Optimal Monetary Policy with Uncertain
Fundamentals and Dispersed Information

Guido Lorenzoni

March 2008

Optimal Monetary Policy with Uncertain Fundamentals and Dispersed Information

Guido Lorenzoni[*]

March 2008

Abstract

This paper studies monetary policy in an economy where output fluctuations are driven by the private sector's uncertainty about the economy's fundamentals. I consider an economy where information on aggregate productivity is dispersed across agents and there are two aggregate shocks: a standard productivity shock and a "noise shock" affecting public beliefs about aggregate productivity. Neither the central bank nor individual agents can distinguish the two shocks when they hit the economy. The main results are: (1) despite the lack of superior information, an appropriate monetary policy rule can change the economy's response to the two aggregate shocks; (2) monetary policy can achieve "full aggregate stabilization," that is, an equilibrium where aggregate activity is the same as in the case of full information; (3) under optimal monetary policy, the economy achieves a constrained efficient allocation; (4) optimal monetary policy is typically different from full aggregate stabilization. Behind these results there are two crucial ingredients. First, agents are forward looking. Second, as time passes, better information on past fundamentals becomes available. The central bank can then adopt a backward-looking policy rule, based on more precise information about past fundamentals. By announcing its response to future information, the central bank can influence the expected real interest rate faced by agents with different beliefs and thus induce an optimal use of the information dispersed in the economy.

Keywords: Optimal monetary policy, imperfect information, consumer sentiment.
JEL Codes: E52, E32, D83.

1   Introduction

Suppose a central bank observes an unexpected expansion in economic activity. This could be due to a shift in fundamentals, say an aggregate productivity shock, or to a shift in public beliefs with no actual change in the economy's fundamentals. If the central bank could tell apart the two shocks the optimal response would be simple: accommodate the first shock and offset the second. In reality, however, central banks can rarely tell apart these shocks when they hit the economy. What can the central bank do in this case? What is the optimal monetary policy response? In this paper, I address these questions in the context of a model with dispersed information, which allows for a micro-founded treatment of fundamental shocks and "sentiment shocks."

The US experience in the second half of the 90s has fueled a rich debate on these issues. The run up in asset prices has been taken by many as a sign of optimistic expectations about widespread technological innovations. In this context, the advice given by different economists has been strongly influenced by the assumptions made about the ability of the central bank to identify the economy's *actual* fundamentals. Some, e.g. Cecchetti et al. (2000) and Dupor (2002), attribute to the central bank some form of superior information and advocate early intervention to contain an expansion driven by incorrect beliefs. Others, e.g. Bernanke and Gertler (2001), emphasize the uncertainty associated with the central bank's decisions and advocate sticking to a simple inflation-targeting rule. In this paper, I explore the idea that, even if the central bank does not have superior information, a policy rule can be designed to take into account, and partially offset, aggregate mistakes by the private sector regarding the economy's fundamentals.

I consider an economy with heterogeneous agents and monopolistic competition, where aggregate productivity is subject to unobservable random shocks. Agents have access to a noisy public signal of aggregate productivity, which summarizes public news about technological advances, aggregate statistics, and information reflected in stock market prices and other financial variables. The error term in this signal introduces aggregate "noise shocks," that is, shocks to public beliefs which are uncorrelated with actual productivity shocks. In addition to the public signal, agents have access to private information regarding the realized productivity in the sector where they work. Due to cross-sectional heterogeneity, this information is not sufficient to identify the value of the aggregate shock. Therefore, agents combine public and private sources of information to forecast the aggregate behavior of the economy. The central

bank has access only to public information.

In this environment, I obtain two sets of results. First, I show that the monetary authority, using a policy rule which responds to past aggregate shocks, *can* affect the relative response of the economy to productivity and noise shocks. Actually, there exists a policy rule which achieves "full aggregate stabilization," that is, an equilibrium where aggregate activity is the same as in the case of full information. Second, I derive the optimal policy rule and show that full aggregate stabilization is typically suboptimal. As long as the coefficient of relative risk aversion is greater or equal than 1, it is optimal to let output respond less than one for one to underlying changes in aggregate fundamentals and to let it respond positively to noise shocks. At the optimal policy rule, the economy achieves a constrained efficient allocation, where agents make optimal use of public and private sources of information.

The fact that monetary policy can tackle the two shocks separately is due to two crucial ingredients. First, agents are forward looking. Second, productivity shocks are unobservable when they are realized, but become public knowledge in later periods. At that point, the central bank can respond to them. By choosing an appropriate policy rule the monetary authority can then alter the way in which agents respond to private and public information. In particular, the monetary authority can announce that it will increase the target for aggregate nominal spending tomorrow, following an actual increase in aggregate productivity today, so as to generate inflation. Under this policy, consumers observing an increase in productivity in their own sector expect higher inflation than consumers who only observe a positive public signal. Therefore, they expect a lower real interest rate and choose to consume more. This makes consumption more responsive to private information and less to public information and moderates the economy's response to noise shocks. This result points to an idea which applies more generally in models with dispersed information. If future policy is set contingent on variables that are imperfectly observed today, this can change the agents' reaction to different sources of information, and thus affect the equilibrium allocation.

In the model presented, the power of policy rules to shape the economy's response to aggregate shocks is surprisingly strong. Namely, by adopting the appropriate rule the central bank can support an equilibrium where aggregate output responds one for one to fundamentals and does not respond at all to noise in public news. However, such a policy is typically suboptimal, since it has undesirable consequences in terms of the cross-sectional allocation. In particular, full stabilization generates an inefficient compression in the distribution of relative prices.

To define the appropriate benchmark for constrained efficiency, I consider a social planner who can dictate the way in which individual consumers respond to the information in their hands, but cannot change their access to information.[1] My constrained efficiency result shows that, in a general equilibrium environment with isoelastic preferences and Gaussian shocks, a simple linear monetary policy rule, together with a non-state-contingent production subsidy, are enough to eliminate all distortions due to dispersed information and monopolistic competition. In particular, a policy rule which only depends on *aggregate* variables is enough to induce agents to make an optimal use of public and private information.[2]

Finally, I use the model to ask whether better public information regarding the economy fundamentals can have destabilizing effects on the economy and whether it can lead to social welfare losses. [3] I show that increasing the precision of the public signal increases the response of aggregate output to noise shocks and this can potentially increase output gap volatility.[4] However, as agents receive more precise information on average productivity, they can set relative prices that are more responsive to their idiosyncratic productivity shocks. Therefore, a more precise public signal can improve welfare by allowing a more efficient allocation of consumption and labor effort across sectors. What is the total welfare effect of increasing the public signal's precision? If monetary policy is kept constant, then a more precise public signal may, for some set of parameters, reduce total welfare. This provides an interesting general equilibrium counterpart to Morris and Shin's (2002) "anti-transparency" result. However, if monetary policy is chosen optimally, then a more precise signal is always welfare improving. This follows from the fact that, as pointed out by Angeletos and Pavan (2007a), more precise information is always desirable when the equilibrium is constrained efficient.

In this paper, equilibrium allocations and welfare are derived in closed form. This is possible thanks to an assumption about the random selection of consumption baskets. In particular, I maintain the convenience of a continuum of goods in each basket, while, at the same time, I allow for baskets that differ from consumer to consumer. This technical solution may be usefully adapted to other models of information diffusion with random matching, as it allows

---

[1]See Hellwig (2005) and Angeletos and Pavan (2007a) for similar notions of constrained efficiency.

[2]Angeletos and Pavan (2007b) derive a similar result in the context of quadratic games with optimal policy. See Angeletos, Lorenzoni, and Pavan (2008) for an application of the same principle to a model of investment and financial markets.

[3]On the social value of public information in game-theoretic settings, see Morris and Shin (2002), Angeletos and Pavan (2007a, 2007b), Hellwig and Veldkamp (2007), Amador and Weill (2007). This literature has sparked a lively debate on the merits of transparency in monetary policy, see Amato, Morris, and Shin (2002), Svensson (2005), Hellwig (2005), Morris and Shin (2005).

[4]Here the output gap is measured with respect to the equilibrium under full information.

to construct models where each agent interacts with a large number of other agents, but does not fully learn about aggregate behavior.

A number of recent papers, starting with Woodford (2002) and Sims (2003), have revived the study of monetary models with imperfect common knowledge, in the tradition of Phelps (1969) and Lucas (1972).[5] In particular, this paper is closely related to Hellwig (2005) and Adam (2006), who study monetary policy in economies where money supply is imperfectly observed by the public. In both papers consumers' decisions are essentially static, as a cash-in-advance constraint is present and always binding. Therefore, the forward-looking element, which is crucial in this paper, is absent in their models. In the earlier literature, King (1982) was the first to recognize the power of policy rules in models with imperfect information. He noticed that "*prospective* feedback actions" responding to "disturbances that are currently imperfectly known by agents" can affect real outcomes.[6] However, the mechanism in King (1982) is based on the fact that different policy rules change the informational content of prices. As I will show below, that channel is absent in this paper. Here, policy rules matter because they affect agents' incentives to respond to private and public signals.

The existing literature on optimal monetary policy with uncertain fundamentals has focused on the case of common information in the private sector. This includes Aoki (2003), Svensson and Woodford (2003, 2005), and Reis (2003). A distinctive feature of the environment in this paper is that private agents have access to superior information about fundamentals in their local market but not in the aggregate. The presence of dispersed information generates a novel tension between aggregate efficiency and cross-sectional efficiency in the design of optimal policy.

There is a growing literature on the effect of expectations and news on the business cycle. In particular, Christiano, Motto, and Rostagno (2006) and Lorenzoni (2006) show that shocks to expectations about productivity can generate realistic aggregate demand disturbances in business cycle models with nominal rigidities.[7] In Christiano, Motto, and Rostagno (2006) the monetary authority has full information regarding aggregate shocks and can adjust the nominal interest rate in such a way so as to essentially offset the effect of the news shock and replicate the behavior of the corresponding flexible price economy. Moreover, this offsetting is optimal

---

[5]See also Moscarini (2004), Milani (2005), Nimark (2005), Bacchetta and Van Wincoop (2005), Luo (2006), Maćkowiak and Wiederholt (2006). Mankiw and Reis (2002) and Reis (2006) explore the complementary idea of lags in informational adjustment as a source of nominal rigidity.

[6]King (1982), p. 248.

[7]See Beaudry and Portier (2006) and Jaimovich and Rebelo (2006) for flexible price models of cycles driven by news about future productivity.

in their model.[8] This leads to the question: are expectations-driven cycles merely a symptom of a suboptimal monetary regime, or is there some amount of expectations-driven volatility that survives under optimal monetary policy? This paper addresses this question in a setup with dispersed information, as in Lorenzoni (2006), and shows that optimal monetary policy does not eliminate noise-driven cycles. One may think that this result comes immediately from the assumption that the monetary authority has limited information. That is, it would seem that the central bank cannot intervene to bring output towards its "natural" level, given that this natural level is unknown. The analysis in this paper shows that the argument is subtler. The monetary authority *could* eliminate the aggregate effect of news shocks by announcing an appropriate monetary rule. However, this rule is not optimal due to its undesirable cross-sectional consequences.

Finally, from a methodological point of view, this paper is related to a set of papers who exploit isoelastic preferences and Gaussian shocks to derive closed-form expressions for social welfare in heterogeneous agent economies, e.g., Benabou (2002) and Heathcote, Storosletten, and Violante (2008). The main novelty here is the presence of differentiated goods and consumer-specific consumption baskets.

The model is introduced in Section 2. In Section 3, I characterize stationary, linear rational expectations equilibria. In Section 4, I show how the choice of the monetary policy rule affects the equilibrium allocation. In Section 5, I derive the welfare implications of different policies, characterize optimal monetary policy and prove constrained efficiency. In Section 6, I study the welfare effects of public information. Section 7 concludes. All the proofs not in the text are in the appendix.

2 The Model

2.1 Setup

I consider a dynamic model of monopolistic competition à la Dixit-Stiglitz with heterogeneous productivity shocks and imperfect information regarding aggregate shocks. Prices are set at the beginning of each period, but are, otherwise, flexible.

There is a continuum of infinitely lived households uniformly distributed on the unit interval $[0, 1]$. Each household i is made of two agents: a consumer and a producer who is specialized

---

[8]See Appendix B of Christiano, Motto, and Rostagno (2006).

in the production of good i. Preferences are represented by the utility function

$$\mathbb{E}\left[\sum_{t=0}^{\infty}\beta^t U\left(C_{it}, N_{it}\right)\right],$$

with

$$U\left(C_{it}, N_{it}\right) = \frac{1}{1-\gamma}C_{it}^{1-\gamma} - \frac{1}{1+\eta}N_{it}^{1+\eta},$$

where C_{it} is a consumption index and N_{it} is the labor effort of producer i.[9] The consumption index is given by

$$C_{it} = \left(\int_{J_{it}} C_{ijt}^{\frac{\sigma-1}{\sigma}} dj\right)^{\frac{\sigma}{\sigma-1}},$$

where C_{ijt} denotes consumption of good j by consumer i in period t, and $J_{it} \subset [0,1]$ is a random consumption basket, which is described in detail below. The elasticity of substitution between goods, σ, is greater than 1.

The production function for good i is

$$Y_{it} = A_{it}N_{it}.$$

Productivity is household-specific and labor is immobile across households. The productivity parameters A_{it} are the fundamental source of uncertainty in the model. Let a_{it} denote the log of individual productivity, $a_{it} = \log\left(A_{it}\right)$.[10] Individual productivity has an aggregate component a_t and an idiosyncratic component ϵ_{it},

$$a_{it} = a_t + \epsilon_{it},$$

with $\int_0^1 \epsilon_{it} di = 0$. Aggregate productivity a_t follows the AR1 process

$$a_t = \rho a_{t-1} + \theta_t,$$

with $\rho \in [0,1]$.

At the beginning of period t, all households observe the value of aggregate productivity in the previous period, a_{t-1}. Next, the shocks ϵ_{it} and θ_t are realized. Agents in household

---

[9]If $\gamma = 1$, the per-period utility function is

$$U\left(C_{it}, N_{it}\right) = \log C_{it} - \frac{1}{1+\eta}N_{it}^{1+\eta}.$$

[10]Throughout the paper, a lowercase variable will denote the natural logarithm of the corresponding uppercase variable.

i cannot observe ϵ_{it} and θ_t separately, they only observe the sum of the two, that is, the individual productivity innovation

$$x_{it} = \theta_t + \epsilon_{it}.$$

Moreover, all agents observe a noisy public signal of the aggregate innovation

$$s_t = \theta_t + e_t.$$

The random variables ϵ_{it}, θ_t and e_t are independent, serially uncorrelated, and normally distributed with zero mean and variances σ_ϵ^2, σ_θ^2, and σ_e^2.[11] I assume throughout the paper that σ_ϵ^2 and σ_θ^2 are strictly positive, and I study separately the cases $\sigma_e^2 = 0$ and $\sigma_e^2 > 0$, corresponding, respectively, to full information and imperfect information on θ_t.

Summarizing, there are two aggregate shocks: the productivity shock θ_t and the "noise shock" e_t. Both are unobservable during period t, but are fully revealed at the beginning of $t + 1$, when a_t is observed. The second shock is the source of correlated mistakes in this economy, as it induces households to temporarily overstate or understate the current value of θ_t. The vector of past aggregate shocks is denoted by

$$h_t \equiv (\theta_{t-1}, e_{t-1}, \theta_{t-2}, e_{t-2}, ..., \theta_0, e_0).$$

Let me turn now to the random consumption baskets. Each period, nature selects a random set of goods $J_{it} \subset [0, 1]$ with correlated productivity shocks. In this way, even though each consumer consumes a large number of goods (a continuum), the law of large numbers does not apply, and consumption baskets differ across consumers. In the appendix, I give a full description of the matching process between consumers and producers. Here, I summarize the properties of the consumption baskets that arise from the process. Each consumer receives a "sampling shock" v_{it} (unobserved by the consumer) and the goods in J_{it} are selected so that the distribution of the shocks ϵ_{jt} for $j \in J_{it}$ is normal with mean v_{it} and variance $\sigma_{\epsilon|v}^2$. The sampling shocks v_{it} are normally distributed across consumers, with zero mean and variance σ_v^2. They are independent of all other shocks and satisfy $\int_0^1 v_{it} di = 0$. To ensure consistency of the matching process the variances $\sigma_v^2, \sigma_{\epsilon|v}^2$ and σ_ϵ^2 have to satisfy $\sigma_v^2 + \sigma_{\epsilon|v}^2 = \sigma_\epsilon^2$. Therefore, the variance σ_v^2 is restricted to be in the interval $[0, \sigma_\epsilon^2]$. Let me introduce here the parameter

---

[11]In the cases where $\gamma \neq 1$ and productivity is a random walk, $\rho = 1$, it is necessary to impose a bound on σ_θ^2 to ensure that expected utility is finite, namely

$$\sigma_\theta^2 < 2 \left(\frac{\gamma + \eta}{(1 - \gamma)(1 + \eta)} \right)^2 (-\log \beta).$$

$\chi = \sigma_v^2/\sigma_\epsilon^2$ which lies in $[0, 1]$ and reflects the degree of heterogeneity in consumption baskets. The limit case $\chi = 0$ corresponds to the standard case where all consumers consume the same representative sample of goods.

2.2 Trading, financial markets and monetary policy

The central bank acts as an account keeper for the agents in the economy. Each household holds an account denominated in dollars, directly with the central bank. The account is debited whenever the consumer makes a purchase and credited whenever the producer makes a sale. The balance of household i at the beginning of the period is denoted by B_{it}. All households begin with a zero balance at date 0. At the beginning of each period t, the bank sets the (gross) nominal interest rate R_t, which will apply to end-of-period balances. Households are allowed to hold negative balances at the end of the period and the same interest rate applies to positive and negative balances. However, there is a lower bound on nominal balances, which rules out Ponzi schemes.

To describe the trading environment, it is convenient to divide each period t in three stages, $(t, 0)$, (t, I), and (t, II). In stage $(t, 0)$, everybody observes a_{t-1}, the central bank sets R_t, and the households trade one-period state-contingent claims on a centralized financial market. These claims will be paid in $(t + 1, 0)$. In stages (t, I) and (t, II), the market for state-contingent securities is closed and the only assets traded are dollar balances in the central bank's payment system and non-state-contingent bonds payable in $(t + 1, 0)$. By arbitrage, the price of the bonds must be equal to $1/R_t$ at all stages. Since balances with the central bank and non-state contingent bonds are perfect substitutes and are in zero net supply, I simply assume that holdings of non-state-contingent bonds are always zero. In stage (t, I), all aggregate and individual shocks are realized, producer i observes s_t and x_{it}, sets the dollar price of good i, P_{it}, and stands ready to deliver any quantity of good i at that price. In stage (t, II), consumer i observes the prices of the goods in his consumption basket, $\{P_{jt}\}_{j \in J_{it}}$, chooses his consumption vector, $\{C_{ijt}\}_{j \in J_{it}}$, and buys C_{ijt} from each producer $j \in J_{it}$. In this stage, consumer i and producer i are spatially separated, so the consumer does not observe the current production of good i. Figure 1 summarizes the events taking place during period t.

