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Working Papers Series Research Department WP-99-19

Federal Reserve Bank of Chicago

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#### Abstract

This paper illustrates a particular limited information strategy for assessing the empirical plausibility of alternative quantitative general equilibrium business cycle models. The basic strategy is to test whether a model economy can account for the response of the actual economy to an exogenous shock. Here we concentrate on the response of aggregate hours worked and real wages to a fiscal policy shock. The fiscal policy shock is identified with the dynamic response of government purchases and average marginal income tax rates to an exogenous increase in military purchases. Burnside, Eichenbaum and Fisher (1999) show that standard Real Business Cycle models cannot account for the salient features of how hours worked and after - tax real wages respond to a fiscal policy shock. In this paper we show that this failure extends to a class of business cycle models in which the labor market is characterized by efficiency wages.

<sup>\*</sup>The views expressed in this paper do not necessarily represent the views of the Federal Reserve Bank of Chicago, the Federal Reserve System or the World Bank. Martin Eichenbaum gratefully acknowledges the financial support of a grant from the National Science Foundation to the National Bureau of Economic Research.

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#### 1. Introduction

This paper illustrates a particular limited information strategy for assessing the empirical plausibility of alternative quantitative general equilibrium business cycle models. The basic strategy is to test whether a model economy can account for the response of the actual economy to an exogenous shock. To be useful, this strategy requires that we know how the actual economy responds to the shock in question and that different models generate different predictions for that response. Here we concentrate on the response of aggregate hours worked and real wages to a fiscal policy shock.<sup>1</sup> The fiscal policy shock is identified with the dynamic response of government purchases and average marginal income tax rates to an exogenous increase in military purchases.

Burnside, Eichenbaum and Fisher (1999) (BEF) show that standard Real Business Cycle (RBC) models can account for the salient features of how hours worked and after - tax real wages respond to a fiscal policy shock, but only if it is assumed that marginal tax rates are constant. When this counterfactual assumption is abandoned, RBC models cannot account for the response of the economy to a fiscal policy shock. For example, high labor supply elasticity versions of these models counterfactually predict that after a fiscal policy shock, government purchases are negatively correlated with hours worked. In reality, after a fiscal policy shock, government purchases and hours worked are strongly positively correlated. Low labor supply elasticity versions of these models greatly understate the conditional volatility of hours worked. So regardless of what is assumed about the elasticity of labor supply, the model cannot account for the facts. Ramey and Shapiro (1998) show that various two sector versions of the RBC model generate predictions for aggregate hours worked and real wages that are very similar to those of the one sector model. So presumably these models too would fail our diagnostic test. Rotemberg and Woodford (1992) and Devereux, Head and Lapham

<sup>&</sup>lt;sup>1</sup>See Christiano, Eichenbaum and Evans (1998) for a similar approach to evaluating alternative models of the monetary transmission mechanism.

(1996) study the effects of changes in government purchases in stochastic general equilibrium models which incorporate increasing returns and oligopolistic pricing. Since their models imply that a positive shock to government purchases raises real wages, they would fail our test.

In this paper, we examine a variant of Alexopoulos' (1998) model of efficiency wages. We find that, like the other models discussed above, the efficiency wage model cannot account for the quantitative responses of hours worked and of real wages to a fiscal policy shock. In particular it shares the strengths and weaknesses of high labor supply elasticity RBC models. So the model can account for the conditional volatility of real wages and hours worked. But it cannot account for the temporal pattern of how these variables respond to a fiscal policy shock and generates a counterfactual negative conditional correlation between government purchases and hours worked. Integrating over the results we have obtained with the different models, we conclude there is a puzzle. Measurement is ahead of theory.

To identify exogenous changes to government purchases and tax rates we build on the approach used by Ramey and Shapiro (1998) who focus on exogenous movements in defense spending. To isolate such movements, Ramey and Shapiro (1998) identify three political events that led to large military buildups which were arguably unrelated to developments in the domestic U.S. economy. We refer to these events as 'Ramey-Shapiro episodes'. Controlling for other shocks, we explore how the U.S. economy behaved after the onset of the Ramey-Shapiro episodes and use the results in two ways. First, we use it to construct the basic experiment that is conducted in the model. Specifically we confront agents in the model with a sequence of changes in total government purchases and marginal income tax rates that coincides with the estimated dynamic response of those variables to a Ramey and Shapiro episode. Second, we use the estimated dynamic response paths of aggregate hours worked and after - tax real wages as the standard against which we assess our model's performance.

The remainder of this paper is organized as follows. Section 2 summarizes our

evidence regarding the dynamic effects of a fiscal shock. Section 3 discusses our procedure for results to assess the empirical plausibility of competing business cycle models. Section 4 presents a version of Alexopoulos' (1998) efficiency wage model, modified to allow for fiscal shocks. Section 5 assesses the quantitative properties of the model. Finally, Section 6 contains concluding remarks.

# 2. Evidence on the Effects of a Shock to Fiscal Policy

In this section we accomplish two tasks. First, we describe our strategy for estimating the effects of an exogenous shock to fiscal policy. Second, we present the results of implementing this strategy.<sup>2</sup>

## 2.1. Identifying the Effects of A Fiscal Policy Shock

Ramey and Shapiro (1998) use a 'narrative approach' to isolate three arguably exogenous events that led to large military buildups and total government purchases: the beginning of the Korean War (1950:3), the beginning of major U.S. involvement in the Vietnam War (1965:1) and the beginning of the Carter-Reagan defense buildup (1980:1).

