The Riemann interpretation of the integral is that it is the limit of the above sums, as $M \to \infty$. The relationship between the above finite sum and the integral resembles that between equations 2 and 3 if we adopt $y_j = \omega_j = 2\pi j/T$, $\Delta_M = 2\pi/T$, M = T/2, $f(y_j) = a(\omega_j) \cos(\omega_j t) + b(\omega_j) \sin(\omega_j t)$, $a(\omega_j) = a_j T/2\pi$, $b(\omega_j) = b_j T/2\pi$.

³Actually, the theory as we summarized it here technically does not accommodate nonstationary processes like random walks. Christiano and Fitzgerald (1998) discuss standard ways of extending the theory to this case. Also, optimizing the mean square error criterion, equation 7, requires a constant term in equation 8. See Christiano and Fitzgerald (1998) for more details.

occur for plausible parameter values. These results suggest that attempts to account for comovement by adjusting the elasticity of substitution in production in a standard real business cycle model are unlikely to be successful.

We begin by describing a version of the standard real business cycle model. We assume that households have identical preferences of the following form:

$$E_0\sum_{t=0}^{\infty}\beta^t \frac{\left[C_t(1-L_t)^{\Psi}\right]^{1-\sigma}}{1-\sigma},$$

¹Note that $\sin(t\omega_{T^2}) = 0$ for all integers *t*. Since the right column of *X* is zero in this case, *X* is singular and so cannot be inverted. In practice, the last column of *X* is replaced by a column of ones, to accommodate a non-zero sample mean in x_i . Under these conditions, the columns of *X* are orthogonal, so that $X^{-1}Y$ is trivial to compute. In particular, for j = 1, ..., T/2 - 1:

where σ , $\psi > 0$ satisfy the various conditions required for utility to be strictly concave. Also, $C_t > 0$ denotes per capita consumption, and L_t denotes per capita hours worked. We require $0 \le L_t \le 1$. The resource constraint is

$$c_t + k_{t+1} - (1 - \delta)k_t \leq \left[(1 - \alpha)k_t^{\frac{\nu - 1}{\nu}} + \alpha L_t^{\frac{\nu - 1}{\nu}} \right]^{\frac{\nu}{\nu - 1}} z_t.$$

Here, $0 \le \delta \le 1$ is the rate of depreciation on capital and $0 \le \alpha \le 1$ is a parameter. The elasticity of substitution between capital and labor is $v \ge 0$. Also,

$$\log(z_t) = \rho \log(z_{t-1}) + \varepsilon_t, 0 < \rho < 1,$$

where ε_t is a zero mean random variable, that is independently distributed over time. Finally, $k_t > 0$ denotes the beginning of period *t* stock of capital, which is a given quantity at time *t*.

As noted in the body of the article, it is possible to interpret this as a two sector model: one in which consumption goods, c_p and investment goods, $k_{t+1} - (1-\delta)k_p$, are produced in different sectors by different firms. It is assumed that both sectors use the same production function, the one stated above, and that capital and labor can move freely between the two sectors, subject only to the obvious constraint that the sum of capital and labor in the two sectors equals k_t and L_p respectively. Thus, letting L_{cl} , k_{cl} and L_{ll} , k_{ll} denote the amount of labor and capital, respectively, used in the consumption and investment sectors, we require

$$L_{ct} + L_{it} = L_{t}, k_{ct} + k_{it} = k_{t}$$

As mentioned in note 16, the marginal product of labor in the sector producing the consumption good equals households' marginal rate of substitution between consumption and leisure.

$$\alpha \left(\frac{C}{L_c}\right)^{\frac{1}{\nu}} z^{\frac{\nu-1}{\nu}} = \frac{\psi C}{(1-L_c-L_i)}.$$

We drop the time subscripts to simplify the notation. Rearranging this equation, we obtain

$$\left(\frac{\alpha}{\psi}\right)^{\nu} \left(\frac{C}{z}\right)^{1-\nu} (1-L_c-L_i)^{\nu} = L_c.$$

Note first that when v = 1, we reproduce the result in note 16, which indicates that L_c and L_i cannot move in the same direction. When $v \neq 1$, this reasoning no longer holds. We can see intuitively that employment in the two sectors might move together with v > 1. In particular, consider the case v = 1. In this case, we have found in many numerical examples that C/z falls with a rise in z due to a positive shock in ε . Continuity suggests that this also happens when v is a little above unity. We conclude that if the resulting rise in $(C/z)^{1-v}$ is sufficiently large, then it is possible for both L_c and L_i to increase in response to a positive shock in ε (see note 16 for the sort of reasoning used here).