In stages (t, I) and (t, II), households are exposed to idiosyncratic uncertainty and do not have access to state-contingent claims. Therefore, they will generally end up with different end-of-period balances. However, households can fully insure against these shocks ex ante, by trading contingent claims in $(t, 0)$. This implies that the nominal balances B_{it} will be constant
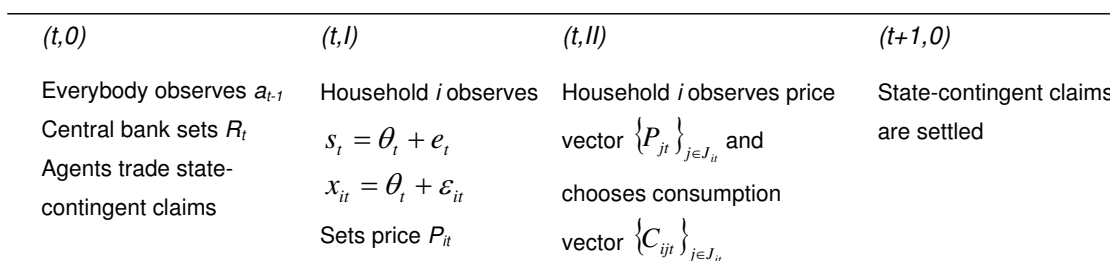
(t,0)	(t,I)	(t,II)	(t+1,0)
Everybody observes a_{t-1} Central bank sets R_t Agents trade state-contingent claims	Household i observes $s_t = \theta_t + e_t$ $x_{it} = \theta_t + \varepsilon_{it}$ Sets price P_{it}	Household i observes price vector $\{P_{jt}\}_{j \in J_{it}}$ and chooses consumption vector $\{C_{ijt}\}_{j \in J_{it}}$	State-contingent claims are settled

Figure 1: Timeline

and equal to 0 in equilibrium.[12] In this way, I can eliminate the wealth distribution from the state variables of the problem, which greatly simplifies the analysis.[13]

Let $Z_{it+1}(\omega_{it})$ denote the state-contingent claims purchased by household i in $(t,0)$, where $\omega_{it} \equiv (\epsilon_{it}, v_{it}, \theta_t, e_t)$. The price of these claims is denoted by $Q_t(\omega_{it})$. The household balances at the beginning of period $t+1$ are then given by

$$B_{it+1} = R_t \left[B_{it} - \int_{\mathbb{R}^4} Q_t(\tilde{\omega}_{it}) Z_{it+1}(\tilde{\omega}_{it}) d\tilde{\omega}_t + (1+\tau) P_{it} Y_{it} - \int_{J_{it}} P_{jt} C_{ijt} dj - T_t \right] + Z_{it+1}(\omega_{it}),$$

where τ is a proportional subsidy on sales and T_t is a lump-sum tax.

Let me define aggregate indexes for nominal prices and real activity. For analytical conve-

---

[12]The balances B_{it} are computed after the claims from $t-1$ have been settled.
[13]The use of this type of assumption to simplify the study of monetary models goes back to Lucas (1990).

nience, I use simple geometric means,[14]

$$P_t \equiv \exp\left(\int_0^1 \log P_{it} di\right),$$

$$C_t \equiv \exp\left(\int_0^1 \log C_{it} di\right).$$

The behavior of the monetary authority is described by a policy rule. In period $(t,0)$, the central bank sets R_t based on the past realizations of the exogenous shocks θ_t and e_t, and on the past realizations of P_t and C_t. The monetary policy rule is described by the map $\mathcal{R}$, where $R_t = \mathcal{R}(h_t, P_{t-1}, C_{t-1}, ..., P_0, C_0)$. Allowing the monetary policy to condition R_t on the current public signal s_t would not alter any of the results. The only other policy instrument available is the subsidy τ, which is financed by the lump-sum tax T_t. The government runs a balanced budget so

$$T_t = \tau \int_0^1 P_{it} Y_{it} di.$$

As usual in the literature, the subsidy τ will be used to eliminate the distortions due to monopolistic competition.

2.3 Equilibrium definition

Household behavior is captured by three functions, $\mathcal{Z}$, $\mathcal{P}$ and $\mathcal{C}$. The first gives the optimal holdings of state-contingent claims as a function of the initial balances B_{it} and of the vector of past aggregate shocks h_t, that is, $Z_{it+1}(\omega_{it}) = \mathcal{Z}(\omega_{it}; B_{it}, h_t)$. The second gives the optimal price for household i, as a function of the same variables plus the current realization of individual productivity and of the public signal, $P_{it} = \mathcal{P}(B_{it}, h_t, s_t, x_{it})$. The third gives optimal consumption as a function of the same variables plus the observed price vector, $C_{it} = \mathcal{C}(B_{it}, h_t, s_t, x_{it}, \{P_{ij}\}_{j \in J_{it}})$. Before defining an equilibrium, I need to introduce two other objects. Let $\mathcal{D}(.|h_t)$ denote the distribution of nominal balances B_{it} across households, conditional on the history of past aggregate shocks h_t. The price of a ω_{it}-contingent claim in period $(t,0)$, given the vector of past shocks h_t, is given by $\mathcal{Q}(\omega_{it}; h_t)$.

---

[14]Alternative price and quantity indexes are

$$\hat{P}_t \equiv \left(\int_0^1 P_{it}^{1-\sigma} di\right)^{\frac{1}{1-\sigma}},$$

$$\hat{Y}_t \equiv \frac{\int_0^1 P_{it} Y_{it} di}{\hat{P}_t}.$$

All results stated for P_t and C_t hold for $\hat{P}_t$ and $\hat{Y}_t$, modulo multiplicative constants.

A *symmetric rational expectations equilibrium* under the policy rule $\mathcal{R}$ is given by an array $\{\mathcal{Z}, \mathcal{P}, \mathcal{C}, \mathcal{D}, \mathcal{Q}\}$ that satisfies three conditions: optimality, market clearing, and consistency. Optimality requires that the individual rules $\mathcal{Z}, \mathcal{P}$ and $\mathcal{C}$ are optimal for the individual household, taking as given: the exogenous law of motion for h_t, the policy rule $\mathcal{R}$, the prices $\mathcal{Q}$, and the fact that all other households follow $\mathcal{Z}, \mathcal{P}$, and $\mathcal{C}$, and that their nominal balances are distributed according to $\mathcal{D}$. Market clearing requires that the goods markets and the market for state-contingent claims clear for each h_t. Consistency requires that the dynamics of the distribution of nominal balances, described by $\mathcal{D}$, are consistent with the individual decision rules.

3 Linear equilibria

In this section, I characterize the equilibrium behavior of output and prices. Given the assumption that agents trade state-contingent claims in periods $(t, 0)$, I can focus on equilibria where beginning-of-period nominal balances are constant and equal to zero for all households. That is, the distribution $\mathcal{D}(. | h_t)$ is degenerate for all h_t. Moreover, thanks to the assumption of separable, isoelastic preferences and Gaussian shocks, I can analyze linear rational expectations equilibria in closed form. In particular, I will characterize stationary linear equilibria where the logs of individual prices and consumption levels take the following form

$$
p_{it} \quad = \quad \phi_a a_{t-1} + \phi_s s_t + \phi_x x_{it}, \tag{1}
$$

$$
c_{it} \quad = \quad \psi_0 + \psi_a a_{t-1} + \psi_s s_t + \psi_x x_{it} + \psi_{\overline{x}} \overline{x}_{it}, \tag{2}
$$

where $\phi \equiv \{\phi_a, \phi_s, \phi_x\}$ and $\psi \equiv \{\psi_0, \psi_a, \psi_s, \psi_x, \psi_{\overline{x}}\}$ are vectors of constant coefficients to be determined and $\overline{x}_{it}$ is the average productivity innovation for the goods in the basket of consumer i,

$$
\overline{x}_{it} \equiv \int_{J_{it}} x_{jt} dj = \theta_t + v_{it}.
$$

I will explain in a moment why this variable enters (2). Summing (1) and (2) across agents, I obtain the aggregate price and quantity indexes

$$
p_t \quad = \quad \phi_a a_{t-1} + \phi_\theta \theta_t + \phi_s e_t, \tag{3}
$$

$$
c_t \quad = \quad \psi_0 + \psi_a a_{t-1} + \psi_\theta \theta_t + \psi_s e_t, \tag{4}
$$

where $\phi_\theta \equiv \phi_s + \phi_x$ and $\psi_\theta \equiv \psi_s + \psi_x + \psi_{\overline{x}}$.

In the rest of this section, I first characterize the optimal behavior of an individual household, assuming that the all other households follow (1) and (2). Then, I introduce a linear

monetary policy rule, and show that, under that rule, (1) and (2) form a rational expectations equilibrium.

3.1 Optimal consumption and prices

As useful preliminary steps, let me derive the appropriate price index for household i and the demand curve for good i. Individual optimization implies that, given C_{it}, the consumption of good j by consumer i is

$$C_{ijt} = \left(\frac{P_{jt}}{\overline{P}_{it}} \right)^{-\sigma} C_{it}, \tag{5}$$

where $\overline{P}_{it}$ is the price index

$$\overline{P}_{it} \equiv \left(\int_{j \in J_{it}} P_{jt}^{1-\sigma} dj \right)^{\frac{1}{1-\sigma}}.$$

The assumptions made on consumption baskets imply that the productivity innovations x_{jt} for the goods in J_{it} are normally distributed with mean $\overline{x}_{it}$ and variance $\sigma_{\epsilon|v}^2$. Using my conjecture (1) for individual prices, I then obtain an exact expression for the log of the price index of consumer i,[15]

$$\overline{p}_{it} = \kappa_{\overline{p}} + p_t + \phi_x v_{it}. \tag{6}$$

Consider now the demand curve for good i. Let $\tilde{J}_{it}$ denote the set of consumers buying good i at time t. Aggregating their demand, gives $Y_{it} = D_{it} P_{it}^{-\sigma}$, where D_{it} is the demand index

$$D_{it} \equiv \int_{j \in \tilde{J}_{it}} \overline{P}_{jt}^{\sigma} C_{jt} dj.$$

Also for the demand index D_{it}, I can use my assumptions on consumption baskets and conjectures (1) and (2) to obtain an exact linear expression

$$d_{it} = \kappa_d + c_t + \sigma p_t + (\psi_{\overline{x}} + \sigma \phi_x) \chi \epsilon_{it}. \tag{7}$$

Using these expressions, I can then derive the household's first-order conditions for P_{it} and C_{it}. The first takes the form

$$p_{it} = \kappa_p + \mathbb{E}_{i,(t,I)} \left[\overline{p}_{it} + \gamma c_{it} + \eta n_{it} \right] - a_{it}, \tag{8}$$

where $\mathbb{E}_{i,(t,I)} [.]$ denotes the expectation of household i at date (t, I). The labor effort n_{it} is determined by the technological constraint $n_{it} = y_{it} - a_{it}$, and the output y_{it} by the demand relation derived above, $y_{it} = d_{it} - \sigma p_{it}$. The expression on the right-hand side of (8) captures

---

[15]This expression and expressions (7)-(9) below, are derived formally in the proof of Proposition 1, in the appendix, where I also derive the constant terms $\kappa_{\overline{p}}$, κ_d, κ_p, and κ_c

the expected nominal marginal cost for producer i. This depends positively on the price of the consumption basket of consumer i, $\bar{p}_{it}$, and on the marginal rate of substitution between consumption and leisure, $\gamma c_{it} + \eta n_{it}$, and it depends negatively on the productivity a_{it}. To compute the expectation in (8), notice that all the relevant information at (t, I) is summarized by a_{t-1}, s_t and x_{it}, so $\mathbb{E}_{i,(t,II)}[.]$ can be replaced by $\mathbb{E}[.|a_{t-1}, s_t, x_{it}]$.

The optimality condition for C_{it} takes the form

$$c_{it} = \kappa_c + \mathbb{E}_{i,(t,II)}[c_{it+1}] - \gamma^{-1}\left(r_t - \mathbb{E}_{i,(t,II)}[\bar{p}_{it+1}] + \bar{p}_{it}\right), \qquad (9)$$

where $\mathbb{E}_{i,(t,II)}[.]$ denotes the expectation of household i at date (t, II). Apart from the fact that expectations and price indexes are consumer-specific, this is a standard consumer's Euler equation: current consumption depends positively on future expected consumption and negatively on the expected real interest rate. To compute the expectations in (9), notice that the consumer can observe the price vector $\{p_{jt}\}_{j \in J_{it}}$. However, if all producers follow (1), these prices are normally distributed with mean $\phi_a a_{t-1} + \phi_x \bar{x}_{it} + \phi_s s_t$ and variance $\phi_x^2 \sigma_{\epsilon|v}^2$. Given that the consumer already knows a_{t-1} and s_t, he can back out $\phi_x \bar{x}_{it}$ from the mean of this distribution and this is a sufficient statistic for all the information on θ_t contained in the observed prices. Therefore, $\mathbb{E}_{i,(t,II)}[.]$ can be replaced by $\mathbb{E}[.|a_{t-1}, s_t, x_{it}, \phi_x \bar{x}_{it}]$. This result is summarized in a lemma.

Lemma 1 *If prices are given by (1), then the information of consumer i regarding the current shock θ_t is summarized by the three independent signals s_t, x_{it} and $\phi_x \bar{x}_{it}$.*

This confirms my initial conjecture that the individual consumption policy (2) is a linear function of a_{t-1}, s_t, x_{it}, and $\bar{x}_{it}$.

3.2 Policy rule and equilibrium

To find an equilibrium, I substitute (1) and (2) in the optimality conditions (8) and (9), and obtain a system of equations in ϕ and ψ.[16] This system of equations does not determine ϕ and ψ uniquely. In particular, for any choice of the parameter ϕ_a in $\mathbb{R}$, there is a unique pair $\{\phi, \psi\}$ which is compatible with individual optimality. To complete the equilibrium characterization and pin down ϕ_a, I need to define a monetary policy rule.

Consider an interest rate rule which targets aggregate nominal spending. The nominal interest rate is set to

$$r_t = \xi_0 + \xi_a a_{t-1} + \xi_m (m_{t-1} - \hat{m}_{t-1}), \qquad (10)$$

---

[16]See (32)-(39) in the appendix.

where $m_t \equiv p_t + c_t$ is an index of aggregate nominal spending and $\hat{m}_t$ is the central bank's target

$$\hat{m}_t = \mu_0 + \mu_a a_{t-1} + \mu_\theta \theta_t + \mu_e e_t. \tag{11}$$

The parameters $\xi = \{\xi_0, \xi_a, \xi_m\}$ and $\mu = \{\mu_a, \mu_\theta, \mu_e\}$ are chosen by the monetary authority.

The central bank's behavior can be described as follows. At the beginning of period t, the monetary authority observes a_{t-1} and announces its current target $\hat{m}_t$ for nominal spending. The target $\hat{m}_t$ has a forecastable, backward-looking component $\mu_a a_{t-1}$, and a state-contingent part which is allowed to respond to the current shocks θ_t and e_t. During trading, each agent i sets his price and consumption responding to the variables in his information set. At the beginning of period $t + 1$, the central bank observes the realized level of nominal spending m_t and the realized shocks θ_t and e_t. If m_t deviated from target in period t, in the next period the nominal interest rate is adjusted according to (10).

Given this policy rule, I can complete the equilibrium characterization and prove the existence of stationary linear equilibria. In particular, the next proposition shows that the choice of μ_a by the monetary authority pins down ϕ_a in the system of equations described above, and thus the equilibrium coefficients ϕ and ψ. In the proposition, I exclude one possible value for μ_a, denoted by μ_a^0, which corresponds to the pathological case where the equilibrium construction would give $\phi_x = 0$. This case is discussed in the appendix.

Proposition 1 *For each $\mu_a \in \mathbb{R} / \{\mu_a^0\}$ there is a pair $\{\phi, \psi\}$ and a vector $\{\xi_0, \xi_a, \mu_0, \mu_\theta, \mu_e\}$ such that the prices and consumption levels in (1)-(2) form a rational expectations equilibrium under the policy rule (10)-(11), for any value of $\xi_m \in \mathbb{R}$. If $\xi_m > 1$ the equilibrium is locally determinate. The value of ψ_a is independent of the policy rule and equal to*

$$\psi_a = \frac{1+\eta}{\gamma+\eta}\rho.$$

In equilibrium, monetary policy always achieves its nominal spending target, that is, $m_t = \hat{m}_t$, and the nominal interest rate is equal to

$$r_t = \xi_0 - (\mu_a + (\gamma - 1)\psi_a)(1 - \rho)a_{t-1}.$$

4 The effects of monetary policy

Let me turn now to the effects of different monetary policy rules on the real equilibrium allocation. By Proposition 1, the choice of the policy rule is summarized by the parameter

μ_a, so, from now on, I will simply refer to the "policy rule μ_a." Proposition 1 shows that in equilibrium $m_t = \hat{m}_t$ and the central bank always achieves its desired target for nominal output. In particular, by choosing μ_a the central bank determines the response of nominal output to *past* realizations of aggregate productivity. This is done by inducing price setters to adjust their nominal prices in equilibrium. Since a_{t-1} is common knowledge, price setters simply coordinate on setting prices proportionally to $\exp\{\mu_a a_{t-1}\}$.[17]

The first question raised in the introduction can now be stated in formal terms. How does the choice of μ_a affects the equilibrium response of aggregate activity to fundamental and noise shocks, that is, the coefficients ψ_θ and ψ_s in (4)? More generally, how does the choice of μ_a affects the vectors ϕ and ψ, which determine the cross-sectional allocation of goods and labor effort across households? The rest of this section addresses these questions.

4.1 Full information

Let me begin with the case where households have full information on the aggregate shock θ_t. This happens when s_t is a noiseless signal, $\sigma_e^2 = 0$. In this case, households can perfectly forecast current aggregate prices and consumption, p_t and c_t, by observing a_{t-1} and s_t. Using (6) and (7) to substitute for $\bar{p}_{it}$ and d_{it} in the optimal pricing condition (8), and taking the expectation $\mathbb{E}[.|a_{t-1}, s_t]$ on both sides, gives

$$p_t = (\kappa_p + \kappa_{\bar{p}} + \eta \kappa_d) + p_t + \gamma c_t + \eta (c_t - a_t) - a_t.$$

This implies that aggregate consumption is

$$c_t^{fi} = \psi_0 + \frac{1+\eta}{\gamma+\eta} a_t, \tag{12}$$

that is $\psi_\theta = (1+\eta)/(\gamma+\eta)$. In the next proposition, I show that also ψ_0 and the other coefficients which determine the real equilibrium allocation, are uniquely determined and independent of μ_a. This is a baseline neutrality result: under full information the real equilibrium allocation is independent of the monetary policy rule.[18]

Proposition 2 *If households have full information on θ_t, the allocation of consumption goods and labor effort in all stationary linear equilibria is independent of the monetary policy rule μ_a.*

---

[17]The response of real output to a_{t-1}, instead, is independent of the policy rule, as shown in Proposition 1.