To estimate the exogenous movements in government purchases,  $G_t$ , and average marginal tax rates,  $\tau_t$ , induced by the onset of a Ramey-Shapiro episode and the corresponding movements in other variables, we use the following procedure. Define the set of dummy variables  $D_t$ , where  $D_t = 1$  if  $t = \{1950:3, 1965:1, 1980:1\}$  and zero otherwise. We include  $D_t$  as an explanatory variable in a vector autoregression (VAR). Suppose that the  $k \times 1$  vector stochastic process  $Z_t$  has the representation:

$$Z_t = A_0 + A_1(L)Z_{t-1} + A_2(L)D_t + u_t, (2.1)$$

where  $A_1(L)$  and  $A_2(L)$  are finite ordered matrix polynomials in nonnegative

<sup>&</sup>lt;sup>2</sup>This section is a condensed version of a similar section in BEF.

powers of the lag operator,  $Eu_t = 0$ ,

$$Eu'_t u_{t-s} = \begin{cases} 0 \text{ for all } s \neq 0 \\ \Sigma \text{ for } s = 0, \end{cases}$$

and  $\Sigma$  is a positive definite  $k \times k$  matrix. The  $A_i$  can be consistently estimated using least squares. The response of  $Z_{it+k}$ , the  $i^{th}$  element of  $Z_{t+k}$ , to the onset of a Ramey-Shapiro episode at date t, is given by the coefficient on  $L^k$  in the expansion of  $[I - A_1(L)L]^{-1} A_2(L)$ . Note that this procedure assumes that the Ramey-Shapiro episodes are of equal intensity.<sup>3</sup>

### 2.2. Empirical Results

In this subsection we present the results of implementing the procedure discussed above. Unless otherwise noted, the vector  $Z_t$  contains the log level of time t real GDP, the three month Treasury bill rate, the log of the producer price index of crude fuel, the log level of a measure of the average marginal income tax rate, the log level of real government purchases and either the log level of real wages or the log of aggregate hours worked.<sup>4</sup> Our measure of the tax rate, taken from Stephenson (1998), is an updated version of the average marginal statutory tax rate constructed by Barro and Sahasakul (1983). It is a weighted average of statutory marginal tax rates, where the weights are the shares of adjusted gross income subject to each statutory rate.<sup>5</sup> In all cases we included six lagged values of all variables in the VAR. All estimates are based on quarterly data from 1947:1 to 1994:4.

Figure 1 reports the responses of real government purchases and the tax rate to the onset of a Ramey-Shapiro episode. The solid lines display point esti-

<sup>&</sup>lt;sup>3</sup>BEF modify this procedure to allow the different episodes to have different intensities. They show that the qualitative nature of the estimated impulse response functions does not depend on whether one imposes the equal intensity assumption or not.

<sup>&</sup>lt;sup>4</sup>An appendix available from us describes the data in more detail.

<sup>&</sup>lt;sup>5</sup>See Stephenson (1998) for refinements to the Barro-Sahasakul measure. We found that our results were insensitive to ignoring these refinements, and to using another tax rate measure suggested by Seater (1985).

mates of the coefficients of the dynamic response functions.<sup>6</sup> The dashed lines are sixty-eight percent confidence intervals.<sup>7</sup> Consistent with results in Ramey and Shapiro (1998) and Edelberg, Eichenbaum and Fisher (1999), the onset of a Ramey-Shapiro episode leads to a large, persistent, hump-shaped rise in government purchases, with a peak response of 13 percent roughly 6 quarters after the shock. In addition, the tax rate rises in a hump-shaped pattern, mirroring the hump-shaped dynamic response function of government purchases, The peak response of 2.3 percentage points occurs roughly 7 quarters after the onset of a Ramey-Shapiro episode. This represents a rise of roughly 13 percent in the tax rate relative to its value in 1949.

Figure 1 also displays the responses of aggregate hours worked in the private sector and the after - tax real wage in the manufacturing sector to the onset of a Ramey-Shapiro episode. Two key results emerge here. First, hours worked has a delayed, hump-shaped response with a peak response of over 2 percent occurring roughly 6 periods after the fiscal shock. Second, the after-tax real wage falls after the fiscal policy shock.<sup>8</sup>

# 3. A Limited Information Diagnostic Procedure

The previous section displayed our estimates of the dynamic consequences of a fiscal policy shock to government purchases, average marginal tax rates, hours worked and real wages. In this section we provide a short discussion, taken from BEF, of a limited information procedure for using these results to assess the

<sup>&</sup>lt;sup>6</sup>The impulse response function of the tax rate is reported in percentage points. The other impulse response functions are reported in percentage deviations from each variable's unshocked path.

<sup>&</sup>lt;sup>7</sup>See BEF for details of the construction of these confidence bands. These confidence bands assume that the dates marking the onset of the Ramey Shapiro episodes are known with certainty. We have conducted experiments to quantify the importance of "date uncertainty". Specifically we take into account the possibility that the exact Ramey-Shapiro dates might be off by up to 3 quarters each. We find our results to be robust to these experiments.

<sup>&</sup>lt;sup>8</sup>In BEF we find qualitatively similar results for other measures of hours worked and real wages.

empirical plausibility of competing models.