We approximated the solution to this model using the undetermined coefficient method in Christiano (1991). We assigned parameter values in the following way. For our baseline parameterization, we set $\beta = 0.99$, $\delta = 0.025$, $\rho = 0.95$, and $\sigma = 1$. We chose ψ and α to guarantee that, in the model's steady state, labor's share of income is 0.64 and steady state hours worked is 0.30. An empirical defense for the choice of values for β , δ , ρ , labor's share, and steady state hours worked may be found in the real business cycle literature. For the calculations reported below, we set $\sigma =$ 0.01, and drew 1,000 observations on ε_t from a normal distribution with mean zero and standard deviation, σ_c .

In our first experiment, we considered values of v on a grid between 0.7 and 20. For each value of v, 1,000 observations on L_{ct} and L_{μ} and

$$I_{c,t} = k_{c,t+1} - (1 - \delta)k_{c,t}, I_{i,t} = k_{i,t+1} - (1 - \delta)k_{i,t}$$

were generated using our approximation to the model's solution. The 1,000 observations were then used to compute the correlations, \mathbf{r}_{ic}^{l} between L_{ct} and L_{it} and the correlations, $\boldsymbol{\rho}_{ic}^{i}$, between $I_{c,t}$ and $I_{i,t}$. A model exhibits comovement in employment and investment if both $\boldsymbol{\rho}_{ic}^{i}$, $\boldsymbol{\rho}_{ic}^{l} > 0$. We found $\boldsymbol{\rho}_{ic}^{i}$, $\boldsymbol{\rho}_{ic}^{l} < 0$ for each value of n using the benchmark parameterization.

We repeated these calculations several times, each time perturbing one, and only one, of the parameters in the benchmark parameterization. We considered the following alternatives: $\sigma = 5$; $\rho = 0.99$; $\rho = 0.0$; steady state hours equal to 0.1; steady state labor's share equal to 0.3; $\delta = 0.05$, $\delta = 0.01$; and $\beta = 0.97$, $\beta = 0.995$. The perturbations in σ , ρ , β , and δ did not produce a parameterization exhibiting comovement. The reduction in labor's share resulted in comovement in employment, but not investment, for values of v between about 3 and 13. Lowering steady state hours to 0.10 also resulted in comovement in employment but not investment. Hence, we conclude that altering the elasticity of factor substitution in production does not improve the standard real business cycle model's ability to reproduce full comovement for reasonable parameter values.

TECHNICAL APPENDIX 3

Analysis of the input-output tables

Our analysis of the input–output tables is based on the 1987 benchmark, 95 variable input–output table for the U.S. economy. Our main objective here is to define the fraction of a sector's final output which is used directly or indirectly in the production of final investment goods. Let Y denote the vector of gross outputs for the production sectors of the economy. Let $A = [a_{ij}]$ be the matrix of input–output coefficients. That is, a_{ij} is the quantity of the *i*th industry's output used to produce one unit of the *j*th industry's output. Let F, C, G, O denote the vectors of gross private fixed investment, personal consumption expenditures, government (federal, state, and local) purchases, and 'other' for each sector. Here, 'other' is essentially exports minus imports. Total output, Y, is broken down into a part allocated to intermediate inputs, AY, and a part allocated to final output F + C + G + Q as follows:

row must be unity.row must be unity.row must be unity.ports minus imports. Total output,
$$Y$$
, is broken downinto a part allocated to intermediate inputs, AY , and apart allocated to final output, $I' + C + G + O$ as follows:AY + $I' + C + G + O = Y$.TABLE A1Results for consumptionI-Oindustry title $Y_{I'}$ Y_c 1+2+3+4Agriculture, forestry, and fisheries0.0600.895+6+7+8+9+10Mining0.2070.89

Solving this for *Y*, we get

$$Y = Y_{If} + Y_{C} + Y_{G} + Y_{O'}$$

$$Y_{i} = [I - A]^{-1} i, i = I', C, G, O.$$

For convenience, we report Y_i , i = I, C, G, O for the 95 sectors of the U.S. economy which are included in the input–output table underlying the analysis reported in figure 6. Table A1 reports results for the 17 sectors of the nondurable goods industry, as defined in the Hornstein and Praschnik (1997) analysis. That table reports the input–output table industry numbers that make up the industries whose name is in the middle column. Table A2 reports the numbers for the other sectors. The sum of the numbers in a row must be unity.