[18]See McCallum (1979) for an early neutrality result in a model with pre-set prices.

4.2 Imperfect information: a special case

Let me turn now to the case of imperfect information on θ_t, which arises when σ_e^2 is positive. In this case, the choice of μ_a affects equilibrium prices and quantities. To understand how monetary policy operates, it is useful to start from a special case.

Consider the case where the intertemporal elasticity of substitution γ is 1, the disutility of effort is linear, $\eta = 0$, and productivity is a random walk, $\rho = 1$. In this case, the consumer's Euler equation can be rewritten as[19]

$$\overline{p}_{it} + c_{it} = \mathbb{E}_{i,(t,II)} \left[\overline{p}_{it+1} + c_{it+1} \right],$$

that is, nominal spending is a random walk. Under the nominal output target (11), the forecastable part of future nominal spending is equal to

$$\mathbb{E}_{i,(t,II)} \left[p_{t+1} + c_{t+1} \right] = \mu_0 + \mu_a \mathbb{E}_{i,(t,II)} \left[a_t \right].$$

Moreover, assume that χ is zero, so that all the consumers consume all the goods.[20] Then, as I will check below, the consumers can perfectly infer the value of θ_t from the observed values of p_t and s_t, so $\mathbb{E}_{i,(t,II)} \left[a_t \right] = a_t$. Putting together these results and using the fact that $\mu_0 = \psi_0$,[21] it follows that all consumers choose the same consumption

$$c_t = \psi_0 + \mu_a a_t - p_t. \tag{13}$$

To complete the equilibrium characterization, let me turn to price setting. Households still have imperfect information when they set prices, since they only observe s_t and x_{it} in $(t,0)$. Given that $\eta = 0$ and substituting the optimal consumption derived above, the optimality condition for prices (8) boils down to[22]

$$p_{it} = \mu_a \mathbb{E} \left[a_t | s_t, x_{it} \right] - a_{it}.$$

The expectation of a_t can be written as $\mathbb{E} \left[a_t | s_t, x_{it} \right] = a_{t-1} + \beta_s s_t + \beta_x x_{it}$, where β_s and β_x are positive inference coefficients which satisfy $\beta_s + \beta_x < 1$.[23] Aggregating across producers and

---

[19]To obtain this expression from (9), notice that, when $\rho = 1$, r_t is constant and equal to ξ_0, by Proposition 1. Moreover, in the proof of the same proposition I show that $\xi_0 = \gamma \kappa_c$, so the terms κ_c and $\gamma^{-1} r_t$ in (9) cancel out.

[20]This shows that my basic positive results can be derived without introducing heterogeneous consumption baskets. However, Proposition 6 below shows that heterogeneous consumption baskets are necessary to obtain interesting welfare trade-offs.

[21]See equation (42) in the appendix.

[22]Equation (32) in the appendix shows that $\kappa_p + \kappa_{\overline{p}} + \psi_0 = 0$ when $\gamma = 1$ and $\eta = 0$, so there is no constant term in this expression.

[23]See (30) in the appendix.

rearranging gives

$$p_t = (\mu_a - 1) a_{t-1} + \mu_a \beta_s s_t + (\mu_a \beta_x - 1) \theta_t. \tag{14}$$

This shows that observing p_t and s_t fully reveals θ_t, except in the knife-edge case where $\mu_a = 1/\beta_x$. The following discussion disregards this case.

Combining (13) and (14), I get the equilibrium value of aggregate consumption

$$c_t = \psi_0 + a_{t-1} + (1 + \mu_a (1 - \beta_s - \beta_x)) \theta_t - \mu_a \beta_s e_t, \tag{15}$$

that is, in this economy $\psi_\theta = 1 + \mu_a (1 - \beta_s - \beta_x)$ and $\psi_s = -\mu_a \beta_s$. The choice of the policy rule μ_a is no longer neutral. In particular, increasing μ_a increases the output response to fundamental shocks and reduces its response to noise shocks. To interpret this result, it is useful to look separately at consumers' and price setters' behavior. If the monetary authority increases μ_a, (13) shows that, for a given price level p_t, the response of consumer spending to θ_t increases. A larger value of μ_a implies that, if a positive productivity shock materializes at date t, the central bank will target a higher level of nominal spending in the following period. This, given the consumers' forward looking behavior, translates into higher nominal spending in the current period. On the other hand, the consumers' response to a noise shock e_t, for given p_t, is zero, irrespective of μ_a, given that consumers have perfect information on a_t and place zero weight on the signal s_t.

Consider now the response of price setters. If the monetary authority chooses a larger value for μ_a, price setters tend to set higher prices following a positive productivity shock θ_t as they observe a positive s_t and, on average, a positive x_{it}, and thus expect higher consumer spending. However, due to imperfect information, they tend to underestimate the spending increase. Therefore, their price increase is not enough to undo the direct effect on consumers' demand, and, on net, real consumption goes up. Formally, this is captured by

$$\frac{\partial \psi_\theta}{\partial \mu_a} = 1 - \beta_s - \beta_x > 0.$$

On the other hand, following a positive noise shocks, price setters mistakenly expect an increase in demand, following their observation of a positive s_t, and tend to raise prices. Consumers' demand, however, is unchanged. The net effect is a reduction in output, that is,

$$\frac{\partial \psi_s}{\partial \mu_a} = -\beta_s < 0.$$

A further result which is easily established, is that monetary policy can achieve the full information benchmark for aggregate activity, by picking the right μ_a. When $\gamma = 1$, (12) shows

17

that aggregate consumption under full information is $c_t^{fi} = \psi_0 + a_t$. Moreover, (15) shows that the central bank can achieve the same aggregate consumption path by setting $\mu_a = 0$. That is, there is a value of μ_a which, at the same time, achieves $\psi_\theta = 1$ and $\psi_s = 0$.[24] This may seem the outcome of the special assumptions made here and, in particular, of the fact that consumers have full information. In fact, it is a result that holds more generally, as I will show below.

4.3 Imperfect information: general results

The following two propositions extend the results derived above to the general case. First, I extend the non-neutrality result and show that increasing μ_a increases the response of aggregate consumption to fundamental shocks, ψ_θ, and reduces its response to noise shocks, ψ_s.

Proposition 3 *If households have imperfect information on θ, the real equilibrium allocation depends on μ_a. The equilibrium coefficients $\{\phi, \psi\}$ are linear functions of the policy parameter μ_a, with*

$$\frac{\partial \psi_\theta}{\partial \mu_a} > 0, \qquad \frac{\partial \psi_s}{\partial \mu_a} < 0, \qquad \frac{\partial \phi_x}{\partial \mu_a} > 0.$$

The simple example presented above helps to build the intuition for the general result. Now, consumers no longer have perfect information on θ_t and form expectations based on the imperfect signals s_t, x_{it}, and $\overline{x}_{it}$. Consider two hypothetical scenarios. In case A, there is a positive fundamental shock, both s_t and θ_t are positive, and the typical consumer receives both a positive public signal and positive private signals $x_{it}, \overline{x}_{it} > 0$. In case B, there is a positive noise shock e_t, s_t is positive, θ_t is zero, and the typical consumer receives a positive public signal and neutral private signals $x_{it} = \overline{x}_{it} = 0$.

Suppose that μ_a increases. In both scenarios, consumers expect an increase in nominal output at $t+1$ and higher future prices. The increase in $\mathbb{E}_{i,(t,II)}\left[\overline{p}_{it+1}\right]$ on the right-hand side of (9) leads to an increase in consumer demand at time t, for given prices $\overline{p}_{it}$. Under both scenarios, the producers forecast a demand increase and tend to raise current prices. However, in case A the producers tend to underestimate the increase in $\mathbb{E}_{i,(t,II)}\left[\overline{p}_{it+1}\right]$ which is driving up demand, while in case B they tend to overestimate it. The reason for this is that, in case A, the typical consumer is using both public and private information, while, in case B, he is only using public information. In the first case, the producers can perfectly forecast the

demand increase associated to a positive s_t, but can only partially foresee the demand increase due to the private signals. In the second case, they think that θ_t is positive and erroneously forecast a demand increase driven by both public and private signals, while, in the aggregate, only the public signal is operating. The underreaction of current prices in case A means that $\mathbb{E}_{i,(t,II)}\left[\bar{p}_{it+1}\right] - \bar{p}_{it}$ tends to increase. The overreaction of current prices in case B leads to the opposite result. Therefore, consumers' expected inflation goes up in case A and down in case B, leading to an increase in real consumption in the first case and to a reduction in the second.

There are three crucial ingredients behind this result: dispersed information, forward-looking agents, and a backward-looking policy based on the observed realization of past shocks. The different information sets of consumers and price setters play a central role in the mechanism described above. The presence of forward-looking agents is clearly needed so that announcements about future policy affect current behavior. The backward-looking policy rule works because it is based on past shock realizations which were not observed by the agents at the time they hit. To clarify this point, notice that the results above would disappear if the central bank based its intervention at $t+1$ on any variable that is common knowledge at date t, for example on s_t. Suppose, for example, that the backward-looking component of the nominal spending target (11) took the form $\mu_s s_{t-1}$ instead of $\mu_a a_{t-1}$. Then, any adjustment in the backward-looking parameter μ_s would lead to identical and fully-offsetting effects on current prices and expected future prices, with no effects on the real allocation.[25]

The next proposition, extends the second result obtained in 4.2. There exists a policy rule μ_a which achieves full aggregate stabilization, that is, an equilibrium where aggregate activity perfectly tracks the full information benchmark derived in 4.1.

Proposition 4 *There exists a monetary policy rule μ_a^{fs} which, together with the appropriate subsidy τ^{fs}, achieves full aggregate stabilization, that is, an equilibrium with $c_t = c_t^{fi}$.*

To achieve the full information benchmark for c_t, the central bank has to eliminate the effect of noise shocks, setting ψ_s equal to zero, and ensure, at the same time, that the output response to the fundamental shocks ψ_θ is equal to $(1+\eta)/(\gamma+\eta)$. Given that, by Proposition 3, there is a linear relation between μ_a and ψ_s and $\partial \psi_s/\partial \mu_a \neq 0$, it is always possible to find a μ_a such that ψ_s is equal to zero.[26] The surprising result is that the value of μ_a that sets ψ_s

---

[25]On the other hand, it is not crucial that the central bank can observe θ_t perfectly in period $t+1$. In fact, it is possible to generalize the result above to the case where the central bank receives noisy information about θ_t at $t+1$, as long as this information is not in the agents' information sets at time t.

[26]In the proof of Proposition 4, I check that μ_a^{fs} is different from μ_a^0.

to zero does, at the same time, set ψ_θ equal to $(1 + \eta) / (\gamma + \eta)$. This result is an immediate corollary of the following lemma.

Lemma 2 *In any linear equilibrium, ψ_θ and ψ_s satisfy*

$$\psi_\theta \sigma_\theta^2 + \psi_s \sigma_e^2 = \frac{1 + \eta}{\gamma + \eta} \sigma_\theta^2. \tag{16}$$

Proof. Starting from the optimal pricing condition (8), take the conditional expectation $\mathbb{E}\left[.|a_{t-1}, s_t\right]$ on both sides and use the law of iterated expectations, to obtain

$$\mathbb{E}\left[p_{it}|a_{t-1}, s_t\right] = \kappa_p + \mathbb{E}\left[\bar{p}_{it} + \gamma c_{it} + \eta d_{it} - \eta \sigma p_{it} - (1 + \eta) a_{it}|a_{t-1}, s_t\right].$$

I can then substitute for $\bar{p}_{it}$ and d_{it} using (6) and (7), and exploit the fact that all idiosyncratic shocks have zero mean ex ante. Then, optimal pricing implies that c_t satisfies

$$\mathbb{E}\left[c_t - \psi_0 - \frac{1 + \eta}{\gamma + \eta} a_t|a_{t-1}, s_t\right] = 0. \tag{17}$$

Using (4) to substitute for c_t and using $\mathbb{E}\left[a_t|a_{t-1}, s_t\right] = \rho a_{t-1} + \mathbb{E}\left[\theta_t|s_t\right]$ and $\psi_a = \rho (1 + \eta) / (\gamma + \eta)$, this equation boils down to

$$\mathbb{E}\left[\psi_\theta \theta_t + \psi_s e_t|s_t\right] = \frac{1 + \eta}{\gamma + \eta} \mathbb{E}\left[\theta_t|s_t\right].$$

Substituting for $\mathbb{E}\left[\theta_t|s_t\right] = (\sigma_\theta^2/(\sigma_\theta^2 + \sigma_e^2)) s_t$ and $\mathbb{E}\left[e_t|s_t\right] = (\sigma_e^2/(\sigma_\theta^2 + \sigma_e^2)) s_t$, gives the linear restriction (16). ∎

The point of this lemma is that the output responses to the two shocks are tied together by the fact that ex ante, conditional on a_{t-1} and s_t, price setters must expect their prices to be in line with their nominal marginal costs. This implies that aggregate consumption and output are expected to be, on average, at their full information level, as shown by (17). In turns, this implies that when ψ_θ increases ψ_s must decrease, otherwise the sensitivity of expected output to s_t would be inconsistent with optimal pricing. This also implies that, if aggregate consumption moves one for one with $((1 + \eta) / (\gamma + \eta)) \theta_t$, then the effect of the signal s_t (and thus of the noise e_t) must be zero.

To conclude this section, let me remark that the choice of μ_a also affect the sensitivity of individual consumption and prices to idiosyncratic shocks. That is, the policy rule has implications not only for aggregate responses, but also for the cross-sectional distribution of consumption and relative prices. This observation will turn out to be crucial in evaluating the welfare consequences of different monetary rules.

5 Optimal monetary policy

5.1 Welfare

I now turn to the welfare analysis and to the characterization of optimal monetary policy. In a linear equilibrium, the consumption of good j by consumer i is given by

$$C_{ijt} = \exp\left\{\psi_0 + \sigma\kappa_{\overline{p}} + \psi_a a_{t-1} + \psi_s s_t + \psi_x x_{it} + \psi_{\overline{x}}\overline{x}_{it} - \sigma\phi_x\left(x_{jt} - \overline{x}_{it}\right)\right\},$$

which follows substituting the equilibrium price and consumption decisions, (1) and (2), and the price index (6), in equation (5). The equilibrium labor effort of household i is then given by the market clearing condition

$$N_{it} = \frac{\int_{j\in\tilde{J}_{it}} C_{jit}dj}{A_{it}}. \tag{18}$$

Using these expressions and exploiting the normality of the shocks, it is then possible to compute the value of the expected utility of a representative household at the beginning of period 0, as shown in the following lemma.

Lemma 3 *Take any monetary policy $\mu_a \in \mathbb{R}/\mu_a^0$ and consider the associated linear equilibrium, characterized by the coefficients $\{\phi, \psi\}$. Assume that the subsidy τ is chosen optimally. Then the expected utility of a representative household is given by*

$$\mathbb{E}\left[\sum_{t=0}^{\infty}\beta^t U\left(C_{it}, N_{it}\right)\right] = \frac{1}{1-\gamma}W_0 e^{(1-\gamma)\frac{1+\eta}{\gamma+\eta}w(\mu_a)},$$

if $\gamma \neq 1$, and by

$$\mathbb{E}\left[\sum_{t=0}^{\infty}\beta^t U\left(C_{it}, N_{it}\right)\right] = w_0 + \frac{1}{1-\beta}w(\mu_a),$$

if $\gamma = 1$. W_0 and w_0 are constant terms independent of μ_a, W_0 is positive, and $w(.)$ is a known quadratic function, which depends on the model's parameters.

The function $w(\mu_a)$ can be used to evaluate the welfare effects of different policies in terms of equivalent consumption changes. Suppose I want to compare two policies μ_a' and μ_a'' by finding the Δ such that

$$\mathbb{E}\left[\sum_{t=0}^{\infty}\beta^t U\left((1+\Delta)C_{it}', N_{it}'\right)\right] = \mathbb{E}\left[\sum_{t=0}^{\infty}\beta^t U\left(C_{it}'', N_{it}''\right)\right],$$

where C_{it}', N_{it}' and C_{it}'', N_{it}'' are the equilibrium allocations arising under the two policies. The value of Δ represents the proportional increase in lifetime consumption which is equivalent, in

welfare terms, to a policy change from μ'_a to μ''_a. The following lemma shows that $w\left(\mu''_a\right) - w\left(\mu'_a\right)$ provides a first-order approximation for Δ.[27]

Lemma 4 *Let $\Delta\left(\mu'_a, \mu''_a\right)$ be the welfare change associated to the policy change from μ'_a to μ''_a, measured in terms of equivalent proportional change in lifetime consumption. The function $\Delta\left(.,.\right)$ satisfies*

$$\left.\frac{d\Delta\left(\mu_a, \mu_a + u\right)}{du}\right|_{u=0} = w'\left(\mu_a\right).$$

5.2 Constrained efficiency

To characterize optimal monetary policy, I will show that it achieves an appropriately defined social optimum. I consider a planner who can choose the consumption and labor effort levels C_{ijt} and N_{it} facing only two constraints: the resource constraint (18) and the informational constraint that C_{ijt} be measurable with respect to $a_{t-1}, s_t, x_{it}, \overline{x}_{it}$ and x_{jt}. This requires that, when selecting the consumption basket of consumer i at time t, the planner can only use the information that would be available to the consumer in the market economy. Specifically, I allow the planner to use the same information available to consumers in linear equilibria with $\phi_x \neq 0$. [28] An allocation that solves the planner problem is said to be "constrained efficient."

The crucial assumption here is that the planner can determine how consumers respond to various sources of information, but cannot intervene to change this information. This notion of constrained efficiency is developed and analyzed in a broad class of quadratic games in Angeletos and Pavan (2007a, 2007b). Here, it is possible to apply it in a general equilibrium environment, even though agents extract information from prices and prices are endogenous, because the matching environment is such that the information sets are essentially exogenous.

The following result shows that, with the right choice of μ_a and τ, the equilibrium found in Proposition 1 is constrained efficient.

Proposition 5 *There exist a monetary policy μ^*_a and a subsidy τ^* such that the associated stationary linear equilibrium is constrained efficient.*

This proposition shows that a simple backward-looking policy rule, which is only contingent on *aggregate* variables, is sufficient to induce agents to make the best possible use of the public

---

[27]I am grateful to Kjetil Storesletten for suggesting this result.