We partition  $Z_t$  as

$$Z_t = \left(\begin{array}{c} \bar{Z}_t \\ F_t \end{array}\right),$$

where  $F_t = (G_t \ \tau_t)'$  and  $\bar{Z}_t$  is a  $(k-2) \times 1$  vector of the other variables. For the class of models that we consider the equilibrium law of motion for  $Z_t$  takes the form of a system of linear difference equations:

$$B_0 Z_t = \kappa + B_1(L) Z_{t-1} + B_2(L) D_t + \varepsilon_t. \tag{3.1}$$

Here  $B_1(L)$  is a finite-ordered matrix polynomial in the lag operator,  $B_2(L) = [0'_{k-2} \quad M(L)']'$ ,  $0_{k-2}$  is a k-2 vector of zeroes, and the elements of  $\varepsilon_t = (\varepsilon'_{\bar{z}t} \quad \varepsilon'_{Ft})'$  are uncorrelated with each other, with  $D_t$  and with lagged values of  $Z_t$ . The last two rows of (3.1) are the policy rule for the fiscal variables,  $F_t$ . With this specification the only variables that are directly affected by  $D_t$  are those in  $F_t$ . The onset of a Ramey-Shapiro episode  $(D_t = 1)$ , sets off a chain of exogenous movements in  $F_t$  which leads to movements in  $\bar{Z}_t$  through the mechanisms embedded in the particular model under consideration.

Our theoretical model, (3.1), and a VAR of the form (2.1) are equivalent when

$$A_0 = B_0^{-1} \kappa, \quad A_1(L) = B_0^{-1} B_1(L), \quad A_2(L) = B_0^{-1} B_2(L), \quad u_t = B_0^{-1} \varepsilon_t.$$
 (3.2)

To characterize impulse response functions we use the moving average representation (MAR) corresponding to (3.1) given by

$$Z_t = \Pi_0 + \Pi(L)\varepsilon_t + H(L)D_t. \tag{3.3}$$

By assumption,  $\{\Pi_i\}$  and  $\{H_i\}$  form square summable sequences. Note that H(L) completely characterizes the dynamic response path of the vector  $Z_t$  to the time t realization of  $D_t$ . In particular, the response of  $Z_{t+j}$  is given by the coefficient on  $L^j$  in H(L).

It is useful to write the last two rows of (3.3) as

$$F_t = \Pi_0^2 + \Pi^2(L)\varepsilon_t + H^2(L)D_t \tag{3.4}$$

We do not identify the elements of  $\varepsilon_t$  in our empirical analysis. However, under our assumptions,  $D_t$  is orthogonal to  $\varepsilon_t$ . So we can study the effects of a change in  $D_t$  abstracting from movements in  $\varepsilon_t$ . This is equivalent to working with the exogenous variable policy rule

$$F_t = \Pi_0^2 + H^2(L)D_t. \tag{3.5}$$

To assess the empirical plausibility of a model's implications for an exogenous shock to fiscal policy we can proceed as follows.

- 1. Estimate the VAR given by (2.1) using U.S. data. This yields estimates  $\hat{A}_0$ ,  $\hat{A}_1(L)$  and  $\hat{A}_2(L)$ .
- 2. Use the estimates  $\hat{A}_0$ ,  $\hat{A}_1(L)$  and  $\hat{A}_2(L)$  to obtain a moving average representation for  $Z_t$  that is equivalent to (3.3):

$$Z_{t} = \left[I - \hat{A}_{1}(1)\right]^{-1} \hat{A}_{0} + \left[I - \hat{A}_{1}(L)L\right]^{-1} \hat{A}_{2}(L)D_{t} + \left[I - \hat{A}_{1}(L)L\right]^{-1} u_{t}$$
$$= \Pi_{d0} + H_{d}(L)D_{t} + \tilde{\Pi}_{d}(L)u_{t}.$$

Notice that  $H_d^1(L)$ , the first  $(k-2) \times 1$  sub-block of  $H_d(L)$ , characterizes the dynamic responses of the non-fiscal variables,  $\bar{Z}_t$ , to the onset of a Ramey-Shapiro episode.

- 3. Use  $H_d^2(L)$ , the last  $2 \times 1$  sub-block of  $H_d(L)$ , to characterize the exogenous variable fiscal policy rule in the theoretical model given by (3.5).
- 4. Using this rule, and calibrating the remaining model parameters, study the theoretical model's implications for the dynamic responses of the non-fiscal variables to the onset of the Ramey-Shapiro episode. Denote the polynomial in the lag operator that characterizes these responses as  $H_m^1(L)$ .
- 5. Compare these responses, obtained using the theoretical model, to their empirical counterparts, estimated in the second step. Abstracting from sampling uncertainty and the linearity assumptions implicit in the VAR analysis,

the two sets of response functions should be the same, i.e. it should be the case that  $H_m^1(L) = H_d^1(L)$ .

Results in Burnside and Eichenbaum (1996) suggest that uncertainty about the structural parameters of the model describing preferences and technology are unlikely to significantly affect inference for the models we consider. Hence, we ignore this source of uncertainty. We do take into account sampling uncertainty pertaining to the estimated response of the U.S. economy to a fiscal policy shock. Sampling uncertainty about the  $\hat{A}_i$  from our VAR generates uncertainty about our data-based estimates of the impulse responses  $H^1(L)$  denoted  $H^1_d(L)$  and our data-based estimates of  $H^2(L)$  denoted  $H^2_d(L)$ . In addition sampling uncertainty in  $H^2_d(L)$  feeds into uncertainty about our model-based estimates of  $H^1(L)$ , denoted  $H^1_m(L)$ .