Results for consumption					
I-O industry number	I–O industry title	Y ₁ f	Yc	Y _g	Y _o
1+2+3+4	Agriculture, forestry, and fisheries	0.060	0.894	0.052	-0.006
5+6+7+8+9+10	Mining	0.207	0.893	0.181	-0.282
14	Food and kindred products	0.018	0.962	0.041	-0.021
15	Tobaccoproducts	0.000	0.914	0.000	0.086
16+17	Textile mill products	0.185	0.995	0.072	-0.252
18+19	Apparel and other textile products	0.037	1.284	0.041	-0.362
24+25	Paper and allied products	0.103	0.833	0.112	-0.047
26A+26B	Printing and publishing	0.058	0.795	0.121	0.026
27A+27B+28	Chemicals and allied products	0.180	0.698	0.138	-0.016
31	Petroleum refining and related products	0.105	0.782	0.145	-0.032
32	Rubber and miscellaneous plastics products	0.246	0.761	0.127	-0.134
33+34	Footwear, leather, and leather products	0.031	2.154	0.037	-1.222
65A++68C	Transportation, communications, and utilities	0.107	0.740	0.123	0.029
69A	Wholesaletrade	0.232	0.589	0.098	0.082
69B	Retail trade	0.066	0.919	0.015	0.000
70A+70B+71A+71B	Finance, insurance, and real estate	0.061	0.877	0.043	0.020
72A++77B	Services	0.076	0.879	0.044	0.002
Row numbers are scaled Source: Authors' calculat	unt of gross output of industry in indicated row sent directly so they sum to unity. ions based on data from U.S. Department of Commerce, Bu ss, Volume 72, Number 4, April.		,).	

	TABLE A2 Results for nonconsumption				
I-0 industry number	I–O industry title	Y _l f	Y _c	Y _G	Yo
11	Newconstruction	0.805	0.000	0.195	0.00
12	Maintenance and repair construction	0.180	0.574	0.243	0.00
13	Ordnance and accessories	0.011	0.051	0.838	0.10
20+21	Lumber and wood products	0.542	0.340	0.169	-0.0
22+23	Furniture and fixtures	0.477	0.585	0.067	-0.12
29A	Drugs	0.018	0.963	0.125	-0.10
29B	Cleaning and toilet preparations	0.018	0.949	0.033	0.00
30	Paints and allied products	0.422	0.445	0.168	-0.03
35	Glass and glass products	0.202	0.798	0.116	-0.1
36	Stone and clay products	0.575	0.314	0.206	-0.0
37	Primary iron and steel manufacturing	0.515	0.501	0.207	-0.22
38	Primary nonferrous metals manufacturing	0.400	0.485	0.247	-0.1
39	Metal containers	0.057	0.891	0.064	-0.0
40	Heating, plumbing, and fabricated structural metal products	0.604	0.201	0.198	-0.0
41	Screw machine products and stampings	0.358	0.641	0.132	-0.1
42	Other fabricated metal products	0.377	0.573	0.175	-0.1
43	Engines and turbines	0.377	0.362	0.246	0.01
44+45	Farm, construction, and mining machinery	0.731	0.151	0.097	0.02
44+4J 46	Materials handling machinery and equipment	0.876	0.134	0.105	-0.1
47	Metalworking machinery and equipment	0.779	0.261	0.108	-0.1
48	Special industry machinery and equipment	0.962	0.154	0.028	-0.1
40 49	General industrial machinery and equipment	0.729	0.305	0.028	-0.1
47 50		0.309	0.438	0.130	-0.0
50 51	Miscellaneous machinery, except electrical				
52	Computer and office equipment	0.786	0.148 0.289	0.156	-0.0
	Service industry machinery	0.636		0.120	-0.04
53	Electrical industrial equipment and apparatus	0.639	0.308	0.168	-0.1
54	Household appliances	0.242	0.842	0.045	-0.12
55	Electric lighting and wiring equipment	0.471	0.447	0.185	-0.10
56	Audio, video, and communication equipment	0.626	0.564	0.206	-0.3
57	Electronic components and accessories	0.338	0.437	0.322	-0.0
58	Miscellaneous electrical machinery and supplies	0.321	0.687	0.148	-0.1
59A	Motor vehicles (passenger cars and trucks)	0.478	0.776	0.051	-0.30
59B	Truck and bus bodies, trailers, and motor vehicles parts	0.437	0.746	0.080	-0.20
60	Aircraft and parts	0.134	0.049	0.546	0.27
61	Other transportation equipment	0.145	0.543	0.336	-0.02
62	Scientific and controlling instruments	0.442	0.166	0.372	0.02
63	Ophthalmic and photographic equipment	0.347	0.590	0.228	-0.10
64	Miscellaneous manufacturing	0.175	1.121	0.071	-0.3
78	Federal government enterprises	0.079	0.814	0.104	0.00
79	State and local government enterprises	0.033	0.928	0.043	-0.0
30	Noncomparable imports	0.000	0.000	0.000	0.00
31	Scrap, used, and secondhand goods	-9.699	7.493	1.830	1.37
32	General government industry	0.000	0.000	1.000	0.00
33	Rest of the world adjustment to final uses	0.000	0.000	0.000	0.00
84	Household industry	0.000	1.000	0.000	0.00
35	Inventory valuation adjustment	0.000	0.000	0.000	1.00