[28]In the proof of Proposition 5, I show that $\phi_x \neq 0$ in the best linear equilibrium. When $\phi_x \neq 0$, the consumer can recover $\overline{x}_{it}$ and x_{jt} for all $j \in J_{it}$ from $\{P_{jt}\}_{j \in J_{it}}$. Therefore, I could allow each C_{ijt} to be a function of the entire distribution $\{x_{jt}\}_{j \in J_{it}}$. Making it just a function of $\overline{x}_{it}$ and x_{jt} simplifies the analysis and is without loss of generality.

and private information available to them.

The resource constraint and the measurability constraint are satisfied by all the linear equilibrium allocations. Therefore, an immediate corollary of Proposition 5 is that μ_a^* is the optimal monetary policy which maximizes $w(\mu_a)$. However, the set of feasible allocations for the planner is larger than the set of linear equilibrium allocations, since in the planner's problem C_{ijt} is allowed to be any function, possibly non-linear, of $a_{t-1}, s_t, x_{it}, \bar{x}_{it}$ and x_{jt}, and there are no restrictions on the responses of C_{ijt} to these variables.[29]

5.3 Optimal accommodation of noise shocks

Having obtained a general characterization of optimal monetary policy, I can turn to more specific questions: what is the economy's response to the various shocks at the optimal monetary policy? In particular, is full aggregate stabilization optimal? That is, should monetary policy completely eliminate the aggregate disturbances due to noise shocks, setting $\psi_s = 0$? The next proposition shows that, typically, full aggregate stabilization is suboptimal.

Proposition 6 *Suppose there is imperfect information and $\eta > 0$, $\chi \in (0,1)$. If $\sigma\gamma > 1$ full aggregate stabilization is suboptimal and $\mu_a^* < \mu_a^{fs}$. At the optimal policy, aggregate consumption is less responsive to fundamental shocks than under full information and noise shocks have a positive effect on aggregate consumption,*

$$\psi_\theta^* < \frac{1+\eta}{\gamma+\eta}, \qquad \psi_s^* > 0.$$

If $\sigma\gamma < 1$ full aggregate stabilization is also suboptimal, but the opposite inequalities apply. Full stabilization is optimal if one of the following conditions hold: $\eta = 0$, $\chi = 0$, $\chi = 1$, $\sigma\gamma = 1$.

To interpret this result, I use the following expression for the welfare index $w(\mu_a)$ defined in Lemma 3,

$$
\begin{aligned}
w &= -\frac{1}{2}(\gamma + \eta)\,\mathbb{E}[(c_t - c_t^{fi})^2] + \\
&\quad + \frac{1}{2}(1-\gamma)\int_0^1 (c_{it} - c_t)^2\,di - \frac{1}{2}(1+\eta)\int_0^1 (n_{it} - n_t)^2\,di + (c_t - a_t - n_t), \quad (19)
\end{aligned}
$$

where n_t is the employment index $n_t \equiv \int_0^1 n_{it}di$. This expression is derived in the appendix. The first term in (19) captures the welfare effects of aggregate volatility. In particular, it shows that social welfare is negatively related to the volatility of the "output gap" measure

---

[29]In fact, it is possible to further generalize the result in Proposition 5, allowing the planner to use a general time-varying rule for C_{ijt}, conditional on all past shocks' realizations.

$c_t - c_t^{fi}$, which captures the distance between c_t and the full-information benchmark analyzed in Section 4.1. A policy of full aggregate stabilization maximizes this expression, setting it to zero. However, the remaining terms are also relevant to evaluate social welfare. Once they are taken into account, full aggregate stabilization is no longer desirable. These terms capture welfare effects associated to the cross-sectional allocation of consumption goods and labor effort, conditional on the aggregate shocks θ_t and e_t. Let me analyze them in order.

The second and third term in (19) capture the effect of the idiosyncratic variances of c_{it} and n_{it}. Since c_{it} and n_{it} are the logs of the original variables, these expressions capture both level effects and volatility effects. In particular, focusing on the first one, when the distribution of c_{it} is more dispersed, C_{it} is, on average, higher, given that

$$\mathbb{E}[C_{it}|a_{t-1}, \theta_t, e_t] = \exp\left\{ c_t + \frac{1}{2} \int_0^1 (c_{it} - c_t)^2 \, di \right\},$$

but C_{it} is also more volatile as

$$Var[C_{it}|a_{t-1}, \theta_t, e_t] = \exp\left\{ \frac{1}{2} \int_0^1 (c_{it} - c_t)^2 \, di \right\}.$$

This explains why the term $\int_0^1 (c_{it} - c_t)^2 \, di$ is multiplied by $(1 - \gamma)$. When the coefficient of relative risk aversion γ is greater than 1, agents care more about the volatility effect than about the level effect. In this case, an increase in the dispersion of c_{it} reduces consumers' expected utility. The opposite happens when γ is smaller than 1. A similar argument applies to the third term in (19), although there both the level and the volatility effects reduce expected utility, given that the utility function is convex in labor effort.

The last term, $c_t - a_t - n_t$, reflects the effect of monetary policy on the economy's average productivity *in consumption terms*. Due to the heterogeneity in consumption baskets, a given average level of labor effort, with given productivities, translates into different levels of the average consumption index c_t depending on the distribution of quantities across consumers and producers. The following expression is also derived in the appendix.

$$c_t - a_t - n_t = -\frac{1}{2} Var[c_{jt} + \sigma \bar{p}_{jt}|j \in \tilde{J}_{it}, a_{t-1}, \theta_t, e_t] + \frac{\sigma(\sigma - 1)}{2} Var[p_{jt}|j \in J_{it}, a_{t-1}, \theta_t, e_t]. \quad (20)$$

To interpret the first term, notice that $c_{jt} + \sigma \bar{p}_{jt}$ is the intercept in the log demand for good i by consumer j. A producer who faces more volatile log demand has on average to put in higher effort, to achieve the same average log output. To interpret the second term, notice that consumers like price dispersion in their consumption basket, given that when prices are more variable they can reallocate their expenditure from more expensive goods to cheaper ones.

Therefore, a given average effort by the producers translates into higher consumption indexes when relative prices are more dispersed. [30] Summing up, when the dispersion in demand is lower and the dispersion in prices is higher, a given average effort n_t generates a higher average consumption index c_t.

5.4 A numerical example

To illustrate the various welfare effects just described, I will use a numerical example. The parameters for the example are in Table 1. The coefficient of relative risk aversion γ is set to 1. The values for σ and η are chosen in the range of values used in the sticky-price literature. The values for the variances $\sigma_\theta^2, \sigma_\epsilon^2$ and σ_e^2 are set at 1. The variance of the sampling shocks σ_v^2 must then be in $[0, 1]$. I pick the intermediate value $\sigma_v^2 = 1/2$.

γ	1	η	2
σ	7		
σ_θ^2	1	σ_ϵ^2	1
σ_e^2	1	σ_v^2	0.5

Table 1: Parameters for the numerical example.

Figure 2 shows the relation between μ_a and total welfare w. Figure 3 illustrates how μ_a affects the various terms in (19). In particular, panel (a) plots the relation between μ_a and the first term in (19), capturing the negative effect of aggregate volatility. Not surprisingly, the maximum of this function is reached at the full-stabilization policy μ_a^{fs}, where it reaches zero. With $\gamma = 1$, the second term in (19) is always zero, so I leave it aside. Panel (b) shows the effects of monetary policy on the third term, the negative effect due to the dispersion in labor supply. Panels (c) and (d) show the effect on the "productivity" term $c_t - a_t - n_t$, which is further decomposed into two effects, using equation (20). In panel (c), I report the negative effect of the demand dispersion faced by a given producer, in panel (d), the positive effect of the price dispersion faced by a given consumer.

Figure 2 shows that the optimal monetary policy is to the left of the full-stabilization policy, which is consistent with Proposition 6, given that $\gamma\sigma > 1$. Figure 3 shows that the crucial effect behind this result is the effect on price dispersion in panel (d). When moving from μ_a^{fs} to μ_a^*, there are welfare losses both in terms of aggregate volatility and in terms of labor supply and demand dispersion, as shown in panels (a)-(c). But the welfare gain due to increased

---

[30]Since prices are expressed in logs, an increase in the volatility of p_{jt} has both a level and a volatility effect. Given that $\sigma > 1$, the second always dominates.
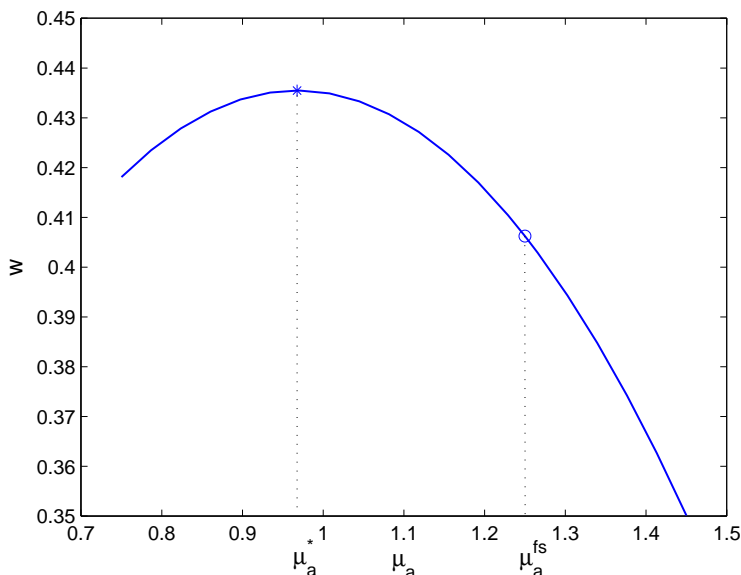
Figure 2: Welfare effects of monetary policy.

price dispersion more than compensate for these losses. Let me provide some intuition for the mechanism behind this picture.

In a neighborhood of μ_a^*, increasing μ_a has the effect of reducing price dispersion by reducing the value of $|\phi_x|$, which determines the response of individual prices to individual productivity shocks. [31] This reduction in price dispersion can be interpreted as follows. At the optimal equilibrium, producers with higher productivity must set lower prices, to induce consumers to buy more of their goods. This requires $\phi_x^* < 0$. By increasing μ_a, the central bank induces household consumption to be more responsive to the private productivity signal x_{it}.[32] This implies that a more productive household faces a lower marginal utility of consumption, and, at the price-setting stage, has a weaker incentive to lower the price of its good. Through this channel an increase in μ_a induces relative prices to be less responsive to differences in individual productivities, leading to a more compressed price distribution, as shown in panel (d).

Under the parametric assumptions made, this mechanism also leads to a reduction in labor supply dispersion and in demand dispersion, as shown in panels (b) and (c). In the economy considered, at the optimal policy, individual labor supply, n_{it}, is increasing in individual productivity, x_{it}. When relative prices become less responsive to individual productivity, the

---

[31]In the proof of Proposition 6, I show that $\phi_x^* < 0$ at μ_a^*. This, together with $\partial\phi_x/\partial\mu_a > 0$, from Proposition 3, implies that $|\phi_x|$ is decreasing in μ_a, in a neighborhood of μ_a^*.

[32]Expression (48) in the appendix shows that $\partial\psi_x/\partial\mu_a > 0$.
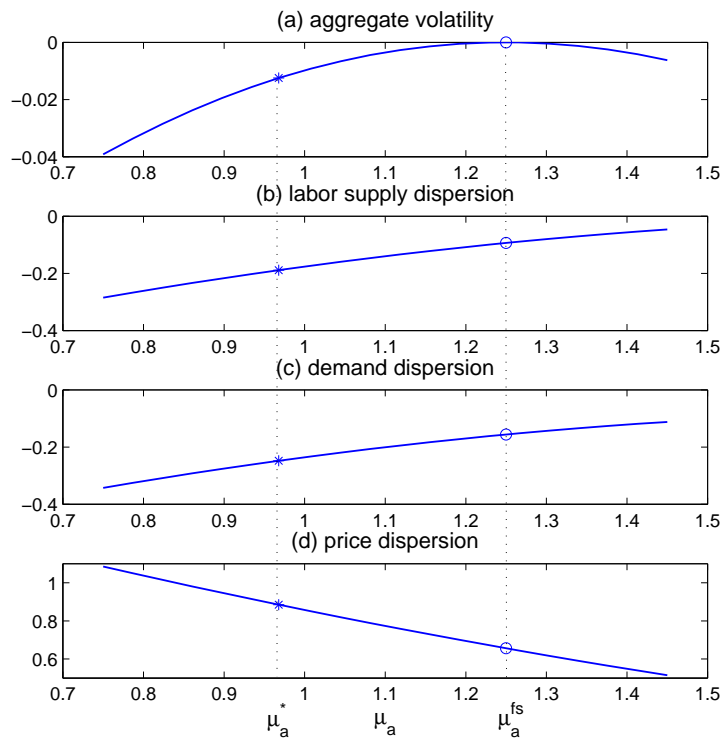
26

Figure 3: Welfare effects of monetary policy. Decomposition.

relation between x_{it} and n_{it} becomes flatter and this reduces the cross-sectional dispersion in n_{it}. Finally, an increase in μ_a leads to a compression in the distribution of demand indexes $c_{jt} + \sigma \bar{p}_{jt}$ faced by a given producer, because the price indexes $\bar{p}_{jt}$ become less dispersed.

Summing up, if the central bank wants to reach full stabilization it has to induce households to rely more heavily on their private productivity signals x_{it} when making their consumption decisions. By inducing them to concentrate on private signals the central bank can mute the effect of public noise. However, in doing so, the central bank reduces the sensitivity of individual prices to productivity, generating an inefficiently compressed distribution of relative prices.

Notice that in standard new Keynesian models, relative price dispersion is typically harmful for social welfare, because producers have identical productivities. Things are different here, because there is heterogeneity in individual productivities. This does not mean that more price dispersion is always better. An increase in price dispersion eventually leads to an excessive increase in the dispersion of labor supply (as captured by panel (b) of Figure 3).

Using Lemma 4, it is possible to quantify the welfare costs associated to suboptimal policies. Panel (a) in Figure 3 shows that, focusing purely on the aggregate output gap, the planner finds that going from μ_a^* to μ_a^{fs} leads to an approximate welfare gain of 1% in equivalent consumption. However, when all cross-sectional terms are considered, Figure 3 shows that, in fact, this policy change generates welfare losses of more than 2% in equivalent consumption. Although this is just an example, these numbers show that disregarding the cross-sectional implications of policy, in an environment with heterogeneity, can lead to serious welfare miscalculations.

5.5 The role of strategic complementarity in pricing

Proposition 6 identifies a set of special cases where full stabilization is optimal. An especially interesting case is when $\eta = 0$, that is, when utility is linear in labor effort. In this case, there is no strategic complementarity in price setting, under the nominal spending target (10)-(11). Substituting the consumer's Euler equation (9) on the right-hand side of the pricing condition (8), and using the law of iterated expectations, after some manipulations, I obtain

$$p_{it} = \left(\mu_a - \frac{\rho}{\gamma} \right) a_{t-1} + \left(\mu_a + \frac{\gamma - 1}{\gamma} \rho \right) \mathbb{E}_{i,(t,I)} \left[\theta_t \right] - x_{it}. \tag{21}$$

This shows that in this case prices only depend on the agents' first-order expectations regarding the fundamental shock θ_t. The analysis in Section 4.2 shows that even in this simple case an interesting form of non-neutrality is present, because of asymmetric information between price-

setters and consumers. However, in this case there is no significant interaction *among* price-setters. That is, the strategic complementarity emphasized in Woodford (2002) and Hellwig (2005) is completely muted.

When $\eta = 0$, the planner can reach the constrained efficient allocation by letting $\mu_a = -((\gamma - 1)/\gamma)\rho$.[33] This policy implies that the marginal utility of expenditure, which is proportional to $\exp\{-\bar{p}_{it} - \gamma c_{it}\}$, is perfectly equalized across households. At the same time, by (21), relative prices are perfectly proportional to individual productivities. When $\eta = 0$ these relative prices achieve an efficient cross-sectional allocation of labor effort. That is, in this economy there is no tension between aggregate efficiency and cross-sectional efficiency. Actually, it is possible to prove that, under the optimal monetary policy, this economy achieves the full-information first-best allocation.[34]

Once $\eta \neq 0$, producers must forecast their sales to set optimal prices and these sales depend on the prices set by other producers. Now the pricing decisions of the producers are fully interdependent. On the planner's front, when $\eta \neq 0$, it is necessary to use individual estimates of θ_t when setting efficient "shadow" prices. In this case, the optimal policy can no longer achieve the unconstrained first-best. Therefore, the presence of strategic complementarity in pricing is tightly connected to the presence of an interesting trade-off between aggregate and cross-sectional efficiency.

6 The welfare effects of public information

So far, I have assumed that the source of public information, the signal s_t, is exogenous and outside the control of the monetary authority. Suppose now that the central bank has some control on the information received by the private sector. For example, it can decide whether or not to systematically release some aggregate statistics, which would increase the precision of public information. What are the welfare effects of this decision? To address this question I look at the effects of changing the precision of the public signal, measured by $\pi_s \equiv 1/\sigma_e^2$, on total welfare. This exercise connects this paper to a growing literature on the welfare effects of public information.[35] I consider two possible versions of this exercise. First, I assume that when π_s changes the monetary policy rule μ_a is kept constant, while the subsidy τ is adjusted

---

[33]This can be derived from equation (72) in the appendix.

[34]To prove this, follow the same steps as in the proof of Proposition 5.1, but allow the consumption rule to be contingent on θ_t. Then, it is possible to show that the optimal allocation is supported by the equilibrium described above.

[35]See the references in footnote 3.

to its new optimal level. Second, I assume that for each value of π_s both μ_a and τ are chosen optimally.

Suppose the economy's parameters are those in Table 1 and suppose that μ_a is fixed at its optimal value for $\pi_s = 1$. Figure 4 shows the effect of changing π_s on welfare. The solid line represents total welfare, measured by w in (19).[36] The dashed line represents the relation between μ_a and the first component in (19), which captures the negative welfare effects of aggregate volatility. Let me begin by discussing this second relation. When the signal s_t is very imprecise agents disregard it and the coefficient ψ_s goes to zero. When the signal becomes more precise, agents rely more on the public signal. So, although the volatility of e_t is falling, the increase in ψ_s can lead to an increase in aggregate volatility. In the example considered, this happens whenever $\log(\pi_s)$ is smaller than 1.8. In that region, more precise public information has a destabilizing effect on the economy. Eventually, when the signal precision is sufficiently large, the economy converges towards the full information equilibrium and output gap volatility goes to zero. Therefore, there is a non-monotone relation between μ_a and aggregate volatility. However, this only captures the first piece of the welfare function (19). The solid line in Figure 4 shows that, when all the other pieces are taken into account, welfare is increasing everywhere in π_s.