BEF show how to account for these sources of sampling uncertainty when assessing the ability of the model to account for various conditional moments of the data, i.e. moments pertaining to the behavior of the economy conditional on a fiscal shock having occurred. One way to estimate such a moment is to use a point estimate,  $\hat{\theta}_H$ , of the vector of coefficients,  $\theta_H$ , characterizing H(L), in a way that does not involve the use of an economic model. We let  $d(\hat{\theta}_H)$  denote the point estimate of a conditional moment obtained in this way. A different way to estimate the conditional moment is to use an economic model along with values for the parameters describing agents' preferences and technology and an estimate of the coefficients characterizing the exogenous variable policy rule,  $\theta_H^2$ . Note that  $\theta_H^2$  is a subset of  $\theta_H$ . We denote by  $m(\hat{\theta}_H^2)$  the point estimate of the conditional moment in question derived from the economic model.

<sup>&</sup>lt;sup>9</sup>As discussed in BEF the previous conclusion depends on the following simplification regarding agents' views about the law of motion for  $D_t$ : agents expect  $D_t = 0$  for all t. In addition, a realization of  $D_t = 1$  does not affect agents' future expectations of  $D_t$ , i.e. they continue to expect that future values of  $D_t$  will equal zero. So from their perspective, a realization of  $D_t = 1$  is just like the realization of an iid exogenous shock to  $F_t$ . But once such a shock occurs, the expected response of  $F_{t+j}$  is given by the coefficient on  $L^j$  in the polynomial  $H^2(L)$ .

Let

$$s(\theta_H) = d(\theta_H) - m(\theta_H^2).$$

We are interested in testing hypothesis of the form:

$$H_0: s(\theta_H) = 0.$$

An implication of results in Eichenbaum, Hansen and Singleton (1988) and Newey and West (1987) is that the test statistic

$$J = s(\widehat{\theta}_H)' \widehat{\text{var}} \left[ s(\widehat{\theta}_H) \right]^{-1} s(\widehat{\theta}_H)$$
(3.6)

is asymptotically distributed as a chi-squared distribution with 1 degree of freedom, where  $\widehat{\text{var}}\left[s(\widehat{\theta}_H)\right]$  is a consistent estimator of  $\text{var}\left[s(\widehat{\theta}_H)\right]$ . Below we use this test statistic to formally assess the ability of an efficiency wage model to account for various conditional moments of the data.

# 4. A General Equilibrium Efficiency Wage Model

In this section we describe a version of Alexopoulos' (1998) efficiency wage model, modified to allow for distortionary income taxes. The basic structure of the model is similar to a standard RBC model with the exception of the labor market. In contrast to RBC models, we assume that a worker's effort is imperfectly observable by firms. Competitive firms offer contracts that induce workers not to shirk on the job. These contracts specify a real wage, an effort level, and a specification that a worker will be dismissed and paid only a fraction of the wage if he is

$$\widehat{\operatorname{var}}\left[s(\widehat{\theta})\right] = \frac{1}{N-1} \sum_{i=1}^{N} \left(s(\theta_{Hi}) - \overline{s}(\theta_{Hi})\right)^{2},$$

where  $\overline{s}(\theta_{Hi}) = (1/N) \sum_{i=1}^{N} s(\theta_{Hi})$ , is a consistent estimate of var  $\left[s(\widehat{\theta}_{H})\right]$ .

 $<sup>^{10}</sup>$ To generate an estimate of var  $[s(\theta_H)]$  we use the same bootstrap procedure employed to compute confidence intervals for the impulse response functions estimated in the data. Specifically, let  $\theta_{Hi}$  be the point estimate of the moving average coefficients of  $Z_t$  implied by the VAR coefficients generated by the ith bootstrap draw, i = 1, ..., N, where N = 500. Then

caught shirking on the job. Given a no bonding constraint, the supply for labor will in general exceed the demand for labor, resulting in unemployment. Whether the ex-post utility of employed workers exceeds the utility of unemployed individuals depends on the nature of risk sharing among members of the household. In the version of Alexopoulos' model discussed below, risk sharing is imperfect (by assumption) and unemployed workers are worse off, ex-post, than employed workers.<sup>11</sup>

#### 4.1. The Government

The government faces the flow budget constraint

$$G_t < \tau_t (r_t - \delta) K_t + \tau_t W_t n_t h + \Phi_t$$

where  $G_t$  is real government purchases,  $\tau_t$  is the marginal tax rate,  $r_t$  is the rental rate of capital,  $0 < \delta < 1$  is the depreciation rate,  $W_t$  is the real wage rate,  $n_t$  is employment, and  $\Phi_t$  is lump-sum taxes. By assumption h, hours worked per worker, is constant so that hours and employment move in proportion to one another. The fiscal policy rule is of the form given by the last two rows of (3.1).

## 4.2. Households

The representative household owns the stock of capital, makes all capital related decisions, and pays both capital income taxes and lump-sum taxes. The household consists of a unit measure continuum of individuals. If individuals earn labor income, they must pay taxes on it. Employed members of the household partly insure the income of unemployed members of the household.