Row numbers are scaled so they sum to unity. Source: Authors' calculations based on U.S. Department of Commerce, Bureau of Economic Analysis, 1992, *Survey of Current Business*, Volume 72, Number 4, April.

NOTES

¹See Burns and Mitchell (1946, p. 3), Lucas (1981, p. 217), and Sargent (1979, p. 215).

²This definition was taken from the NBER's web address, http://www.nber.org/cycles.html.

³An important exception is Long and Plosser (1983), which does allow for multiple sectors. Their model economy is straightforward to analyze because they adopt several key simplifying assumptions. For example, they assume the entire stock of capital in each sector wears out within three months. However, these assumptions make the model ill-suited for quantitative, empirical analysis. It took many years before economists undertook a systematic empirical analysis of versions of the Long and Plosser model without the key simplifying assumptions (see Horvath [1998a, b]).

⁴Employment data are taken from DRI Basic Economics database. The hours worked data are indexes of aggregate weekly hours of production or nonsupervisory workers on private nonagricultural payrolls by industry. The data on numbers of workers are workers on nonagricultural payrolls, by industry. All data are monthly and seasonally adjusted and cover 1964:Q1– 95:Q3.

⁵Other studies of this question include Baxter (1996), Cooper and Haltiwanger (1990), Hornstein and Praschnik (1997), Huffman and Wynne (1998), and Murphy, Shleifer, and Vishny (1989).

⁶Our statistic is the regression R^2 obtained by regressing the business cycle component of that series on the business cycle component of total hours worked, at lags 0, 1, and -1. We allow next month's employment and the previous month's employment to enter this relationship because we do not want our measure of comovement to be low just because a variable may be out of phase with total private hours worked by only one month. If we did not include these lags, our regression R^2 would coincide exactly with the square correlation referred to in the text. We construct our statistic as follows. Let y_i denote the business cycle component of a given sector's employment. Let x_i denote the corresponding measure of total hours worked. We consider the regression of y_i on x_i , x_{i+1} , and x_{i+1} , $y_i = \hat{\alpha}_0 x_i + \hat{\alpha}_1 x_{i+1} + \hat{\alpha}_1 x_{i+1}$ $\hat{\alpha}_{2}x_{t+1} + \varepsilon_{t}$, where $\hat{\alpha}_{t}$ represents the estimated coefficients. Then, the R^2 's reported in the table are var $(\hat{\alpha}_0 x_t + \hat{\alpha}_1 x_{t-1} + \hat{\alpha}_2 x_{t+1})/2$ var(y).

⁷Table 1 shows the volatility in each of these data series.

⁸The correlation between y_i and y_u is $corr(y, y_u) = Cov(y, y_u)/[Var(y_i)Var(y_u)]^{s_i}$. But, $Cov(y, y_u) = \sigma^2$ and $Var(y_i) = 2\sigma^2$, $Var(y_u) = \sigma^2$. Substituting these results into the formula, we get $corr(y_e, y_u) = 1/\sqrt{2} \approx 0.71$.

⁹Suppose $y_i = y_{1i} + \dots + y_{ni}$. The logic of the previous note leads to $corr(y_i, y_{ni}) = 1/\sqrt{n}$. With n = 33, this is 0.17, after rounding.

 10 The midpoints are $-0.35, -0.25, -0.15, \ldots, 0.85, 0.95.$ In each case, the interval has length 0.1 and extends 0.05 above and below the midpoint.