To understand the relationship between the two graphs in Figure 4, notice that, when the public signal is very imprecise, agents have to use their own individual productivity to estimate aggregate productivity. This makes them underestimate the idiosyncratic component of productivity and leads to a compressed distribution of relative prices. An increase in the signal precision helps producers set relative prices that reflect more closely the underlying productivity differentials. The associated gain in allocative efficiency is always positive and more than compensates for the welfare losses due to higher aggregate volatility, in the region where $\log(\pi_s) \leq 1.8$.

The notion that more precise information about aggregate variables has important cross-sectional implications is also highlighted in Hellwig (2005). In that paper, agents face uncertainty about monetary policy shocks and there are no idiosyncratic productivity shocks. Therefore, the cross-sectional benefits of increased transparency are reflected in a *reduction* in price dispersion. Here, instead, more precise public information tends to generate higher price dispersion. However, the underlying principle is the same: in both cases a more precise public

---

[36]To improve readability, the values of w in the plot are normalized subtracting the value of w at $\pi_s = 0$, and I use a log scale for π_s.
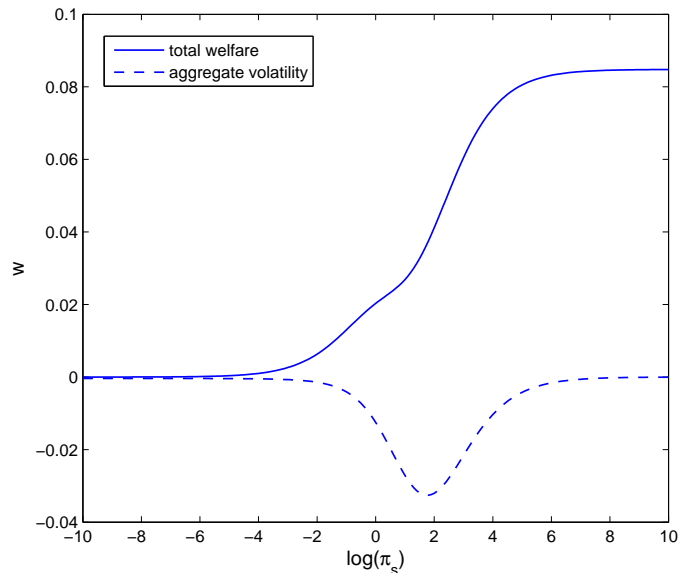
Figure 4: Welfare effects of changing the public signal precision, $\eta = 2$.

signal leads to relative prices more in line with productivity differentials.

Let me now consider a more intriguing example, where social welfare can be decreasing in π_s. In Figure 5, I plot the relation between π_s and w for an economy identical to the one above, except that the inverse Frisch elasticity of labor supply is set to a much higher value, $\eta = 5$. When η is larger, the costs of aggregate volatility are bigger, and, it is possible to have a non-monotone relationship between π_s and total welfare, as shown by the solid line in Figure 5. For example, when $\log(\pi)$ increases from 0 to 1, social welfare falls by about one-half percent in consumption equivalent terms. This result mirrors the result obtained by Morris and Shin (2002) in a simple quadratic game. As stressed by Angeletos and Pavan (2007a), their result depends crucially on the form of the agents' objective function and on the nature of their strategic interaction. In my model, the possibility of welfare-decreasing public information depends on the balance between aggregate and cross-sectional effects. When η is large the negative welfare effects of aggregate volatility become a dominant concern, and increases in public signal precision can be undesirable.

This result disappears when I allow the central bank to adjust the monetary policy rule to changes in the informational environment. In this case, more precise public information is unambiguously good for social welfare. This is illustrated by the dotted line in Figure 5, which shows the relation between π_s and w, when μ_a is chosen optimally. By Proposition
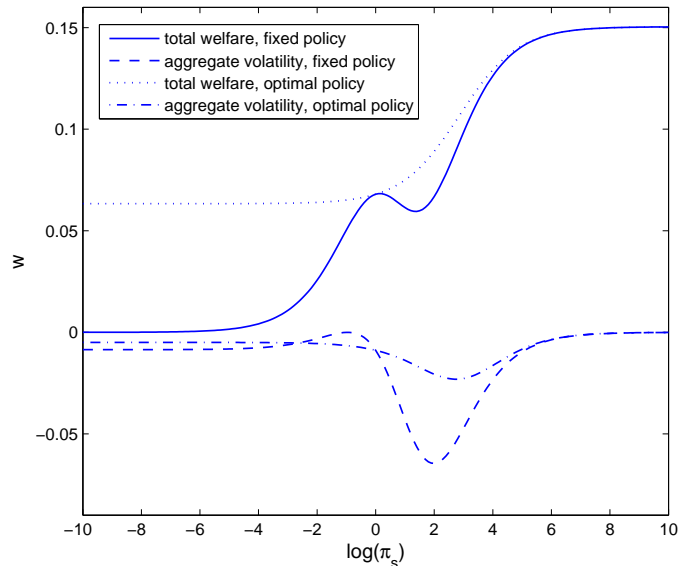
Figure 5: Welfare effects of changing the public signal precision, $\eta = 5$.

5, the optimal μ_a induces agents to use information in a socially optimal way. Therefore, at the optimal policy, better information always leads to higher social welfare. This provides a general equilibrium counterpart to the results in Angeletos and Pavan (2007a), who apply the same principle to quadratic coordination games. The following proposition summarizes.

Proposition 7 *If μ_a is kept fixed, an increase in π_s can lead to a welfare gain or to a welfare loss, depending on the model's parameters. If μ_a is chosen optimally for each π_s, an increase in π_s never leads to a welfare loss.*

7 Conclusions

In this paper, I have explored the role of monetary policy rules in an economy where information about macroeconomic fundamentals is dispersed across the economy. The emphasis has been on the ability of the policy rule to shape the economy's response to different shocks. In particular, the monetary authority is able to reduce the economy's response to noise shocks by manipulating agents' expectations about the real interest rate. The principle behind this result goes beyond the specific model used in this paper: by announcing that policy actions will respond to future information, the monetary authority can affect differently agents with different pieces of information. In this way, it can change the aggregate response to fundamental and noise shocks even if it has no informational advantage over the private sector. A second

general lesson that comes from the model is that, in the presence of heterogeneity and dispersed information, the policy maker will typically face a trade-off between aggregate efficiency and cross-sectional efficiency. Inducing agents to be more responsive to perfectly observed local information can lead to aggregate outcomes that are less sensitive to aggregate noise shocks, but it can also lead to a worse cross-sectional allocation.

The optimal policy rule used in this paper can be implemented both under commitment and under discretion. To offset an expansion driven by optimistic beliefs, the central bank announces that it will make the realized real interest rate higher if good fundamentals do not materialize. With flexible prices, this effect is achieved with a downward jump in the price level between t and $t+1$. Since a_t is common knowledge at time $t+1$, this jump only affects nominal variables, but has no consequences on the real allocation in that period. Therefore, the central bank has no incentive to deviate ex post from its announced policy. In economies with sluggish price adjustment, a similar effect could be obtained by a combination of a price level change and an increase in nominal interest rates. In that case, however, commitment problems are likely to arise, because both type of interventions have additional distortionary consequences ex post. The study of models where lack of commitment interferes with the central bank's ability to deal with informational shocks is an interesting area for future research.

A strong simplifying assumption in the model is that the only financial assets traded in subperiods (t, I) and (t, II) are non-state-contingent claims on dollars at $(t+1, 0)$. Introducing a richer set of traded financial assets would increase the number of price signals available to both consumers and the monetary authority. In a simple environment with only two aggregate shocks, this will easily lead to a fully revealing equilibrium. Therefore, to fruitfully extend the analysis in that direction requires the introduction of a larger number of shocks, which make financial prices noisy indicators of the economy's fundamentals.

Finally, in the model presented, the information sets of consumers, producers, and of the central bank, are independent of the monetary rule chosen. Morris and Shin (2005) have recently argued that stabilization policies may have adverse effects, if they reduce the informational content of prices. Here this concern does not arise, as the information in the price indexes observed by consumers is essentially independent of monetary policy. A natural extension of the model in this paper would be to introduce additional sources of noise in prices, so as to make their informational content endogenous and sensitive to policy.

8 Appendix

8.1 Random consumption baskets

At the beginning of each period, household i is assigned two random variables, ϵ_{it} and v_{it}, independently drawn from normal distributions with mean zero and variances, respectively, σ_ϵ^2 and σ_v^2. These variables are not observed by the household. The first random variable represents the idiosyncratic productivity shock, the second is the sampling shock that will determine the sample of firms visited by consumer i. Consumers and producers are then randomly matched so that the probability that a producer with shock ϵ_{jt} is matched to a consumer with shock v_{it} is given by the bivariate normal density $\phi(\epsilon_{it}, v_{jt})$, with covariance matrix

$$
\begin{bmatrix}
\sigma_\epsilon^2 & \sqrt{\chi}\sigma_v\sigma_\epsilon \\
. & \sigma_v^2
\end{bmatrix},
$$

where χ is a parameter in $[0,1]$. Since the variable v_{it} has no direct effect on payoffs, I can normalize its variance and set $\sigma_v^2 = \chi\sigma_\epsilon^2$. Let J_{it} denote the set of producers met by consumer i and $\tilde{J}_{it}$ the set of consumers met by producer i. Given the matching process above the following properties follow. The distribution $\{\epsilon_{jt} : j \in J_{it}\}$ is a normal $N(v_{it}, \sigma_{\epsilon|v}^2)$ with $\sigma_{\epsilon|v}^2 = (1-\chi)\sigma_\epsilon^2$. The distribution $\{v_{jt} : j \in \tilde{J}_{it}\}$ is a normal $N(\chi\epsilon_{it}, \sigma_{v|\epsilon}^2)$, with $\sigma_{v|\epsilon}^2 = \chi(1-\chi)\sigma_\epsilon^2$.

8.2 Proof of Proposition 1

The proof is split in steps. First, I derive price and demand indexes that apply in the linear equilibrium conjectured. Second, I use them to setup the individual optimization problem and derive necessary conditions for individual optimality. Third, I use these conditions to characterize a linear equilibrium. Fourth, I show how choosing μ_a uniquely pins down the coefficients $\{\phi, \psi\}$ and derive the remaining coefficients of the monetary policy rule that implements $\{\phi, \psi\}$. The proof of local determinacy is in the supplementary material.

8.2.1 Price and demand indexes

Lemma 5 *If individual prices and quantities are given by (1) and (2) then the price index for consumer i and the demand index for producer i are equal to (6) and (7) where $\kappa_{\overline{p}}$ and κ_d are constant terms equal to*

$$
\kappa_{\overline{p}} = \frac{1-\sigma}{2}\phi_x^2\sigma_{\epsilon|v}^2, \tag{22}
$$

$$
\kappa_d = \frac{1}{2}\psi_x^2\sigma_\epsilon^2 + \frac{1}{2}(\psi_{\overline{x}} + \sigma\phi_x)^2\sigma_{v|\epsilon}^2 + \sigma\frac{1-\sigma}{2}\phi_x^2\sigma_{\epsilon|v}^2. \tag{23}
$$

Proof. Recall that the shocks ϵ_{jt} for $j \in J_{it}$ have a normal distribution $N(v_{it}, \sigma_{\epsilon|v}^2)$. Then, given (1), the prices observed by consumer i are log-normally distributed, with mean $p_t + \phi_x v_{it}$ and variance $\phi_x^2\sigma_{\epsilon|v}^2$, therefore,

$$
\int_{j \in J_{it}} P_{jt}^{1-\sigma}dj = e^{(1-\sigma)(p_t + \phi_x v_{it}) + \frac{(1-\sigma)^2}{2}\phi_x^2\sigma_{\epsilon|v}^2}.
$$

Taking both sides to the power $1/(1-\sigma)$ gives the desired expression for $\overline{P}_{it}$, from which (6) and (22) follow immediately. Using this result and expression (2), the demand index for producer i can be written as

$$
D_{it} = \int_{j \in \tilde{J}_{it}} C_{jt}\overline{P}_{jt}^\sigma dj = e^{c_t + \sigma p_t}e^{\sigma\kappa_{\overline{p}}}\int_{j \in \tilde{J}_{it}} e^{\psi_x\epsilon_{jt} + \psi_{\overline{x}}v_{jt}}e^{\sigma\phi_x v_{jt}}dj.
$$

Recall that the distribution $\{v_{jt} : j \in \tilde{J}_{it}\}$ is a normal $N(\chi\epsilon_{it}, \sigma_{v|\epsilon}^2)$, and ϵ_{jt} and v_{jt} are independent. It follows that

$$
\int_{j \in \tilde{J}_{it}} e^{\psi_x\epsilon_{jt} + \psi_{\overline{x}}v_{jt} + \sigma\phi_x v_{jt}}dj = e^{\frac{1}{2}\psi_x^2\sigma_\epsilon^2 + (\psi_{\overline{x}} + \sigma\phi_x)\chi\epsilon_{it} + \frac{1}{2}(\psi_{\overline{x}} + \sigma\phi_x)^2\sigma_{v|\epsilon}^2}.
$$

Substituting in the previous expression gives (7) and (23). ∎

8.2.2 Individual optimization

Consider an individual household, who expects all other households to follow (1)-(2) and the central bank to follow (10)-(11). In period $(t,0)$, before all current shocks are realized, the household's expectations about the current and future path of prices, quantities and interest rates depend only on a_{t-1} and R_t. Moreover, the only relevant individual state variable is given by the household nominal balances B_{it}. Therefore, I can analyze the household's problem using the Bellman equation

$$V\left(B_{it}, a_{t-1}, R_t\right) = \max_{\substack{\{Z_{it+1}(.)\}, \{B_{it+1}(.)\}, \\ \{P(.,.)\}, \{C(.,.,.)\}}} \mathbb{E}_t\left[U\left(C_{it}, N_{it}\right) + \beta V\left(B_{it+1}, a_t, R_{t+1}\right)\right]$$

subject to the constraints

$$B_{it+1}\left(\omega_{it}\right) = R_t\left[B_{it} - \int q\left(\tilde{\omega}_{it}\right) Z_{it}\left(\tilde{\omega}_{it}\right) d\tilde{\omega}_{it} + (1+\tau) P_{it}Y_{it} - \overline{P}_{jt}C_{it} - T_t\right] + Z_{it}\left(\omega_{it}\right),$$

$$Y_{it} = D_{it}P_{it}^{-\sigma}, \quad Y_{it} = A_{it}N_{it}, \quad P_{it} = P\left(s_t, x_{it}\right), \quad C_{it} = C\left(s_t, x_{it}, \overline{x}_{it}\right),$$

and the law of motions for a_t and R_{t+1}. $\mathbb{E}_t\left[.\right]$ represents expectations formed at $(t,0)$ and, in the equilibrium conjectured, it can be replaced by $\mathbb{E}\left[.|a_{t-1}\right]$. This problem gives the following optimality conditions for prices and consumption

$$\mathbb{E}_{i,(t,I)}\left[(1+\tau)\overline{P}_{it}^{-1}C_{it}^{-\gamma}Y_{it} - \frac{\sigma}{\sigma-1}A_{it}^{-1}N_{it}^{\eta}P_{it}^{-1}Y_{it}\right] = 0, \tag{24}$$

$$\mathbb{E}_{i,(t,II)}\left[\overline{P}_{it}^{-1}C_{it}^{-\gamma} - \beta R_t\overline{P}_{it+1}^{-1}C_{it+1}^{-\gamma}\right] = 0, \tag{25}$$

where $\mathbb{E}_{i,(t,I)}\left[.\right]$ and $\mathbb{E}_{i,(t,II)}\left[.\right]$ denote the expectations of agent i at (t,I) and (t,II). Given the conjectured equilibrium and, given Lemma 1, they are equal to $\mathbb{E}\left[.|a_{t-1}, s_t, x_{it}\right]$ and $\mathbb{E}\left[.|a_{t-1}, s_t, x_{it}, \phi_x\overline{x}_{it}\right]$. By Lemma 5 all the random variables in the expressions above are log-normal, including the output and labor supply of producer i which are equal to $Y_{it} = D_{it}P_{it}^{-\sigma}$ and $N_{it} = A_{it}^{-1}Y_{it}$. Rearranging and substituting in (24) and (25) gives (8) and (9) in the text, which I report here for completeness,

$$p_{it} = \kappa_p + \eta\left(\mathbb{E}_{i,(t,I)}\left[d_{it}\right] - \sigma p_{it} - a_{it}\right) + \mathbb{E}_{i,(t,I)}\left[\overline{p}_{it} + \gamma c_{it}\right] - a_{it}, \tag{26}$$

$$\overline{p}_{it} + \gamma c_{it} = \gamma\kappa_c - r_t + \mathbb{E}_{i,(t,II)}\left[\overline{p}_{it+1} + \gamma c_{it+1}\right]. \tag{27}$$

The constant terms κ_p and κ_c are equal to

$$\kappa_p = H\left(\psi_s, \psi_x, \psi_{\overline{x}}, \phi_x\right) - \log\left(1+\tau\right), \tag{28}$$

$$\kappa_c = G\left(\psi_s, \psi_x, \psi_{\overline{x}}, \phi_x\right), \tag{29}$$

and H and G are known quadratic functions of $\psi_s, \psi_x, \psi_{\overline{x}}$ and ϕ_x.