The household accumulates capital according to

$$K_{t+1} = (1 - \delta)K_t + I_t, \tag{4.1}$$

<sup>&</sup>lt;sup>11</sup>Alexopolous (1998) shows that in her model, with complete risk sharing, unemployed workers are ex post better off than employed workers. This version of her model is observationally equivalent to Hansen's (1985) RBC model with indivisible labor supply. See Woodford (1994) for a similar argument.

where  $K_t$  is the beginning of period t capital stock and  $I_t$  is time t investment. The household rents capital to firms at the competitively determined rate  $r_t$ , and rental income, net of depreciation, is taxed at the margin. The household uses its rental income net of these taxes and any lump-sum taxes that it pays to buy new capital. It distributes any remaining funds equally among the individual members of the household. We denote this common income as

$$C_t^h = (1 - \tau_t) (r_t - \delta) K_t - \Phi_t - (K_{t+1} - K_t).$$
(4.2)

Members of the household derive their remaining income from selling labor services to firms or from partial unemployment insurance provided by the household.<sup>12</sup> They are assumed to take both the terms of labor contracts and firms' demand for labor parametrically. In addition, from the perspective of firms, all individuals look alike. So we can think of the employment outcome for any individual as being determined completely randomly. Some individuals will be employed, while others will be unemployed. Under our assumptions, no individual would choose to be unemployed, because the ex-post utility of such an individual will be less than or equal to that of an employed individual.

Employed workers will either work and exert the level of effort required by the labor contract, denoted  $e_t$ , or they will shirk. The labor contract stipulates that if a worker is caught shirking, they will be fired and receive only a fraction s of their wages. The technology for detecting shirkers is imperfect, so that a shirker is only caught with probability d.

The household only observes the initial employment status of its members, not whether they shirked or were fired. Each employed member of the household transfers  $\Psi_t$  units of income to a pool which is distributed equally among the unemployed members of the household. By assumption, the household chooses the level of the transfer so that unemployed members of the household will be at least as well off as any shirker caught by the firm would be.<sup>13</sup> Finally, we assume

 $<sup>^{12}</sup>$ It is straightforward to reformulate the model so that a self financing unemployment insurance program is provided by the government rather than the household.

<sup>&</sup>lt;sup>13</sup>Alexopoulos (1998) also considers the case in which there is perfect insurance across members

that labor income is taxed. Members of the household who pay the insurance transfer receive no tax credit for it, while recipients of transfers do not pay taxes on that type of income.

Our assumptions imply that the consumption of an employed individual who does not shirk is constrained by

$$C_t \le C_t^h + (1 - \tau_t) W_t h - \Psi_t.$$
 (4.3)

An employed individual who shirks but does not get caught faces the same constraint. An employed individual who shirks and is caught only receives the fraction s of his contractual wages. Hence, his consumption,  $C_t^s$ , is constrained by

$$C_t^s \le C_t^h + (1 - \tau_t) sW_t h - \Psi_t.$$
 (4.4)

Suppose that  $n_t$  members of the household are employed while  $1-n_t$  are unemployed. This implies that the transfer received by each member of the unemployed is given by  $n_t\Psi_t/(1-n_t)$ . Hence, the consumption of an unemployed individual,  $C_t^u$ , is constrained by

$$C_t^u \le C_t^h + \frac{n_t}{1 - n_t} \Psi_t. \tag{4.5}$$

The instantaneous utility of an individual with consumption level C, and a positive level of effort e, is given by

$$\log(C) + \eta \log(T - \xi - he)$$

while the instantaneous utility of an individual with consumption level C who exerts no work effort is given by

$$\log(C) + \eta \ln(T),$$

where  $\eta > 0$ , T is the time endowment, and  $\xi$  is the fixed cost of exerting nonzero effort.

of the family.

Thus, an employed worker who does not shirk has utility

$$\log(C_t) + \eta \ln(T - \xi - he_t)$$

where  $e_t$  is determined by the contract offered by the firm.

An employed worker who shirks but is not caught has utility

$$\log(C_t) + \eta \ln(T)$$

while a shirker who is caught has utility

$$\log(C_t^s) + \eta \ln(T)$$
.

Finally, an unemployed individual has utility

$$\log(C_t^u) + \eta \ln(T).$$

Let  $n_t^s$  be the number of shirkers and let d be the probability of a shirker being caught. Since there is a continuum of individuals, this implies that  $dn_t^s$  is the number of shirkers caught and  $(1-d)n_t^s$  the number of shirkers not caught.

Notice that the effective leisure time of caught shirkers and unemployed individuals is the same. If the family sets the transfer so that their consumption and utility levels are the same this will imply that

$$\Psi_t = (1 - n_t) (1 - \tau_t) s W_t h. \tag{4.6}$$

The household takes the effort level and wage rate as given in the contracts offered by the firm. The household also takes firms' labor demand as given. The only decisions the household makes are those regarding capital and the level of common income, in order to maximize the expected utility of an individual household member:

$$\max_{\{C_t, K_{t+1}\}_{t=0}^{\infty}} \quad E_0 \sum_{t=0}^{\infty} \beta^t \left\{ (n_t - n_t^s) \left[ \log(C_t) + \eta \log(T - \xi - he_t) \right] + n_t^s \left[ (1 - d) \log(C_t) + d \log(C_t^s) + \eta \log(T) \right] + (1 - n_t) \left[ \log(C_t^u) + \eta \ln(T) \right] \right\}.$$

subject to (4.2), (4.3), (4.4), (4.5) and (4.6).