¹¹Real business cycle theory has evolved considerably in recent years and now encompasses a wide variety of conceptions of the economy. The definition proposed by Prescott (1991, p. 3) reflects this: "Real business cycle theory is the application of general equilibrium theory to the quantitative analysis of business cycle fluctuations."

¹²This section and the next one draw heavily on work by Christiano and Fisher (1998).

¹³Some might want to dismiss the notion of a technology shock that affects all firms simultaneously as too preposterous to deserve consideration. Such a person may find it more plausible to think of technology shocks as things that are idiosyncratic to individual firms. Most of the examples of technology shocks given in the text certainly suggest this. This is the line that Lucas took when he dismissed the idea that a technology shock might be the aggregate shock needed to account for business cycles. He argued that, although technology shocks are no doubt important at the firm level, they could not be important for economy-wide aggregate output: He expected that firms affected by positive productivity shocks would be balanced by firms experiencing negative shocks. Work of Shleifer (1986) and Dupor (1998) suggests that the Lucas reason for dismissing technology shocks as an important impulse to business cycles may be premature. These researchers emphasize the distinction between the time that a new technological idea arrives in the firm, and the time the firm implements it. Consistent with Lucas's intuition, the exact timing of arrival of ideas may well be idiosyncratic at the firm level. In this case, the economywide average rate of arrival of new ideas would be constant: Firms discovering ideas for new products or labor-saving ways to produce output would be balanced by firms experiencing no progress or even regress. What Shleifer and Dupor emphasize, however, is that it is not the arrival of new ideas per se that shifts up production functions. Rather, it is the implementation of the new ideas that does this. They point out that there may well be plausible mechanisms in an economy which lead firms to implement new, technology-shifting ideas at the same time. These mechanisms involve "strategic complementarities," which are discussed further below.

¹⁴See, for example, Benhabib, Rogerson, and Wright (1991).

¹⁵Formally, this is what we have in mind. A standard real business cycle model, with unit elasticity of substitution in production between capital and labor, implies that the value of the output of the sector producing consumption goods, measured in utility units, is proportional to the value of the labor used in that sector, also measured in utility units. The value of the output of the consumption sector is the product of the total output of that sector, Y, and the marginal utility of consumption, u_c . The value of the labor used in producing consumption goods, L_c , and the marginal utility of leisure, u_r . Thus,

$\alpha Y u_c = u_i L_c$

This is just a rearrangement of the usual static efficiency condition that specifies that the marginal product of labor in producing the output of the consumption sector, $\alpha Y/L_e$, must equal the marginal rate of substitution between consumption and leisure, u/u_e . Note that if the term on the left of the equality falls ('the value of the output of the sector producing consumption goods falls') and u_i rises ('the marginal utility of leisure rises'), then L_e must fall.

¹⁶The inability of the standard real business cycle model to produce comovement is surprisingly robust. Standard specifications of that model hold that the marginal rate of substitution between consumption and leisure is $\psi C/(1 - L_e - L_i)^{\xi}$, where L_e is employment in the consumption sector, L_i is employment in the investment good sector, and $1 - L_e - L_i$ is leisure. Also, ψ and ξ are non-negative constants. In Hansen's (1985) indivisible labor model, $\xi = 0$. In his divisible labor model, $\xi = 1$. The standard model assumes a Cobb–Douglas production function, so that the marginal product of labor is proportional to average labor productivity in the consumption good producing sector. Equality between the marginal product of labor and the marginal rate of substitution between consumption and leisure implies:

$$\alpha \frac{C}{L_c} = \frac{\psi C}{(1 - L_c - L_i)^{\xi}}$$

Cancelling consumption on the two sides and rearranging, we get

$$\frac{\alpha}{\Psi}(1-L_c-L_i)^{\xi}=L_c$$

From this it is easy to see that if, for whatever reason, L_i or L_c moves, then the other variable *must* move in the opposite direction. This demonstration summarizes a discussion in Benhabib, Rogerson, and Wright (1991) and in Murphy, Shleifer, and Vishny (1989). The result holds for the entire class of utility functions identified by King, Plosser, and Rebelo (1988) as being consistent with balanced growth. However, the same cannot be said for the entire class of production functions consistent with balanced growth. In particular, the result does *not* hold for production functions in which the elasticity of substitution between capital and labor differs from unity. We demonstrate this in technical appendix 2. We also show, however, that for plausibly parameterized versions of the standard real business cycle model, departures from unit elasticity of substitution in production do not help the model reproduce comovement.