8.2.3 Equilibrium characterization

To check for individual optimality, I will substitute the conjectures made for individual behavior, (1) and (2), in the optimality conditions (26) and (27) and obtain a set of restrictions on $\{\phi, \psi\}$. Notice that all the shocks are i.i.d. so the expected value of all future shocks is zero. Let me assume for now that $\phi_x \neq 0$, so that $\mathbb{E}\left[.|s_t, x_{it}, \overline{x}_{it}\right]$ can replace $\mathbb{E}\left[.|s_t, x_{it}, \phi_x\overline{x}_{it}\right]$. Let β_s, β_x and $\delta_s, \delta_x, \delta_{\overline{x}}$ be coefficients such that $\mathbb{E}\left[\theta_t|s_t, x_{it}\right] = \beta_s s_t + \beta_x x_{it}$ and $\mathbb{E}\left[\theta_t|s_t, x_{it}, \overline{x}_{it}\right] = \delta_s s_t + \delta_x x_{it} + \delta_{\overline{x}}\overline{x}_{it}$. Defining the precision parameters $\pi_\theta \equiv \left(\sigma_\theta^2\right)^{-1}, \pi_s \equiv \left(\sigma_e^2\right)^{-1}, \pi_x \equiv \left(\sigma_\epsilon^2\right)^{-1}$, and $\pi_{\overline{x}} \equiv \left(\sigma_v^2\right)^{-1}$, the coefficients β_s, β_x and $\delta_s, \delta_x, \delta_{\overline{x}}$ are

$$\beta_s = \frac{\pi_s}{\pi_\theta + \pi_s + \pi_x}, \quad \beta_x = \frac{\pi_x}{\pi_\theta + \pi_s + \pi_x}, \tag{30}$$

$$\delta_s = \frac{\pi_s}{\pi_\theta + \pi_s + \pi_x + \pi_{\overline{x}}}, \quad \delta_x = \frac{\pi_x}{\pi_\theta + \pi_s + \pi_x + \pi_{\overline{x}}}, \quad \delta_{\overline{x}} = \frac{\pi_{\overline{x}}}{\pi_\theta + \pi_s + \pi_x + \pi_{\overline{x}}}. \tag{31}$$

I use (6) and (7) to substitute for $\bar{p}_{it}$ and d_{it} in the optimality conditions (26) and (27), and then I use (1)-(4) to substitute for p_{it}, c_{it}, p_t and c_t. Finally, I use $\epsilon_{it} = x_{it} - \theta_{it}$ and $v_{it} = \bar{x}_{it} - \theta_{it}$, and I substitute for $\mathbb{E}[\theta_t|s_t, x_{it}]$ and $\mathbb{E}[\theta_t|s_t, x_{it}, \bar{x}_{it}]$. After these substitutions, (26) and (27) give two linear equations in $a_{t-1}, s_t, x_{it}, \bar{x}_{it}$. Matching the coefficients term by term and rearranging gives me the following set of equations.

$$(\gamma + \eta)\psi_0 = -(\kappa_p + \kappa_{\bar{p}} + \eta\kappa_d), \tag{32}$$

$$(\gamma + \eta)\psi_a = (1 + \eta)\rho, \tag{33}$$

$$r_t = \gamma\kappa_c - (\phi_a + \gamma\psi_a)(1 - \rho)a_{t-1}, \tag{34}$$

$$(1 + \eta\sigma)\phi_s = \eta(\psi_s + \sigma\phi_s + (\psi_x + \psi_{\bar{x}} + \sigma\phi_x)\beta_s - (\psi_{\bar{x}} + \sigma\phi_x)\chi\beta_s) + (\phi_a + \gamma\psi_a)\beta_s, \tag{35}$$

$$(1 + \eta\sigma)\phi_x = \eta((\psi_x + \psi_{\bar{x}} + \sigma\phi_x)\beta_x + (\psi_{\bar{x}} + \sigma\phi_x)\chi(1 - \beta_x) - 1) + (\phi_a + \gamma\psi_a)\beta_x - 1, \tag{36}$$

$$\phi_s + \gamma\psi_s = (\phi_a + \gamma\psi_a)\delta_s, \tag{37}$$

$$\gamma\psi_x = (\phi_a + \gamma\psi_a)\delta_x, \tag{38}$$

$$\phi_x + \gamma\psi_{\bar{x}} = (\phi_a + \gamma\psi_a)\delta_{\bar{x}}. \tag{39}$$

Notice that ψ_a is given immediately by (33) and is independent of all other parameters. Next, notice that to ensure that (34) always holds in equilibrium, for any choice of $\xi_m \in \mathbb{R}$, the following conditions need to be satisfied

$$\xi_0 = \gamma\kappa_c, \tag{40}$$

$$\xi_a = -(\phi_a + \gamma\psi_a)(1 - \rho), \tag{41}$$

$$\mu_0 = \psi_0, \tag{42}$$

$$\mu_a = \phi_a + \psi_a, \tag{43}$$

$$\mu_\theta = \phi_s + \phi_x + \psi_s + \psi_x + \psi_{\bar{x}}, \tag{44}$$

$$\mu_e = \phi_s + \psi_s. \tag{45}$$

8.2.4 Constructing the linear equilibrium for given μ_a

Given μ_a, I immediately get $\phi_a = \mu_a - \psi_a$ from (43). To find the values of the remaining parameters in $\{\phi, \psi\}$ as a function of μ_a, I use (35)-(39) together with the following condition, which follows from (43)

$$\phi_a + \gamma\psi_a = \mu_a + (\gamma - 1)\psi_a. \tag{46}$$

To simplify the notation, the right-hand side of this expression is denoted by

$$\tilde{\mu} \equiv \mu_a + (\gamma - 1)\psi_a.$$

First, using (36), (38) and (39), I solve for ϕ_x, ψ_x and $\psi_{\bar{x}}$,

$$\phi_x = \frac{\left(\beta_x + \eta\gamma^{-1}\left(\delta_x\beta_x + \delta_{\bar{x}}(\beta_x + \chi(1 - \beta_x))\right)\right)\tilde{\mu} - 1 - \eta}{1 + \eta\sigma - \eta(\sigma - \gamma^{-1})(\beta_x + \chi(1 - \beta_x))}, \tag{47}$$

$$\psi_x = \gamma^{-1}\delta_x\tilde{\mu}, \tag{48}$$

$$\psi_{\bar{x}} = \gamma^{-1}(\delta_{\bar{x}}\tilde{\mu} - \phi_x). \tag{49}$$

Note that a solution for ϕ_x always exists since $1 + \eta\sigma - \eta(\sigma - \gamma^{-1})(\beta_x + \chi(1 - \beta_x)) > 0$. This inequality follows from $\beta_x \in [0, 1]$ and $\chi \in [0, 1]$. Next, combining (35) and (37), and using the expressions above for ϕ_x, ψ_x and $\psi_{\bar{x}}$, I find ϕ_s and ψ_s,

$$\phi_s = \frac{\left(\beta_s + \eta\gamma^{-1}\delta_s + \eta\gamma^{-1}(\delta_x + \delta_{\bar{x}}(1 - \chi))\beta_s\right)\tilde{\mu} + \eta(\sigma - \gamma^{-1})(1 - \chi)\phi_x\beta_s}{1 + \eta\gamma^{-1}}, \tag{50}$$

36

$$\psi_s = \gamma^{-1} \left(\delta_s \tilde{\mu} - \phi_s \right). \tag{51}$$

Substituting the values of $\psi_s, \psi_x, \psi_{\overline{x}}$ and ϕ_x thus obtained in (22), (23), (28), I obtain values for $\kappa_{\overline{p}}, \kappa_d$ and κ_p. Substituting these values in (32), shows that ψ_0 takes the form

$$\psi_0 = J \left(\psi_s, \psi_x, \psi_{\overline{x}}, \phi_x \right) + \frac{\log \left(1 + \tau \right)}{\gamma + \eta}, \tag{52}$$

where J is a known quadratic function of $\psi_s, \psi_x, \psi_{\overline{x}}$ and ϕ_x. To find the remaining parameters of the monetary policy rule $\xi_0, \mu_0, \mu_a, \mu_s, \mu_\theta$, use (40)-(42) and (44)-(45).

To show that the prices and quantities above form an equilibrium, I need to check that the market for state-contingent claims clears and B_{it} is constant and equal to 0. Let $f \left(\epsilon_{it}, v_{it}, \theta_t, e_t \right)$ denote the joint density of the shocks ϵ_{it}, v_{it}, θ_t, and e_t. Recall that $\omega_{it} \equiv \left(\epsilon_{it}, v_{it}, \theta_t, e_t \right)$ and let the prices of state-contingent claims at $(t, 0)$ be

$$Q(\omega_{it}) = R_t^{-1} f \left(\epsilon_{it}, v_{it}, \theta_t, e_t \right) g \left(\theta_t \right), \tag{53}$$

where $g \left(. \right)$ is a function to be determined. Suppose $B_{it} = 0$. Let the portfolio of state-contingent claims be the same for each household and equal to

$$Z_{it+1} \left(\omega_{it} \right) = R_t \left[\overline{P}_{it} C_{it} - \left(1 + \tau \right) Y_{it} P_{it} + T_t \right].$$

For each realization of the aggregate shocks θ_t and e_t, goods markets clearing and the government budget balance condition imply that

$$\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} Z_{it+1} \left(\{ \epsilon, v, \theta_t, e_t \} \right) f \left(\epsilon, v, \theta_t, e_t \right) \, d\epsilon \, dv = R_t \int_0^1 \left(P_{it} Y_{it} - \overline{P}_{it} C_{it} \right) di = 0.$$

This implies that the market for state-contingent claims clears for each aggregate state θ_t. It also implies that the portfolio $\{ Z_{it+1} \left(\omega_{it} \right) \}$ has zero value at date $(t, 0)$ given that

$$\int_{\mathbb{R}^4} Q \left(\omega_{it} \right) Z_{it+1} \left(\omega_{it} \right) d\omega_{it} = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} Z_{it+1} \left(\{ \epsilon, v, \theta, e \} \right) f \left(\epsilon, v, \theta, e \right) g \left(\theta \right) \, d\epsilon \, dv \, d\theta \, de = 0.$$

Substituting in the household budget constraint shows that $B_{it+1} = 0$. Let me check that the portfolio just described is optimal. The first-order conditions for $Z_{it+1} \left(\omega_{it} \right)$ and $B_{it+1} \left(\omega_{it} \right)$ are, respectively,

$$\lambda \left(\omega_{it} \right) = R_t Q \left(\omega_{it} \right) \int_{\mathbb{R}^3} \lambda \left(\tilde{\omega}_{it} \right) d\tilde{\omega}_{it},$$

$$\lambda \left(\omega_{it} \right) = \frac{\partial V \left(0, a_t, R_{t+1} \right)}{\partial B_{it+1}} f \left(\omega_{it} \right),$$

where $\lambda \left(\omega_{it} \right)$ is the Lagrange multiplier on the budget constraint. Combining them and substituting for $\partial V / \partial B_{it+1}$, using the envelope condition

$$\frac{\partial V \left(0, a_t, R_{t+1} \right)}{\partial B_{it+1}} = \mathbb{E} \left[\overline{P}_{it+1}^{-1} C_{it+1}^{-\gamma} | a_t \right],$$

I then obtain

$$\mathbb{E} \left[\overline{P}_{it+1}^{-1} C_{it+1}^{-\gamma} | a_t \right] f \left(\omega_{it} \right) = R_t Q \left(\omega_{it} \right) \int_{-\infty}^{\infty} \mathbb{E} \left[\overline{P}_{it+1}^{-1} C_{it+1}^{-\gamma} | \rho a_{t-1} + \tilde{\theta} \right] f(\tilde{\theta}) d\tilde{\theta}.$$

Substituting (1), (2), and (53), and eliminating the constant factors on both sides, this becomes

$$e^{-(\phi_a + \gamma \psi_a)(\rho a_{t-1} + \theta_t)} = g \left(\theta_t \right) \int_{-\infty}^{\infty} e^{-(\phi_a + \gamma \psi_a)\left(\rho a_{t-1} + \tilde{\theta} \right)} f(\tilde{\theta}) d\tilde{\theta},$$

which is satisfied as long as the function $g\left(.\right)$ is given by

$$g\left(\theta_t\right) \equiv \exp\left\{-\left(\phi_a + \gamma\psi_a\right)\theta_t - \frac{1}{2}\left(\phi_a + \gamma\psi_a\right)^2\sigma_\theta^2\right\}.$$

Finally, to complete the equilibrium construction, I need to check that $\phi_x \neq 0$. From (47), this requires $\mu_a \neq \mu_a^0$ where

$$\mu_a^0 \equiv \frac{1+\eta}{\beta_x + \eta\gamma^{-1}\left(\delta_x\beta_x + \delta_{\overline{x}}\left(\beta_x + \chi\left(1 - \beta_x\right)\right)\right)} - \rho\left(\gamma - 1\right)\frac{1+\eta}{\gamma + \eta}.$$

Notice that when $\mu_a = \mu_a^0$ a stationary linear equilibrium fails to exists. A stationary equilibrium with $\phi_x = 0$ can arise, but under a policy μ_a which is typically different from μ_a^0. If $\phi_x = 0$ all the derivations above go through, except that $\mathbb{E}\left[\theta_t | s_t, x_{it}, \phi_x\overline{x}_{it}\right] = \beta_s s_t + \beta_x x_{it}$. Therefore, it is possible to derive the analogous of condition (47) and show that $\phi_x = 0$ iff

$$\frac{\left(1 + \eta\gamma^{-1}\beta_x\right)\beta_x\tilde{\mu} - 1 - \eta}{1 + \eta\sigma - \eta\left(\sigma - \gamma^{-1}\right)\left(\beta_x + \chi\left(1 - \beta_x\right)\right)} = 0.$$

This shows that an equilibrium with $\phi_x = 0$ arises when $\mu_a = \hat{\mu}_a$, where

$$\hat{\mu}_a \equiv \frac{1+\eta}{\beta_x\left(1 + \eta\gamma^{-1}\beta_x\right)} - \rho\left(\gamma - 1\right)\frac{1+\eta}{\gamma + \eta}.$$

However, $\hat{\mu}_a$ is also consistent with an equilibrium with $\phi_x \neq 0$. Summing up, if $\mu_a = \mu_a^0$ there is no stationary linear equilibrium; if $\mu_a = \mu_a^a$ there are two stationary linear equilibria, one with $\phi_x \neq 0$ and one with $\phi_x = 0$; if $\mu_a \in \mathbb{R}/\left\{\mu_a^0, \hat{\mu}_a\right\}$, there is a unique stationary linear equilibrium.

8.3 Proof of Proposition 2

If $\sigma_e^2 = 0$ then $\beta_s = \delta_s = 1$ and $\beta_x = \delta_x = \delta_{\overline{x}} = 0$. Substituting in (47)-(51) gives

$$\begin{aligned}\phi_x &= -\frac{1+\eta}{1 + \eta\sigma - \eta\left(\sigma - \gamma^{-1}\right)\chi}, & (54)\\ \phi_s &= \tilde{\mu} + \frac{\eta\left(\sigma - \gamma^{-1}\right)\left(1 - \chi\right)}{1 + \eta\gamma^{-1}}\phi_x,\end{aligned}$$

and

$$\begin{aligned}\psi_x &= 0, \quad \psi_{\overline{x}} = -\gamma^{-1}\phi_x,\\ \psi_s &= \gamma^{-1}\left(\tilde{\mu} - \phi_s\right) = -\gamma^{-1}\frac{\eta\left(\sigma - \gamma^{-1}\right)\left(1 - \chi\right)}{1 + \eta\gamma^{-1}}\phi_x,\end{aligned}$$

and ψ_0 can be determined from (52). Notice ϕ_s is the only coefficient which depends on μ_a. However, the real equilibrium allocation only depends on the consumption levels c_{it} and on the relative prices $p_{it} - p_t$, and, given (1) and (2), these are independent of ϕ_s.

8.4 Proof of Proposition 3

For the following derivations recall that under imperfect information all the coefficients $\beta_s, \beta_x, \delta_s, \delta_x, \delta_{\overline{x}}$ are in $(0,1)$ and $\chi \in [0,1]$. Differentiating (47) with respect to μ_a (recalling that $\tilde{\mu} = \mu_a + \left(\gamma - 1\right)\psi_a$ and ψ_a is constant), gives

$$\frac{\partial\phi_x}{\partial\mu_a} = \frac{\beta_x + \eta\gamma^{-1}\left(\delta_x\beta_x + \delta_{\overline{x}}\left(\beta_x + \chi\left(1 - \beta_x\right)\right)\right)}{1 + \eta\sigma - \eta\left(\sigma - \gamma^{-1}\right)\left(\beta_x + \chi\left(1 - \beta_x\right)\right)} > 0,$$

38

where the denominator is positive since $\beta_x + \chi (1 - \beta_x) < 1$ and $\eta (\sigma - \gamma^{-1}) < \eta\sigma$. Differentiating (50) gives

$$\frac{\partial \phi_s}{\partial \mu_a} = \frac{\beta_s + \eta\gamma^{-1}\delta_s}{1 + \eta\gamma^{-1}} + \left[\frac{\eta\gamma^{-1} (\delta_x + \delta_{\overline{x}} (1 - \chi)) \beta_s}{1 + \eta\gamma^{-1}} + \frac{\eta\beta_s (\sigma - \gamma^{-1}) (1 - \chi)}{1 + \eta\gamma^{-1}} \frac{\partial \phi_x}{\partial \mu_a} \right].$$

Some lengthy algebra, in the supplementary material, shows that the term in square brackets is positive, so $\partial \phi_s / \partial \mu_a > 0$. Next, differentiating (51) gives

$$\frac{\partial \psi_s}{\partial \mu_a} = \gamma^{-1} \left(\delta_s - \frac{\partial \phi_s}{\partial \mu_a} \right).$$

To prove that this quantity is negative notice that

$$\frac{\partial \phi_s}{\partial \mu_a} > \frac{\beta_s + \eta\gamma^{-1}\delta_s}{1 + \eta\gamma^{-1}} > \delta_s,$$

where the last inequality follows from $\beta_s > \delta_s$, which follows immediately from (30) and (31). To prove that $\partial \psi_\theta / \partial \mu_a > 0$ it is sufficient to use of the last result together with Lemma 2, which immediately implies that $\partial \psi_\theta / \partial \mu_a = - (\sigma_e^2 / \sigma_\theta^2) \partial \psi_s / \partial \mu_a$.

8.5   Proof of Proposition 4

The argument in the text shows that there is a μ_a that gives coefficients $\psi_s = 0$ and $\psi_\theta = (1+\eta)/(\gamma+\eta)$, if one assumes that consumers form expectations based on a_{t-1}, s_t, x_{it}, and $\overline{x}_{it}$. It remains to check that this value of μ_a is not equal to μ_a^0, so that $\phi_x \neq 0$ and observed prices reveal $\overline{x}_{it}$. The algebra is presented in the supplementary material.