#### 4.3. The Firm

A perfectly competitive firm produces output using the technology

$$Y_t = K_t^{\alpha} \left( n_t h e_t X_t \right)^{1-\alpha}$$

where  $n_t$  is the number of workers it hires. It maximizes its profits

$$\max_{W_t, n_t, K_t, e_t} K_t^{\alpha} \left( n_t h e_t X_t \right)^{1-\alpha} - W_t n_t h - r_t K_t$$

subject to the 'no shirking' condition:

$$\log(C_t) + \eta \log(T - \xi - he_t) \ge (1 - d) \log(C_t) + d \log(C_t^s) + \eta \ln(T). \tag{4.7}$$

According to (4.7) the expected utility of an employee who does not shirk is at least as great as the expected utility of an employee who shirks. Here we assume that all employed workers are monitored and the exogenous probability of being caught shirking is d. In equilibrium there is no shirking. Given a wage rate,  $W_t$ , we can think of (4.7) as indicating a maximal level of effort the firm will be able to extract from workers. Rearranging the constraint we see that

$$e_t \le e(W_t) = \frac{T - \xi}{h} - \frac{T}{h} \left(\frac{C_t^s}{C_t}\right)^{\frac{d}{\eta}}.$$

The firm takes the level of the intra-household transfer  $\Psi_t$  parametrically. This is what allows us to write the expression on the right-hand side as a function, from the firm's point of view, only of its choice regarding  $W_t$ .

Alexopoulos (1998) shows that the first-order conditions for the firm, along with the expression for  $e(W_t)$  imply that  $C_t/C_t^s$  is a constant given by  $\chi$  where  $\chi$  satisfies

$$Td(1-s\chi)(\chi-1) = \eta(1-s)\left[ (T-\xi)\chi^{1+d/\eta} - T\chi \right].$$
 (4.8)

This is a nonlinear equation in  $\chi$  that can be solved numerically.

The level of employment,  $n_t$ , which characterizes the solution to the firm's problem will not in general coincide with the number of workers who wish to

work at the contract characterized by  $(w_t, e(w_t))$ . As long as the demand for workers is less than the supply of workers, (4.7) will hold with equality and there will be equilibrium unemployment. We confine ourselves to calibrated versions of the model in which this is the case and in which all of the inequality constraints above hold with equality.

We use the log linearization procedure described by Christiano (1998) to solve for the competitive equilibrium of this economy.

## 5. Quantitative Properties of the Model

This sections assesses the quantitative properties of our model. We proceed in three steps. First, we discuss how we calibrate the model's parameters. Second, we discuss how the model responds over time to a fiscal shock. Third, we formally assess the ability of the model to account for various conditional moments of the data.

#### 5.1. Model Calibration

Alexopoulos (1998) estimates and analyzes the lump-sum version of the model above using the Generalized Method of Moments procedures discussed in Christiano and Eichenbaum (1992). Here we simply calibrate the model by choosing parameters to match a number of features in postwar US data. We assume that the time endowment is T=1369 which corresponds to a quarter consisting of 15 hours per day. We assume that the fixed cost of providing nonzero effort is given by  $\xi=16$ . We parameterize  $\chi$  to imply that being unemployed lowers a worker's consumption by about 22 percent (see Gruber 1997). This requires  $\chi=1.285$ . We set  $\beta=1.03^{-1/4}$ ,  $\gamma=1.004$ ,  $\alpha=0.34$ , and  $\delta=0.021$ . We chose s=0.72 so that the steady state value of n would be equal to its sample average, 0.93. Finally, (4.8) can be used to solve for  $d/\eta$  as a function of parameters whose values we have already specified. This results in a value of  $d/\eta$  equal to about 0.062.

## 5.2. Impulse Response Functions

Figure 2 displays the dynamic response functions of hours worked, investment, before and after - tax real wages, household transfers of consumption  $(C_t^h)$ , as well as the marginal tax rate to a fiscal policy shock. Notice that hours worked  $(n_th)$  initially rises roughly 5% in the impact period of the shock (time 0) and then declines, reaching its pre shock level after about 3 quarters. Thereafter  $n_th$  continues to fall, reaching a maximal decline of about 7% around period 8, roughly the same time as the maximal rise in government purchases and the marginal tax rate. So, as in the case of one sector model RBC analyzed in BEF, hours worked decline when income tax rates and government purchases are high. The behavior of the before - tax real wage mirrors the movements in  $n_th$ , initially declining and then rising above its pre shock level roughly 3 periods after the shock. In contrast the after - tax real wage remains below its pre shock level for over twelve periods. Finally notice that the response of investment is qualitatively very similar to that of  $n_th$ , while  $C_t^h$  moves in the opposite manner, falling when  $n_th$  rises, and climbing when  $n_th$  falls.

To see the intuition behind the forces at work in the model, note that equations (4.3) and (4.4), evaluated at equality, imply

$$\frac{C_t}{C_t^s} = \frac{C_t^h + (1 - \tau_t)W_t h - \Psi_t}{C_t^h + (1 - \tau_t)sW_t h - \Psi_t}.$$
(5.1)

It follows that, other things equal, the ratio of an employed worker's consumption to that of a fired worker's consumption,  $(C_t/C_t^s)$  is a decreasing function of  $C_t^h$ .

Suppose that all taxes are lump-sum. Then (4.2) can be written as

$$C_t^h = (r_t - \delta) K_t - \Phi_t - (K_{t+1} - K_t).$$
 (5.2)

With this specification, increases in government purchases are financed by increases in  $\Phi_t$ . So, other things equal, an increase in  $G_t$  causes  $C_t^h$  to fall and  $(C_t/C_t^s)$  to rise. But in equilibrium  $(C_t/C_t^s)$  must be equal to the constant  $\chi$ .

<sup>&</sup>lt;sup>14</sup>This result is reminiscent of the balanced budget case in Baxter and King (1993).