¹⁷One paper that is often mentioned in the comovement literature is Huffman and Wynne (1998). However, their focus is primarily on comovement in investment and output. They largely abstract from comovement in employment by making assumptions that make labor in the consumption sector essentially constant. They specify that the elasticity of substitution between labor and capital in the consumption sector is nearly unity, and that $\xi = 0$. The argument in note 16 explains why their model has the implication that L_{ϵ} is essentially constant.

 $^{18}{\rm Suppose}\,L_{\rm x}$ is the third use of time. Then the equation in note 16 is modified as follows:

$$\frac{\alpha}{\xi}(1-L_c-L_i-L_x)^{\xi}=L_c.$$

Evidently, now it is possible for both L_e and L_i to be procyclical, as long as L_v is sufficiently countercyclical.

¹⁹Closely related to this is their recommendation that economists work with the following utility function in consumption and leisure: $u [c - \psi_0 L^{1+\psi}/(1+\psi)]$, where $\psi, \psi_0 > 0$ and u is a concave, increasing utility function. The marginal rate of substitution between consumption and leisure with this utility function is $\psi_0 L^{\psi}$. Substituting this into the employment condition in note 16 results in

$$\alpha \frac{C}{L_c} = \psi_0 (L_c + L_i)^{\Psi}$$

The argument in that note that L_c and L_i cannot move in the same direction does not work with this utility function.

²⁰Baxter's model is a convenient vehicle for illustrating an issue that has to be confronted in macroeconomic models generally. The text provides an illustration of Baxter's assumption that durable goods and market goods are substitutes. However, it is just as easy to think of examples in which they are complements.

Consider a car, for example. Ownership of a car makes it more attractive to go out on long road trips that require purchasing market goods like hotel and restaurant services. This suggests that cars and market goods are complements. A moment's further thought about this example suggests that most household durables actually cannot be neatly labeled as either complements or substitutes for market consumption. For example, an automobile is also a substitute for market goods because it reduces the need for market services like cab, train, and airplane rides. Similarly, consider the biggest household durable of all, the home. It substitutes for hotel and restaurant services and complements market goods such as party goods, telephone services, and food. Thus, intuition is ultimately not a good guide to assessing Baxter's assumption about the substitutability of durables and market goods. Ultimately, this must be assessed through careful econometric work to determine whether, on average, market goods and durables are more like substitutes or complements.

²¹Consider the limiting case of perfect substitutability, so that consumption is C + D, where C is market consumption and D is the service flow from the stock of home durables. With log utility, the marginal utility of market consumption is 1/(C + D). Suppose D is fixed. Then a given jump in C reduces marginal utility by less, the larger is D.

²²Remarks in note 20 about Baxter's work are obviously relevant here too. Intuition is a very confusing guide, at best, regarding the plausibility of Benhabib, Rogerson, and Wright's assumption that the elasticity of substitution between home-produced and market-produced goods is high. The parameter must be estimated econometrically. This was done in Rupert, Rogerson, and Wright (1995), who report, based on data from the *Panel Study on Income Dynamics*, that the elasticity of substitution indeed is high.

²³Because the model predicts that consumption rises in a boom, the high degree of substitutability between home and market goods causes the marginal value of home goods to drop in a boom. This in turn causes a drop in the value of home durables, leading households to reduce their purchases of new durables. This implication is strongly counterfactual, however, since durables are in fact highly procyclical. Interestingly, Baxter's (1996) model seems to avoid this tension. In particular, her model generates comovement between employment in the consumption and investment industries and simultaneously implies that durable goods purchases are procyclical.

²⁴Boldrin, Christiano, and Fisher (1995) adapt the habit persistence specification of preferences proposed in Constantinides (1990) and Sundaresan (1989).

²⁵See Kocherlakota (1996) for a recent review. Although habit persistence helps to account for the observed average of the premium in equity over risk-free debt, it does not account well for the volatility of these variables. For a further discussion, see Boldrin, Christiano, and Fisher (1997) and Heaton (1995).

²⁶See Constantinides (1990) and Sundaresan (1989) for more evidence on the plausibility of habit persistence preferences.

²⁷In the Hornstein and Praschnik (1997) modification, the output of the consumption sector is C + m, where *m* is intermediate goods sent to the investment good sector. Suppose the marginal utility of market consumption is 1/C. Then, the value of the output of the consumption sector is (C + m)/C = 1 + m/C. Note that this jumps with a rise in *C* as long as *m* rises by a greater percentage than *C*. With *m/C* sufficiently procyclical, it is possible for employment in the investment and consumption good sectors to move up and down together over the cycle.