8.6   Proof of Lemma 3

Let me consider the case $\gamma \neq 1$, the proof for the case $\gamma = 1$ follows similar steps and is presented in the supplementary material. First, I derive expressions for the conditional expectations $\mathbb{E}[C_{it}^{1-\gamma}|a_{t-1}]$ and $\mathbb{E}[N_{it}^{1+\eta}|a_{t-1}]$. Substituting for c_{it} in the first, using (2), I obtain

$$\mathbb{E}\left[C_{it}^{1-\gamma}|a_{t-1}\right] = e^{(1-\gamma)(\psi_0 + \psi_a a_{t-1}) + \frac{1}{2}(1-\gamma)^2 \left(\psi_\theta^2 \sigma_\theta^2 + \psi_s^2 \sigma_e^2 + \psi_x^2 \sigma_\epsilon^2 + \psi_{\overline{x}}^2 \sigma_v^2\right)}.$$

Using (7) to substitute for d_{it}, and the fact that $a_{it} = a_t + \epsilon_{it}$ and $p_{it} - p_t = \phi_x \epsilon_{it}$, I derive the equilibrium labor supply

$$N_{it} = \frac{D_{it} P_{it}^{-\sigma}}{A_{it}} = e^{\kappa_d + \psi_0 + \psi_a a_{t-1} + \psi_\theta \theta_t + \psi_s e_t - a_t - (1 + \sigma\phi_x - (\psi_{\overline{x}} + \sigma\phi_x)\chi)\epsilon_{it}}. \tag{55}$$

From this expression, I obtain

$$\mathbb{E}\left[N_{it}^{1+\eta}|a_{t-1}\right] = e^{(1+\eta)(\kappa_d + \psi_0 + (\psi_a - \rho)a_{t-1}) + \frac{1}{2}(1+\eta)^2 \left((\psi_\theta - 1)^2 \sigma_\theta^2 + \psi_s^2 \sigma_e^2 + (1 + \sigma\phi_x - (\psi_{\overline{x}} + \sigma\phi_x)\chi)^2 \sigma_\epsilon^2\right)}.$$

Using the fact that $\psi_a = \rho (1 + \eta) / (\gamma + \eta)$ to group the terms in a_{t-1}, the instantaneous conditional expected utility takes the form

$$\mathbb{E}\left[U (C_{it}, N_{it}) |a_{t-1}\right] = \left[\frac{1}{1-\gamma} e^{(1-\gamma)(k_1 + \psi_0)} - \frac{1}{1+\eta} e^{(1+\eta)(k_2 + \psi_0)} \right] e^{(1-\gamma)\psi_a a_{t-1}}. \tag{56}$$

where

$$k_1 = \frac{1}{2} (1 - \gamma) \left(\psi_\theta^2 \sigma_\theta^2 + \psi_s^2 \sigma_e^2 + \psi_x^2 \sigma_\epsilon^2 + \psi_{\overline{x}}^2 \sigma_v^2 \right), \tag{57}$$

$$k_2 = \kappa_d + \frac{1}{2} (1 + \eta) \left((\psi_\theta - 1)^2 \sigma_\theta^2 + \psi_s^2 \sigma_e^2 + (1 + \sigma\phi_x - (\psi_{\overline{x}} + \sigma\phi_x)\chi)^2 \sigma_\epsilon^2 \right). \tag{58}$$

The equilibrium equation (52) shows that, for each value of μ_a, there is a one-to-one correspondence between τ and ψ_0, and ψ_0 is the only equilibrium coefficient affected by τ. Therefore, if τ is set optimally, ψ_0 must maximize the term in square brackets on the right-hand side of (56). Solving this problem shows that the term in square brackets must be equal to $\exp\{(((1-\gamma)(1+\eta))/(\gamma+\eta))(k_1 - k_2)\}$ and

$$\psi_0 = \frac{(1-\gamma) k_1 - (1+\eta) k_2}{\gamma + \eta}. \tag{59}$$

Then, I can take the unconditional expectation of (56) and sum across periods to obtain

$$\sum_{t=0}^{\infty} \beta^t \mathbb{E}\left[U\left(C_{it}, N_{it}\right)\right] = \frac{\gamma + \eta}{(1+\eta)(1-\gamma)} e^{\frac{(1-\gamma)(1+\eta)}{\gamma+\eta}(k_1 - k_2)} \sum_{t=0}^{\infty} \beta^t \mathbb{E}\left[e^{\frac{(1-\gamma)(1+\eta)}{\gamma+\eta}\rho a_{t-1}}\right].$$

Letting

$$w \equiv k_1 - k_2, \tag{60}$$

and

$$W_0 \equiv \frac{\gamma + \eta}{1 + \eta} \sum_{t=0}^{\infty} \beta^t \mathbb{E}\left[e^{\frac{(1-\gamma)(1+\eta)}{\gamma+\eta}\rho a_{t-1}}\right],$$

I then obtain the expression in the text. Combining (57), (58), and (60), shows that w can be expressed as follows

$$
\begin{aligned}
w &= -\frac{1}{2}\frac{(1-\gamma)(1+\eta)}{\gamma+\eta}\sigma_\theta^2 - \frac{1}{2}(\gamma+\eta)\left(\left(\psi_\theta - \frac{1+\eta}{\gamma+\eta}\right)^2 \sigma_\theta^2 + \psi_s^2 \sigma_e^2\right) + \\
&\quad + \frac{1}{2}(1-\gamma)\left(\psi_x^2 \sigma_\epsilon^2 + \psi_{\overline{x}}^2 \sigma_v^2\right) - \frac{1}{2}(1+\eta)\left(1 + \sigma\phi_x - (\psi_{\overline{x}} + \sigma\phi_x)\chi\right)^2 \sigma_\epsilon^2, \\
&\quad - \frac{1}{2}\left(\psi_x^2 \sigma_\epsilon^2 + (\psi_{\overline{x}} + \sigma\phi_x)^2 \sigma_{v|\epsilon}^2\right) + \frac{1}{2}\sigma(\sigma-1)\phi_x^2 \sigma_{\epsilon|v}^2.
\end{aligned} \tag{61}
$$

This shows that w is a quadratic functions of the equilibrium coefficients ϕ and ψ. Moreover, Proposition 3 shows that ϕ and ψ are linear functions of μ_a. Therefore, (61) implicitly defines w as a quadratic function of μ_a. The discounted sum in W_0 is always finite if $\rho < 1$, because each term is bounded by $\left|\exp\{((1-\gamma)\psi_a \rho a_{-1} + (1/2)((1-\gamma)^2/(1-\rho^2))\psi_a^2 \sigma_\theta^2\}\right|$. If, instead, $\rho = 1$, to ensure that the sum is finite it is necessary to assume that $\beta \exp\left\{(1/2)(1-\gamma)^2\psi_a^2\sigma_\theta^2\right\} < 1$, which is equivalent to the inequality in footnote 11.

8.7 Derivation of equations (19) and (20)

I will first show that (19) corresponds to (61) in the proof of Lemma 3. For ease of exposition, the expression in the text omits the constant term $-(1/2)((1-\gamma)(1+\eta)/(\gamma+\eta))\sigma_\theta^2$. The first two terms in (19) can be derived from the two terms after the constant in (61), simply using the definitions of c_t and c_{it}. Equation (55), can be used to derive $\int_0^1 (n_{it} - n_t)^2 \, di$, and check that the third term in (19) equals the third term after the constant in (61). Finally, the last line of (61) corresponds to $-\kappa_d$, by (23), while equation (55) implies that $n_t = \kappa_d + c_t - a_t$. This shows that the last terms in (19) and (61) are equal. To derive (20) notice that, as just argued, the last line of (61) is equal to $-\kappa_d$. The derivations in Lemma 5 can then be used to obtain the expression in the text.

8.8 Proof of Lemma 4

I concentrate on the case $\gamma \neq 1$, the case $\gamma = 1$ is proved along similar lines. Given two monetary policies $\mu_a' = \mu_a$ and $\mu_a'' = \mu_a + u$, let C_{it}', N_{it}' and C_{it}'', N_{it}'' denote the associated equilibrium allocations, and define the function

$$f(\delta, u) \equiv \left\{\sum_{t=0}^{\infty} \beta^t \mathbb{E}\left[e^{\frac{(1-\gamma)(1+\eta)}{\gamma+\eta}\rho a_{t-1}}\right]\right\}^{-1} \left\{\sum_{t=0}^{\infty} \beta^t \mathbb{E}\left[U\left(e^\delta C_{it}', N_{it}'\right)\right] - \sum_{t=0}^{\infty} \beta^t \mathbb{E}\left[U\left(C_{it}'', N_{it}''\right)\right]\right\}.$$

Proceeding in as in the proof of Lemma 3, it is possible to show that

$$f\left(\delta, u\right) = \frac{1}{1-\gamma}e^{(1-\gamma)(\delta+k_1+\psi_0)} - \frac{1}{1+\eta}e^{(1+\eta)(k_2+\psi_0)} - \frac{\gamma+\eta}{(1+\eta)(1-\gamma)}e^{\frac{(1+\eta)(1-\gamma)}{\gamma+\eta}w(\mu_a+u)},$$

where k_1 and k_2 are defined in (57) and (58), for the coefficients $\{\phi, \psi\}$ associated to the policy μ_a, and the function $w\left(.\right)$ is defined by (61). Let the function $\delta\left(u\right)$ be defined implicitly by

$$f\left(\delta\left(u\right), u\right) = 0.$$

It is immediate that $\delta\left(0\right) = 0$. Moreover,

$$\left.\frac{\partial f\left(\delta, u\right)}{\partial\delta}\right|_{\delta=u=0} = e^{(1-\gamma)(k_1+\psi_0)},$$

$$\left.\frac{\partial f\left(\delta, u\right)}{\partial u}\right|_{\delta=u=0} = e^{\frac{(1+\eta)(1-\gamma)}{\gamma+\eta}w(\mu_a)}w'\left(\mu_a\right),$$

and (59) implies that

$$e^{(1-\gamma)(k_1+\psi_0)} = e^{\frac{(1+\eta)(1-\gamma)}{\gamma+\eta}w(\mu_a)}.$$

It follows that

$$\delta'\left(u\right) = w'\left(\mu_a\right).$$

Since $\Delta\left(\mu_a, \mu_a + u\right) \equiv \exp\left\{\delta\left(u\right)\right\} - 1$, the result follows from differentiating this expression at $u = 0$.

8.9 Proof of Proposition 5

Let me begin by setting up and characterizing the planner's problem. Then, I will show that there is a monetary policy that reaches the constrained optimal allocation. Let a_- be a given scalar representing productivity in the previous period. Let θ be a normally distributed random variable with mean zero and variance σ_θ^2 and let s be a random variable given by $s = \theta + e$, where e is also a normal random variable with mean zero and variance σ_e^2. Let $x, \overline{x}$ and $\tilde{x}$ be random variables given by $x = \theta + \epsilon$, $\overline{x} = \theta + v$, and $\tilde{x} = \overline{x} + \tilde{\epsilon}$, where ϵ, v and $\tilde{\epsilon}$ are independent random variables with zero mean and variances $\sigma_\epsilon^2, \sigma_v^2, \sigma_{\epsilon|v}^2$.

The planner's problem is to choose functions $\tilde{C}\left(s, x, \overline{x}, \tilde{x}\right)$, $C\left(s, x, \overline{x}\right)$, and $N\left(s, x, \theta\right)$ that maximize

$$\mathbb{E}\left[U\left(C\left(s, x, \overline{x}\right), N\left(s, x, \theta\right)\right)\right]$$

subject to

$$C\left(s, x, \overline{x}\right) = \left(\mathbb{E}\left[\left(\tilde{C}\left(s, x, \overline{x}, \tilde{x}\right)\right)^{\frac{\sigma-1}{\sigma}}|s, x, \overline{x}\right]\right)^{\frac{\sigma}{\sigma-1}} \quad \text{for all } s, x, \overline{x}, \tag{62}$$

$$e^{\rho a_- + \tilde{x}}N\left(s, \tilde{x}, \theta\right) = \mathbb{E}\left[\tilde{C}\left(s, x, \overline{x}, \tilde{x}\right)|s, \tilde{x}, \theta\right] \quad \text{for all } s, \tilde{x}, \theta. \tag{63}$$

Let $\Lambda\left(s, \tilde{x}, \theta\right)$ denote the Lagrange multiplier on constraint (63). Substituting (62) in the objective function, one obtains the following first-order conditions with respect to $\tilde{C}\left(s, x, \overline{x}, \tilde{x}\right)$ and $N\left(s, \tilde{x}, \theta\right)$:

$$\left(\tilde{C}\left(s, x, \overline{x}, \tilde{x}\right)\right)^{-\frac{1}{\sigma}}\left(C\left(s, x, \overline{x}\right)\right)^{\frac{1}{\sigma}-\gamma} = \mathbb{E}\left[\Lambda\left(s, \tilde{x}, \theta\right)|s, x, \overline{x}, \tilde{x}\right], \tag{64}$$

$$\left(N\left(s, \tilde{x}, \theta\right)\right)^{\eta} = e^{\rho a_- + \tilde{x}}\Lambda\left(s, \tilde{x}, \theta\right). \tag{65}$$

The planner's problem is concave, so (64) and (65) are both necessary and sufficient for an optimum. To prove the proposition, I take the equilibrium allocation associated to a generic pair $\left(\mu_a, \tau\right)$, and I derive conditions on μ_a and τ which ensure that it satisfies (64) and (65). An equilibrium allocation immediately satisfies the constraints (62) and (63), the first by construction, the second by market

41

clearing. Take a linear equilibrium allocation characterized by φ and ψ. Let $\tilde{C}\left(.,.,.,.\right)$ and $N\left(.,.,.\right)$ take the form

$$\tilde{C}\left(s,x,\overline{x},\tilde{x}\right) = \exp\left\{\sigma\kappa_{\overline{p}} + \psi_0 + \psi_a a_- + \psi_s s + \psi_x x + \psi_{\overline{x}}\overline{x} - \sigma\phi_x\left(\tilde{x}-\overline{x}\right)\right\}, \tag{66}$$

$$N\left(s,\tilde{x},\theta\right) = \exp\left\{\kappa_d + \psi_0 + \psi_a a_- + \psi_s s + \left(\psi_x + \psi_{\overline{x}}\right)\theta - \rho a_- - \tilde{x} - \left(\sigma\phi_x - \left(\psi_{\overline{x}} + \sigma\phi_x\right)\chi\right)\left(\tilde{x}-\theta\right)\right\}$$

I conjecture that the Lagrange multiplier $\Lambda\left(s,\tilde{x},\theta\right)$ takes the log-linear form

$$\Lambda\left(s,\tilde{x},\theta\right) = \exp\left\{\lambda_0 + \lambda_s s + \lambda_x \tilde{x} + \lambda_\theta \theta\right\}. \tag{67}$$

Let me first check the first-order condition for consumption, (64). Substituting (66) in (62) and using the definition of $\kappa_{\overline{p}}$, I get

$$C\left(s,x,\overline{x}\right) = \exp\left\{\psi_0 + \psi_a a_- + \psi_s s + \psi_x x + \psi_{\overline{x}}\overline{x}\right\}.$$

After some simplifications, the right-hand side of (64) then becomes

$$\tilde{C}\left(s,x,\overline{x},\tilde{x}\right)^{-\frac{1}{\sigma}} C\left(s,x,\overline{x}\right)^{\frac{1}{\sigma}-\gamma} = \exp\left\{-\kappa_{\overline{p}} + \phi_x\left(\tilde{x}-\overline{x}\right)\right\}\exp\left\{-\gamma\left(\psi_0 + \psi_a a_- + \psi_s s + \psi_x x + \psi_{\overline{x}}\overline{x}\right)\right\}.$$

The left-hand side of (64), using (67), is equal to

$$\mathbb{E}\left[\Lambda\left(s,\tilde{x},\theta\right)|s,x,\overline{x},\tilde{x}\right] = \exp\left\{\lambda_0 + \lambda_s s + \lambda_x \tilde{x} + \lambda_\theta \mathbb{E}\left[\theta|s,x,\overline{x}\right] + \frac{1}{2}\lambda_\theta^2 \hat{\sigma}_\theta^2\right\},$$

where $\hat{\sigma}_\theta^2$ is the residual variance of θ, equal to $\left(\pi_\theta + \pi_s + \pi_x + \pi_{\overline{x}}\right)^{-1}$. Therefore, to ensure that (64) holds for all $s,x,\overline{x},\tilde{x}$, the following conditions must hold,

$$\begin{aligned}
\lambda_0 + \frac{1}{2}\lambda_\theta^2 \hat{\sigma}_\theta^2 &= -\kappa_{\overline{p}} - \gamma\left(\psi_0 + \psi_a a_-\right), \\
\lambda_s + \lambda_\theta \delta_s &= -\gamma\psi_s, \\
\lambda_x &= \phi_x, \\
\lambda_\theta \delta_x &= -\gamma\psi_x, \\
\lambda_\theta \delta_{\overline{x}} &= -\gamma\psi_{\overline{x}} - \phi_x.
\end{aligned}$$

Set $\lambda_\theta = -\left(\phi_a + \gamma\psi_a\right)$, $\lambda_s = \phi_s$, $\lambda_x = \phi_x$ and $\lambda_0 = -\kappa_{\overline{p}} - \gamma\left(\psi_0 + \psi_a a_-\right) - (1/2)\lambda_\theta^2 \hat{\sigma}_\theta^2$. Then, the first and the third of these conditions hold immediately. The other three follow from the equilibrium relations (37)-(39). Let me now check the first order condition for labor effort, (65). Substituting (67) and matching the coefficients on both sides, gives

$$\begin{aligned}
\lambda_0 + \rho a_- &= \eta\left(\kappa_d + \psi_0 + \left(\psi_a - \rho\right)a_-\right), \\
\lambda_s &= \eta\psi_s, \\
\lambda_x + 1 &= -\eta\left(1 + \sigma\phi_x - \left(\psi_{\overline{x}} + \sigma\phi_x\right)\chi\right), \\
\lambda_\theta &= \eta\left(\psi_x + \psi_{\overline{x}} + \sigma\phi_x - \left(\psi_{\overline{x}} + \sigma\phi_x\right)\chi\right).
\end{aligned}$$

Substituting, the λ's derived above, using $\psi_a = \rho\left(1+\eta\right)/\left(\gamma+\eta\right)$ and rearranging, gives

$$\begin{aligned}
\left(\gamma+\eta\right)\psi_0 + \eta\kappa_d + \kappa_{\overline{p}} + (1/2)\left(\phi_a + \gamma\psi_a\right)^2 \hat{\sigma}_\theta^2 &= 0, \tag{68} \\
\phi_s - \eta\psi_s &= 0, \tag{69} \\
\left(1 + \eta\sigma\right)\phi_x + 1 + \eta - \eta\left(\psi_{\overline{x}} + \sigma\phi_x\right)\chi &= 0, \tag{70} \\
\phi_a + \gamma\psi_a + \eta\left(\psi_x + \psi_{\overline{x}} + \sigma\phi_x - \left(\psi_{\overline{x}} + \sigma\phi_x\right)\chi\right) &= 0. \tag{71}
\end{aligned}$$

To complete the proof, I need to find μ_a and τ such that the corresponding equilibrium coefficients φ and ψ satisfy (68)-(71). Setting $\tilde{\mu} = \tilde{\mu}^*$ where

$$\tilde{\mu}^* \equiv \frac{\eta\left(\sigma - \gamma^{-1}\right)\left(1-\chi\right)\left(1+\eta\right)}{\left(1 + \eta\gamma^{-1}\left(\delta_x + \delta_{\overline{x}}\left(1-\chi\right)\right)\right)\left(1 + \eta\sigma - \eta\left(\sigma - \gamma^{-1}\right)\chi\right) + \eta^2\left(\sigma - \gamma^{-1}\right)\left(1-\chi\right)\gamma^{-1}\delta_{\overline{x}}\chi}, \tag{72}$$

42

ensures that (69)-(71) are satisfied. To see why these three conditions can be jointly satisfied, notice that the equilibrium conditions (35) and (36) can be rewritten as

$$\phi_s - \eta\psi_s = [\eta((\psi_x + \psi_{\overline{x}} + \sigma\phi_x) - (\psi_{\overline{x}} + \sigma\phi_x)\chi) + \phi_a + \gamma\psi_a]\beta_s,$$
$$(1 + \eta\sigma)\phi_x + 1 + \eta - \eta(\psi_{\overline{x}} + \sigma\phi_x)\chi = [\eta((\psi_x + \psi_{\overline{x}} + \sigma\phi_x) - (\psi_{\overline{x}} + \sigma\phi_x)\chi) + \phi_a + \gamma\psi_a]\beta_x,$$

so that, in equilibrium, (71) implies the other two. Finally, the subsidy τ can be set so as to ensure that (68) is satisfied. The value of ϕ_x at the optimal monetary policy is

$$\phi_x^* = \frac{-1 - \eta + \eta\gamma^{-1}\delta_{\overline{x}}\chi\tilde{\mu}^*}{1 + \eta\sigma - \eta(\sigma - \gamma^{-1})\chi}. \tag{73}$$

Substituting (72) in the expression $\eta\gamma^{-1}\delta_{\overline{x}}\chi\tilde{\mu}^*$, shows that this expression is strictly smaller than $1 + \eta$, which implies that $\phi_x^* < 0$. This confirms that $\mu_a^* \neq \mu_a^0$, so that, by Proposition 1, the associated coefficients φ^* and ψ^* form a linear equilibrium.