So some other factor in (5.1) must adjust. In equilibrium workers are indifferent between shirking and not shirking. So a rise in  $(C_t/C_t^s)$  would cause workers to strictly prefer not to shirk. In such a situation, firms could cut real wages without inducing shirking behavior. Equations (4.6) and (5.1) can be used to show that a decline in  $W_t$  will move  $C_t/C_t^s$  back down towards its constant value  $\chi$ . The net result then of a fiscal shock is a decline in the real wage and an increase in employment.<sup>15</sup>

When the rise in government purchases is persistent there will be a significant rise in the present value of the household's taxes. As in the neoclassical model, this rise induces the household to increase investment. From (5.2) we see that a rise in  $(K_{t+1}-K_t)$  acts like a rise in  $\Phi_t$ . This reinforces the effects discussed above, exerting upward pressure on employment and downward pressure on real wages. For reference Figure 3 presents the dynamic response functions for the model economy under the assumption that the rise in government purchases induced by a fiscal policy shock is entirely financed by lump sum taxes. Consistent with this intuition we see from Figure 3 that in the lump sum tax case, the fiscal policy shock leads to a hump shaped, persistent rise in  $n_t$ , and investment, as well as a persistent, hump shaped fall in before and after - tax real wage rates.

To understand the impact of distortionary taxes on the model, recall that in their presence,  $C_t^h$  is given by

$$C_t^h = (1 - \tau_t) (r_t - \delta) K_t - \Phi_t - (K_{t+1} - K_t).$$

A rise in  $\tau_t$  has two effects: (i) it directly reduces  $C_t^h$  via the term  $(1-\tau_t)(r_t-\delta)K_t$ , and (ii) it indirectly affects  $C_t^h$  via its effect on  $K_{t+1}-K_t$ . The first effect acts much like the increase in  $\Phi_t$  described above. Other things equal then, the rise in  $\tau_t$  tends to magnify the initial fall in real wages and the rise in  $n_t$ . The second effect works through the household's incentive to invest in capital. Other things equal, a higher future value of  $\tau_{t+1}$  reduces the time t return to capital and the incentive to invest at time t. We refer to this as the one period ahead tax effect. In

<sup>&</sup>lt;sup>15</sup>The rise in  $n_t$  is determined by the firm's demand for labor.

addition there is an intertemporal effect associated with movements in  $\tau_t$  which induces the household to shift investment towards periods in which  $\tau_t$  is relatively low.

Recall that according to our estimates,  $\tau_t$  responds in a hump shaped manner to a fiscal period shock, rising by relatively small amounts in the first few periods. So the one period ahead tax effect initially has a relatively small dampening effect on investment. But the intertemporal tax effects are quite large. Agents have an incentive to invest to pay off their higher tax bills and they may as well do so in periods in which  $\tau_t$  is relatively small. Consistent with this, we see from Figures 2 and 3 that initially investment rises by more in the distortionary tax case than in the lump sum tax case. Other things equal this means that  $C_t^h$  falls by more and  $(C_t/C_t^s)$  rises by more in the distortionary tax case. So firms must lower real wages by relatively more in the distortionary case to prevent shirking, which in turn leads to relatively larger initial rises in  $n_t$ .

As marginal tax rates begin to rise significantly, the one period ahead tax effect becomes quantitatively important. By period 3 investment falls to its pre shock level and continues to fall, reaching a maximal decline of roughly 25% in period 8, the period in which  $\tau_t$  peaks. Other things equal the decline in investment causes  $C_t^h$  to rise and  $(C_t/C_t^s)$  to fall. To restore  $(C_t/C_t^s)$  to its equilibrium value of  $\chi$ , real wages rise which induces a fall in  $n_t$ . This explains the sharp decline in hours worked and investment after period 3 in Figure 3. It also accounts for the fact that they are at their lowest levels when government purchases and taxes are at their highest levels.

#### 5.3. Test Statistics

We conclude this subsection by reporting the results of formally testing the models' ability to account for various conditional moments of the data using the Jstatistic defined in (3.6). We begin by discussing four moments pertaining to hours worked. The first two moments relate to the maximal response of hours worked in the aftermath of a Ramey-Shapiro episode:  $R_1(n)$  and  $R_2(n)$  are the peak rise in  $n_th$  and the average response of  $n_th$  in periods 4 through 7 after a fiscal policy shock. The values of these moments for the model and the data,  $R_i^m(n)$  and  $R_i^d(n)$ , i=1,2, respectively, were calculated using estimates of the relevant dynamic response functions. The third moment is the correlation between  $g_t$  and  $n_th$ ,  $\rho(g,n)$ , induced by a fiscal policy shock. We let  $\rho^m(g,n)$  and  $\rho^d(g,n)$  denote the values of this moment implied by the model and the data, respectively. The final moment,  $\sigma_n$ , is the standard deviation of hours worked induced by the onset of a Ramey-Shapiro shock. Below,  $\sigma_n^m$  and  $\sigma_n^d$  denote the values of this moment implied by the model and the data, respectively.