²⁸We are very grateful for instructions and advice from Mike Kouparitsas on how to analyze the input–output data.

²⁹We do not have an index of hours worked for this sector. Instead, we used LHAG, which is Citibase's mnemonic for number of persons employed in the agricultural industry. We obtained a measure of comovement for this variable in the same way as for the other variables.

³⁰The least squares regression line through the data in figure 6 is $\rho_{h,y} = 0.48 + 1.35I_c$. Thus, if a sector was not connected to the investment sector at all (that is, $I_c = 0$), employment in that sector would still exhibit substantial procyclicality (that is, $\rho_{h,y} = 0.48$.)

³¹Such an exercise could be pursued by building on the models in Long and Plosser (1983) and Horvath (1998a, b). To our knowledge, comovement in the sense studied in this article has not been investigated in these models.

³²A slightly different mechanism, whereby a firm's expectation that other firms will be inactive leads all firms to be inactive was analyzed by Shleifer (1986) and Dupor (1998) and summarized in note 13.

³³For example, Benhabib and Farmer (1994, 1996) incorporate strategic complementarities by way of an externality in the production function. Because their production function is of the Cobb–Douglas form, the argument in note 16 applies to these models too. In particular, in these models, employment in the production of consumption and investment goods must move in opposite directions over the business cycle.

³⁴The literature on the potential for expectations to be selffulfilling is large. Influential early papers include Azariadis (1981), Bryant (1983), Cass and Shell (1983), Cooper and John (1988), Diamond (1982), Farmer and Woodford (1984), Shleifer (1986), and Woodford (1986, 1987, 1988, 1991). More recent contributions include Benhabib and Farmer (1994, 1996), Christiano and Harrison (1998), Cooper and Haltiwanger (1990, 1996), Farmer and Guo (1994), Gali (1994), and Schmitt-Grohe (1997).

³⁵An example of a negative externality is the pollution that is generated as a byproduct of a manufacturing process.

³⁶For an analysis of the case where there are information externalities and timing *is* under the control of managers, see Chamley and Gale (1994). They find, as one might expect, that there is a tendency to delay decisions under these circumstances.

³⁷We are grateful to Henry Siu for pointing this out to us.

³⁸The example is similarly sensitive to the assumption that people view the signals they receive as equally reliable to the signals received by others. It is possible that, in practice, the type of individual making investment decisions has greater confidence in her ability to interpret signals than her counterparts at other firms. This is the implication of empirical evidence that suggests that these types of people are overly confident in their own abilities. See Daniel, Hirshleifer, and Subrahmanyam (1998), and the references therein for further discussion. According to them, (p. 5-6): "Evidence of overconfidence has been found in several contexts. Examples include psychologists, physicians and nurses, engineers, attorneys, negotiators, entrepreneurs, managers, investment bankers, and market professionals such as security analysts and economic forecasters. Further, some evidence suggests that experts tend to be more overconfident than relatively inexperienced individuals '

³⁹A small subset of the literature on information externalities includes Banerjee (1992), Bikhchandani, Hirshleifer, and Welch (1994), Caplin and Leahy (1994), and Chamley and Gale (1994).

⁴⁰See Romer (1996) for a review.

⁴¹Let e(w) be the amount of effort a worker expends per hour, given the hourly wage rate, w. The efficiency wage is the value of w that maximizes e(w)/w. One type of e function that guarantees that this has a maximum for $0 \le w \le \infty$ is one in which e, when expressed as a function of w, has an S shape: convex for w near zero and turning concave for larger values of w (see Romer, 1996). The optimal e(w)/w is the slope of the ray drawn from the origin, tangent to the concave part of the e function. At the optimum, the elasticity of effort with respect to the wage is unity, that is, e'(w)w/e(w) = 1. Optimality requires that, when evaluated at the efficiency wage, the second derivative of e with respect to w, is negative.

⁴²The algebra underlying this analysis is simple. Let the production function be f(e(w)L, K, z), where eL is the total amount of effort expended in L hours of work, z is a shock to technology, and K is the stock of capital. We assume that the derivative of f in its first argument is positive and strictly decreasing in eLand increasing in z. Revenues net of labor costs are f(e(w)L, K, z)-wL. The firm maximizes this with respect to w and L. It is convenient, however, to adopt a change of variables, X = wI, and let the firm choose X and w instead. Then, the revenue function is

$$f\left(\frac{e(w)}{w}X,K,z\right)-X.$$

Evidently, maximizing this with respect to w is equivalent to maximizing effort per dollar cost, e(w)/w with respect to w. For a further discussion of this maximization problem, see the previous note. Maximization with respect to X implies:

$$f_1(eL, K, z)e = w,$$

that is, the marginal product of labor must equal the wage rate.