8.10   Proof of Proposition 6

Let me derive the value of ψ_s at the constrained efficient allocation. From condition (69) and the equilibrium condition $\phi_s + \gamma\psi_s = \tilde{\mu}\delta_s$, I get

$$\psi_s^* = \frac{1}{\gamma + \eta}\delta_s\tilde{\mu}^*.$$

If $\chi = 0$, the consumer extracts perfect information from $\overline{x}_{it} = \theta_t$ and $\delta_s = 0$, which implies that $\psi_s^* = 0$. If, instead $\chi > 0$, ψ_s^* inherits the sign of $\tilde{\mu}^*$. Inspecting (72) shows that if $\eta > 0$, $\chi < 1$ and $\sigma\gamma \neq 1$, $\tilde{\mu}^*$ is not zero and has the sign of $\sigma\gamma - 1$. In all other cases, $\tilde{\mu}^* = 0$. Therefore, if $\eta > 0$, $\chi \in (0,1)$ and $\sigma\gamma \neq 1$, ψ_s^* is not zero and has the sign of $\sigma\gamma - 1$. In all remaining cases $\psi_s^* = 0$. The inequalities for ψ_θ^* follow from Lemma 2. To prove the inequalities for μ_a^*, notice that, by Proposition 3 there is a decreasing relation between μ_a and ψ_s, and $\psi_s = 0$ at $\mu_a = \mu_a^{fs}$.

8.11   Proof of Proposition 7

The first part of the Proposition is proved by the two examples discussed in the text. Let me prove the second part. By Proposition 5, social welfare under the optimal monetary policy is the value of a single decision maker's optimization problem (the planner's). For a single decision maker facing a normal signal $s_t = \theta_t + e_t$, increasing the variance σ_e^2 is equivalent to observing the original signal plus an additional independent error, that is, observing $s_t' = s_t + \xi_t$. Then, a decision maker who observes s_t can always replicate the payoff of a decision maker with a less precise signal, by just adding random noise to s_t and following the associated optimal policy. Therefore, the decision maker's payoff cannot increase when σ_e^2 increases.

References

[1] Klaus Adam, 2006, "Optimal Monetary Policy with Imperfect Common Knowledge," *Journal of Monetary Economics* (forthcoming).

[2] Manuel Amador and Pierre-Olivier Weill, 2007, "Learning from Private and Public Observations of Others Actions," mimeo UCLA.

[3] Jeffery Amato, Stephen Morris, and Hyun Song Shin, 2002, "Communication and Monetary Policy," *Oxford Review of Economic Policy* 18, pp. 495-503.

[4] George Marios Angeletos, Guido Lorenzoni, and Alessandro Pavan, 2006, "Wall Street and Silicon Valley: A Delicate Interaction," mimeo, MIT.

[5] George Marios Angeletos and Alessandro Pavan, 2007a, "Efficient Use of Information and Social Value of Information," *Econometrica* 75(4).

[6] George Marios Angeletos and Alessandro Pavan, 2007b, "Policy with Dispersed Information," mimeo, MIT.

[7] Kosuke Aoki, 2003, "On the Optimal Monetary Policy Response to Noisy Indicators," *Journal of Monetary Economics* 50, pp. 501-523.

[8] Philippe Bacchetta and Eric Van Wincoop, 2005, "Higher Order Expectations in Asset Pricing," mimeo, Study Center Gerzensee.

[9] Paul Beaudry and Franck Portier, 2006, "When Can Changes in Expectations Cause Business Cycle Fluctuations in Neo-Classical Settings?," *Journal of Economic Theory*, forthcoming.

[10] Roland Benabou, 2002, "Tax and Education Policy in a Heterogeneous Agent Economy: What Levels of Redistribution Maximize Growth and Efficiency?" *Econometrica*, 70(2), 481-517.

[11] Ben S. Bernanke and Mark Gertler, 2001, "Should Central Banks Respond to Movements in Asset Prices?" *American Economic Review*, Vol. 91, No. 2, pp. 253-257

[12] Stephen Cecchetti, Hans Genberg, John Lipsky, and Sushil Wadhwani, 2000, *Asset Prices and Central Bank Policy.* London, International Center for Monetary and Banking Studies.

[13] Larry Christiano, Roberto Motto, and Massimo Rostagno, 2006, Monetary Policy and Stock Market Boom-Bust Cycles, mimeo, Northwestern.

[14] Jean-Pierre Danthine, John Donaldson, and Thore Johnsen 1998, "Productivity Growth, Consumer Confidence, and the Business Cycle," *European Economic Review* 42, pp. 1113-1140.

[15] Bill Dupor, 2002, "Nominal Price versus Asset Price Stabilization," mimeo.

[16] Bill Dupor, 2005, "Stabilizing Non-Fundamental Asset Price Movements under Discretion and Limited Information," *Journal of Monetary Economics* 62, pp. 727-747.

[17] Jonathan Heathcote, Kjetil Storesletten, and Giovanni L. Violante, 2008, "Insurance and Opportunities: A Welfare Analysis of Labor Market Risk" *Journal of Monetary Economics*, forthcoming.

[18] Christian Hellwig, 2005, "Heterogeneous Information and the Welfare Effects of Public Information Disclosures" UCLA Working Paper.

[19] Christian Hellwig and Laura Veldkamp, 2007, "Knowing What Others Know: Coordination Motives in Information Acquisition," mimeo, NYU.

[20] Nir Jaimovich and Sergio Rebelo, 2005, "Can News about the Future Drive the Business Cycle?" mimeo, Northwestern University.

[21] Robert King, "Monetary Policy and Information Content of Prices," *Journal of Political Economy*, April 1982, pp. 247-279.

[22] Guido Lorenzoni, 2006, "A Theory of Demand Shocks," NBER WP 12477.

[23] Yulei Luo, 2006, "Consumption Dynamics under Information Processing Constraints," mimeo, University of Hong Kong.

[24] Robert E. Lucas Jr.,1972, "Expectations and the Neutrality of Money," *Journal of Economic Theory* 4 (2), pp. 103-24.

[25] Robert E. Lucas Jr.,1990, "Liquidity and Interest Rates." *Journal of Economic Theory*, 1990, 50(2), pp. 237-64.

[26] Bartosz Maćkowiak and Mirko Wiederholt, 2006, "Optimal Sticky Prices under Rational Inattention," mimeo, Humboldt University.

[27] N. Gregory Mankiw and Ricardo Reis, 2002, "Sticky Information Versus Sticky Prices: A Proposal to Replace the New Keynesian Phillips Curve," *Quarterly Journal of Economics* 117, pp. 1295-1328.

[28] Bennett T. McCallum, 1979, "A Monetary Policy ineffectiveness Result in a Model with a Predetermined Price Level," *Economics Letters*, Vol. 3, No. 1, 1979, 1-4.

[29] Fabio Milani, 2005, "Expectations, Learning and Macroeconomic Persistence," mimeo, UC Irvine.

[30] Stephen Morris and Hyun Song Shin, 2002, "The Social Value of Public Information," *American Economic Review* 92, pp. 1521-1534.

[31] Stephen Morris and Hyun Song Shin, 2005, "Central Bank Transparency and the Signal Value of Prices," *Brookings Papers on Economic Activity* 2, pp. 1-66.

[32] Giuseppe Moscarini, 2004, "Limited Information Capacity as a Source of Inertia," *Journal of Economic Dynamics and Control*, vol. 28 (10), pp. 2003-2035.

[33] Kristoffer P. Nimark, 2005, "Calvo Pricing and Imperfect Common Knowledge: A Forward Looking Model of Rational Inflation Inertia," ECB Working Paper No. 474.

[34] Maurice Obstfeld and Kenneth Rogoff, 2000, "New Directions for Stochastic Open Economy Models," *Journal of International Economics* 50, pp. 117-53.

[35] Athanasios Orphanides, 2003, "Monetary Policy Evaluation with Noisy Information," *Journal of Monetary Economics* 50 (3), pp. 605-631.

[36] Edmund S. Phelps et al., 1969, *Microeconomic Foundations of Employment and Inflation Theory*, New York: Norton.

[37] Ricardo Reis, 2003, "Where Is the Natural Rate? Rational Policy Mistakes and Persistent Deviations of Inflation from Target," *Advances in Macroeconomics, The B. E. Journals in Macroeconomics*, vol. 3 (1).

[38] Ricardo Reis, 2006, "Inattentive Producers," *Review of Economic Studies*, vol. 73 (3).

[39] Christopher A. Sims, 2003, "Implications of Rational Inattention," *Journal of Monetary Economics* 50 (3), pp. 665-690.

[40]   Lars E.O. Svensson, 2006, "Social Value of Public Information: Morris and Shin (2002) Is Actually Pro Transparency, Not Con," *American Economic Review* 96, pp. 448-451.

[41]   Lars E.O. Svensson and Michael Woodford, 2003, "Indicator Variables for Optimal Policy," *Journal of Monetary Economics* 50, pp. 691-720.

[42]   Lars E.O. Svensson and Michael Woodford, 2005, "Indicator Variables for Optimal Policy under Asymmetric Information," *Journal of Economic Dynamics and Control.*

[43]   Michael Woodford, 2002, "Imperfect Common Knowledge and the Effects of Monetary Policy", in P. Aghion, R. Frydman, J. Stiglitz, and M. Woodford, eds., *Knowledge, Information, and Expectations in Modern Macroeconomics: In Honor of Edmund S. Phelps*, Princeton: Princeton University Press.

[44]   Michael Woodford, 2003, *Interest and Prices*, Princeton: Princeton University Press.

9 Supplementary Material

[not for publication]

9.1 Proof of Proposition 1: local determinacy

Let variables with a tilde denote deviations from the equilibrium derived above. A first order approximation of the optimality conditions gives

$$\mathbb{E}_{i,(t,I)} \left[(1 + \eta\sigma)\, \tilde{p}_{it} - \eta \tilde{d}_{it} + \widetilde{\overline{p}}_{it} + \gamma \tilde{c}_{it} \right] = 0,$$

$$\mathbb{E}_{i,(t,II)} \left[\widetilde{\overline{p}}_{it} + \gamma \tilde{c}_{it} + \tilde{r}_t - \widetilde{\overline{p}}_{it+1} - \gamma \tilde{c}_{it+1} \right] = 0.$$

Taking expectations at time $(T,0)$ and integrating across agents (in that order) gives

$$\mathbb{E}_T \left[(1 + \eta\sigma)\, \tilde{p}_t - \eta (\tilde{c}_t + \sigma \tilde{p}_t) + \tilde{p}_t + \gamma \tilde{c}_t \right] = 0,$$

$$\mathbb{E}_T \left[\tilde{p}_t + \gamma \tilde{c}_t + \tilde{r}_t - \tilde{p}_{t+1} - \gamma \tilde{c}_{t+1} \right] = 0, \tag{74}$$

for all $t \geq T$. The first condition implies that

$$\mathbb{E}_T \left[\tilde{c}_t \right] = 0 \text{ for all } t \geq T. \tag{75}$$

Moreover, notice that $\tilde{m}_t = m_t - \hat{m}_t$. Therefore, under the policy rule (10)

$$\tilde{r}_t = \xi_m \tilde{m}_{t-1} \text{ for all } t \geq T.$$

Rewrite (74) as

$$\mathbb{E}_T \left[\tilde{m}_t + (\gamma - 1)\, \tilde{c}_t + \tilde{r}_t - \tilde{m}_{t+1} - (\gamma - 1)\, \tilde{c}_{t+1} \right] = 0.$$

Using (75) and defining $h_t \equiv \mathbb{E}_T \left[\tilde{m}_t \right]$, this gives a difference equation for h_t,

$$h_{t+1} - h_t - \xi h_{t-1} = 0 \text{ for all } t \geq T$$

with initial condition $h_{T-1} = \tilde{m}_{T-1}$. The assumption $\xi_m > 1$ ensures that any $h_{T-1} \neq 0$ gives an explosive solution. This shows that any equilibrium in a neighborhood of the original equilibrium must display $m_t = \hat{m}_t$ for all realizations of the aggregate shocks. Using this result one can show that the individual prices and consumption are the same as under the original equilibrium.

9.2 Algebra for the proof of Proposition 3

I need to prove that

$$\gamma^{-1} \left(\delta_x + \delta_{\overline{x}} (1 - \chi) \right) + \left(\sigma - \gamma^{-1} \right) (1 - \chi) \frac{\partial \phi_x}{\partial \mu_a} > 0,$$

which is equivalent to proving

$$\left(\delta_x + \delta_{\overline{x}} (1 - \chi) \right) \left(1 + \eta\sigma - \eta \left(\sigma - \gamma^{-1} \right) \left(\beta_x + \chi (1 - \beta_x) \right) \right) +$$
$$\left(\gamma\sigma - 1 \right) (1 - \chi) \left(\beta_x + \eta\gamma^{-1} \left(\delta_x \beta_x + \delta_{\overline{x}} \left(\beta_x + \chi (1 - \beta_x) \right) \right) \right) > 0.$$

Let me show separately that

$$\left(\delta_x + \delta_{\overline{x}} (1 - \chi) \right) \left(1 + \eta\sigma - \eta\sigma \left(\beta_x + \chi (1 - \beta_x) \right) \right) + \left(\gamma\sigma - 1 \right) (1 - \chi) \beta_x > 0,$$

and

$$\left(\delta_x + \delta_{\overline{x}} (1 - \chi) \right) \left(\beta_x + \chi (1 - \beta_x) \right) + - (1 - \chi) \left(\delta_x \beta_x + \delta_{\overline{x}} \left(\beta_x + \chi (1 - \beta_x) \right) \right) > 0.$$

48

For the first, it is sufficient to observe that

$$\delta_x + \delta_{\overline{x}}\left(1 - \chi\right) - \left(1 - \chi\right)\beta_x > 0,$$

follows from $\delta_x + \delta_{\overline{x}} > \beta_x$. For the second, it is enough to see that

$$\delta_x\left(\left(1 - \chi\right)\beta_x + \chi\right) - \left(1 - \chi\right)\delta_x\beta_x > 0.$$

9.3 Algebra for the proof of Proposition 4

To prove that $\mu_a \neq \mu_a^0$, I suppose the contrary and use the necessary conditions for an equilibrium to obtain a contradiction. By definition, $\mu_a = \mu_a^0$ implies $\phi_x = 0$. Summing (38) and (39) and using

$$\psi_x + \psi_{\overline{x}} = \frac{1 + \eta}{\gamma + \eta}, \tag{76}$$

shows that

$$\phi_a + \gamma\psi_a = \frac{1}{\delta_x + \delta_{\overline{x}}}\frac{\gamma\left(1 + \eta\right)}{\gamma + \eta}.$$

Substituting in (39) then gives

$$\psi_{\overline{x}} = \frac{\delta_x}{\delta_x + \delta_{\overline{x}}}\frac{1 + \eta}{\gamma + \eta}.$$

Substituting the last three expression in (36) then gives

$$0 = \eta\left(\frac{1 + \eta}{\gamma + \eta}\beta_x + \frac{\delta_x}{\delta_x + \delta_{\overline{x}}}\frac{1 + \eta}{\gamma + \eta}\chi\left(1 - \beta_x\right) - 1\right) + \frac{\beta_x}{\delta_x + \delta_{\overline{x}}}\frac{\gamma\left(1 + \eta\right)}{\gamma + \eta} - 1.$$

This leads to a contradiction because

$$\frac{\eta}{\gamma + \eta}\beta_x + \frac{\delta_x}{\delta_x + \delta_{\overline{x}}}\frac{\eta}{\gamma + \eta}\chi\left(1 - \beta_x\right) + \frac{\beta_x}{\delta_x + \delta_{\overline{x}}}\frac{\gamma}{\gamma + \eta} < 1,$$

where the inequality follows because

$$\beta_x + \frac{\delta_x}{\delta_x + \delta_{\overline{x}}}\chi\left(1 - \beta_x\right) < 1$$

and

$$\frac{\beta_x}{\delta_x + \delta_{\overline{x}}} < 1.$$

The last inequality follows from (30) and (31).

9.4 Proof of Lemma 3: case $\gamma = 1$

When $\gamma = 1$ steps analogous to the ones used in case $\gamma \neq 1$ lead to

$$\mathbb{E}\left[U\left(C_{it}, N_{it}\right)|a_{t-1}\right] = \psi_0 + \psi_a a_{t-1} - \frac{1}{1 + \eta}e^{(1+\eta)(k_2 + \psi_0)},$$

and the optimal choice of τ (and ψ_0) gives the first-order condition

$$1 = e^{(1+\eta)(k_2 + \psi_0)},$$

which implies that

$$\psi_0 = k_1 - k_2 = w,$$

as $\gamma = 1$ implies that $k_1 = 0$. The unconditional expected utility is then

$$\sum_{t=0}^{\infty} \beta^t \mathbb{E}\left[U\left(C_{it}, N_{it}\right)\right] = -\frac{1}{1+\eta}\frac{1}{1-\beta} + \sum_{t=0}^{\infty} \beta^t \mathbb{E}\left[\rho a_{t-1}\right] + \frac{1}{1-\beta}w,$$

which gives the expression in the text, with

$$w_0 \equiv -\frac{1}{1+\eta}\frac{1}{1-\beta} + \sum_{t=0}^{\infty} \beta^t \mathbb{E}\left[\rho a_{t-1}\right].$$