Column 1 of Table 1 reports the results of testing the individual hypotheses:  $R_i^d(n) - R_i^m(n) = 0$ , i = 1, 2,  $\rho^d(g, n) - \rho^m(g, n) = 0$ , and  $\sigma_n^d - \sigma_n^m = 0$ . Note that we cannot reject the hypothesis that  $R_1^d(n) - R_1^m(n) = 0$  at conventional significance levels, nor can we reject the hypothesis that  $\sigma_n^d - \sigma_n^m = 0$ . However, while the model can match the overall volatility of  $\sigma_n$  and the peak response of  $n_t h$  it does so in a way that is inconsistent with the timing of the actual movements in  $n_t h$ . Consistent with our discussion above, the model predicts that  $n_t h$  is strongly negatively correlated with  $g_t$  with  $\rho^m(g,n)$  equal to -0.80. But in the data  $g_t$  and  $n_t h$  are strongly positively correlated, with  $\rho^d(g,n) = 0.69$ . Not surprisingly, we can reject the hypothesis that  $\rho^d(g,n) - \rho^m(g,n) = 0$  at the 1% significance level. Consistent with the notion that model mispredicts the timing of the response of  $n_t h$ , we can also reject the hypothesis that  $R_2^d(n) - R_d^m(n) = 0$ . This reflects that the maximal response of  $n_t h$  in the model occurs before period 4 while in the data

 $<sup>^{16}</sup>$  We calculated the last two moments as follows. Let the actual and model implied dynamic response function of a variable  $x_t$  to a fiscal policy shock be given by  $H_x(L)D_t$  and  $\tilde{H}_x(L)D_t$ , respectively,  $x_t = \{n_t, g_t\}$ . The value of  $\sigma_x$  implied by the model and in the data is given by  $\sigma_n^m = \left\{\sum_{i=0}^\infty \left[\tilde{H}_n^1(i)\right]^2\right\}^{1/2}$  and  $\sigma_x^d = \left\{\sum_{i=0}^\infty \left[H_x(i)\right]^2\right\}^{1/2}$ , respectively. Here  $H_x(i)$  and  $\tilde{H}_x(i)$  denote the  $i^{th}$  coefficient in the polynomial lag operator  $H_x(L)$  and  $\tilde{H}_x(L)$ . The value of  $\rho(g_t, n_t)$  implied by the model and in the data is given by  $\rho^m(g,n) = \left\{\sum_{i=0}^\infty \tilde{H}_n(i)H_g(i)\right\}/\sigma_n^m\sigma_g^d$  and  $\rho^d(g,n) = \left\{\sum_{i=0}^\infty H_n(i)H_g(i)\right\}/\sigma_n^d\sigma_g^d$ , respectively. Note that the value of  $\sigma_g$  in the model is equal to  $\sigma_g^d$  by construction. In practice we calculated  $\sigma_n^m$ ,  $\sigma_n^d$ ,  $\rho^m(g,n)$ ,  $\rho^d(g,n)$  and  $\sigma_g^d$  using the first 12 coefficients of the relevant dynamic response functions.

they occur after period 4.

Column 2 of Table 1 provides the results of formally testing the analog hypotheses for real wages. Specifically, the first two moments pertain to the maximal response of after-tax real wages in the aftermath of a Ramey-Shapiro episode:  $R_1[(1-\tau)w]$  and  $R_2[(1-\tau)w]$  denote the maximal declines in  $(1-\tau_t)w_t$  and the average response of  $(1-\tau_t)w_t$  in periods 4 through 7 after a fiscal policy shock. The third moment which we consider is the correlation between  $g_t$  and  $(1-\tau_t)w_t$ ,  $\rho[g,(1-\tau)w]$ , induced by a fiscal policy shock. The final moment,  $\sigma_{(1-\tau)w}$ , is the standard deviation of the after-tax real wage induced by the onset of a Ramey-Shapiro episode.

Notice that we cannot reject the hypotheses that the model accounts for the peak declines in real wages and the conditional volatility in real wages. But as with  $n_t h$ , the model does so in a way that is inconsistent with the timing of the actual movements in  $w_t$ . In the data after - tax wages and government purchases are strongly negatively correlated with  $\rho[g, (1-\tau)w]$  equal to -0.90. In the model these variables are less strongly correlated (-.32). As result we can reject the hypothesis that the model can account for the correlation between after - tax real wages and government purchases at the 1% significance level. We can also easily reject the hypothesis that the average response of real wages during periods in 4 through 7 is the same in the model and in the data.

## 6. Conclusion

This paper implements a particular limited information strategy for assessing the empirical plausibility of competing business cycle models. The basic strategy is to confront models with experiments that we claim to have isolated in the data and whose effects on the actual economy we know. The experiment that we focus on is an exogenous fiscal shock that leads to persistent movements in government purchases and average marginal tax rates. We analyzed the ability of a particular general equilibrium efficiency wage model to account for the actual responses of

hours worked and of real wages to a fiscal policy shock. Our key finding is that the model cannot do so unless we make the counterfactual assumption that marginal tax rates are constant. This failure reflects, to a large extent, the response of investment to the fiscal policy shock. We anticipate addressing this shortcoming in future work.

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| Table 1                               |              |                     |
|---------------------------------------|--------------|---------------------|
| Moment                                | Hours Worked | After-tax Real Wage |
| Peak                                  |              |                     |
| Data                                  | 2.78         | -4.55               |
| Model                                 | 5.27         | -8.71               |
| J-statistic                           | 1.76         | 0.00                |
| P-value                               | 0.19         | 0.99                |
| Average of periods 4, 5, 6, 7         |              |                     |
| Data                                  | 2.51         | -3.77               |
| Model                                 | -4.17        | -1.28               |
| J-statistic                           | 3.63         | 5.29                |
| P-value                               | 0.06         | 0.02                |
| Correlation with government purchases |              |                     |
| Data                                  | 0.81         | -0.97               |
| Model                                 | -0.77        | -0.32               |
| J-statistic                           | 13.0         | 16.3                |
| P-value                               | 0.00         | 0.00                |
| Standard deviation                    |              |                     |
| Data                                  | 5.92         | 11.8                |
| Model                                 | 18.6         | 12.1                |
| J-statistic                           | 2.28         | 0.00                |
| P-value                               | 0.13         | 1.00                |

