⁴³The marginal product of labor curve in figure 7 graphs $f_1(e(w^*)L, K, z)e(w^*)$ as a function of *L*, holding *K* fixed. Here, w^* is the efficiency wage rate. The curve marked marginal product of labor' graphs $f_1(e(w^*)L, K, \tilde{z})e(w^*)$ for $\tilde{z} > z$.

⁴⁴These observations motivate why efficiency wage theory is sometimes viewed as a way to fix another set of counterfactual implications of the standard real business cycle model: that wages tend to fluctuate too much and employment too little over the business cycle.

⁴⁵This argument implicitly assumes that the stock of capital used by a firm, once put in place, cannot be shifted to another firm. The assumption guarantees that a positive technology shock which drives up the marginal productivity of labor curve, *must* be accompanied by a rise in employment if marginal productivity is to remain unchanged. If capital were mobile between sectors, this could even be accomplished with a *fall* in labor, as long as capital in that sector fell by an even greater percentage. The standard real business cycle model assumes that capital is freely mobile between sectors. Thus, the intuition in this article is based on *two* modifications to the real business cycle model: incorporation of efficiency wages and sectoral immobility of capital. The second of these is not sufficient to produce business cycle comovement. This is because the argument in note 16 holds even if capital is immobile between sectors. ⁴⁶In addition to verifying the logical coherence of efficiency wage theory as an explanation of comovement, there are two empirical issues to be investigated. How hard is it to monitor worker effort? If it can be monitored easily, then efficiency wage theory is irrelevant. Also, if the penalty for being fired for shirking is enormous, workers will behave as if they are being monitored continuously, and once again the theory becomes irrelevant. For a further discussion of these issues, see Alexopoulos (1998).

⁴⁷We stress that the intuition developed here relies on *two* assumptions—efficiency wages and sectorally immobile capital.

⁴⁸To be precise, suppose e(w, D) is the effort supplied by workers when the wage rate is w and unemployment duration is D. At the efficiency wage, $e_{11}(w, D) < 0$. Also, we assume $e_{12}(w, D) = 0$. Totally differentiating the first order condition for the efficiency wage, $we_1(w, D)/w = 1$, with respect to w and D, and imposing the restrictions on e_{12} and e_{11} yields the result, dw/dD < 0.

⁴⁹In the literature, what we have called the worker's effort function, *e*, is referred to as the "incentive compatibility constraint."

⁵⁰Alexopoulos and Gomme have reported to us privately that their models are only partially consistent with business cycle comovement. In both cases, employment in the consumption and investment sectors is positively correlated, but investment in these two sectors is negatively correlated. However, both models assume that capital can be transferred instantaneously across sectors in response to a shock. The analysis here suggests

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Benhabib, Jess, and Roger E. A. Farmer, 1996, "Indeterminacy and sector-specific externalities," *Journal of Monetary Economics*, Vol. 37, No. 3, June, pp. 421–443. that sectoral capital immobility may be important for obtaining comovement.

⁵¹For an introduction to the literature on sticky prices and wages, see Romer (1996). To see why countercyclical markups might help, recall the key equation in note 16, used to show why hours worked making consumption goods and hours worked making investment goods in a standard real business cycle model must move in opposite directions. A version of that model with market power, for example, the model of Rotemberg and Woodford (1992), implies that it is the ratio of the marginal product of labor to the markup that must equal the marginal rate of substitution between consumption and leisure. That is, that equation must be modified as follows:

$$\frac{\alpha}{\mu} \frac{C}{L_c} = \frac{\psi C}{(1 - L_c - L_i)^{\xi}},$$

where μ is the markup of price over marginal cost. Cancelling consumption on the two sides and rearranging, we get

$$\frac{\alpha}{\psi}(1-L_c-L_i)^{\xi}=\mu L_c.$$

Suppose a boom occurs, driving up L_i If μ falls, as in the Rotemberg and Woodford model, then it is possible for L_i to rise too. (For another model with countercyclical markups see Gali [1994]). See Murphy, Shleifer and Vishny (1989) for a conjecture about how limited intersectoral labor mobility, together with credit market restrictions, may help account for comovement.

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