A Model of Charles Ponzi

Gadi Barlevy and Inês Xavier

March 18, 2025

WP 2025-05

https://doi.org/10.21033/wp-2025-05

FEDERAL RESERVE BANK *of* CHICAGO

A Model of Charles Ponzi*

Gadi Barlevy
Federal Reserve Bank of Chicago

Inês Xavier
Federal Reserve Board

March 18, 2025

Abstract

We develop a model of Ponzi schemes with asymmetric information to study Ponzi frauds. A long-lived agent offers to save on behalf of short-lived agents at a higher rate than they can earn themselves. The long-lived agent may genuinely have a superior savings technology, but may be an imposter trying to steal from short-lived agents. The model identifies when a Ponzi fraud can occur and what interventions can prevent it. A key feature of Ponzi frauds is that the long-lived agent builds trust over time and improves their reputation by keeping the scheme going.

Key Words: Ponzi scheme, asymmetric information, reputation, fraud
JEL Codes: C73, D82, G51, K42, L14

*The views presented here are solely those of the authors and do not necessarily represent those of the Federal Reserve Bank of Chicago, the Federal Reserve Board, or the Federal Reserve System. We thank David Andolfatto, Yair Antler, Fernando Arce, Marco Bassetto, Daniel Bird, Russ Cooper, Ian Dew-Becker, Bob Hetzel, Ryo Jinnai, Guido Lorenzoni, George Mailath, Alex Monge, Kurt Mitman, Alessandro Pavan, Chris Phelan, Ville Rantala, Manuel Santos, Ofer Setty, David Weiss, Nicolas Werquin, Noah Williams, and Ariel Zetlin-Jones for helpful discussions and various seminar participants at for their comments.

1 Introduction

Economists define a Ponzi scheme as a scenario in which an agent borrows from others and then keeps repaying their debt by taking out new debt rather than drawing on their own resources. For example, a government that rolls over its debt without ever collecting taxes, as in Diamond (1965), is said to run a Ponzi scheme. Likewise, a household that keeps taking on new debt to repay its existing debts is said to run a Ponzi scheme, and restrictions on the total indebtedness of households are usually called no-Ponzi constraints.

Existing models of Ponzi schemes typically assume symmetric information. Examples include O'Connell and Zeldes (1988), who study Ponzi schemes in deterministic settings, and Blanchard and Weil (2001) and Abel and Panageas (2022), who study Ponzi schemes in stochastic environments with equally informed agents. In these symmetric information models, a Ponzi scheme is typically either sustainable indefinitely, meaning the agent can keep rolling over debt as in Diamond (1965), or it cannot take off at all.[1]

In practice, there are many Ponzi schemes that take off even though they cannot be sustained indefinitely. These schemes, including the one hatched by Charles Ponzi after whom these schemes are named, tend to involve an element of fraud: The borrower hides the fact that they are using funds from new investors to repay previous investors and instead purports to be using the proceeds from some real investment activity. Indeed, the legal definition of a Ponzi scheme focuses on misrepresentation as to what the invested funds are used for. Frankel (2012) observes that these frauds "appear with monotonous regularity" across both time and space. Deason et al. (2015) and Marquet (2011) document hundreds of Ponzi scheme prosecutions by the SEC in the past twenty years, while Springer (2020) documents more than a thousand over the past 60 years. The frequency of such frauds highlights the need for a framework to analyze this phenomenon. Symmetric information models of Ponzi schemes, in which all agents know what the borrower is doing, are unsuited for understanding Ponzi *frauds* as distinct from sustainable Ponzi *schemes*.

This paper proposes a model of Ponzi frauds based on private information. The operator of the Ponzi scheme claims that they can achieve higher returns for investors than those investors can achieve on their own. Investors know that the agent may be an imposter who is paying them from funds raised by new investors, and must choose whether to trust the investor with their wealth or proceed to save on their own. The imposter, in turn, chooses

[1] One exception to this dichotomy is Bhattacharya (2003). He considers a symmetric information setting in which scheme participants can pressure the government to bail them out by taxing non-participants. In practice, most Ponzi schemes are too small to concern governments. Even during the 1996-7 Albanian Ponzi crisis that involved nearly half of GDP, Jarvis (2000) reports that the government resisted calls for bailouts.

whether to continue the scheme or abscond with the funds.

We derive conditions under which an imposter with no access to a high return technology can operate a Ponzi scheme in equilibrium. For a Ponzi equilibrium to occur, investors must initially be relatively skeptical that the scheme is legitimate. If the scheme were convincing from the start, it would attract large investment immediately and an imposter would prefer to steal the funds right away rather than try to raise enough funds to pay off initial investors in the hope of attracting more investment later. Another key element is a low probability of detection. Since early investors are skeptical about the legitimacy of the scheme, they must believe the imposter will likely repay them with funds raised from new investors. But this requires that the imposter is unlikely to be exposed as a fraud through investigations. Finally, we find that Ponzi schemes are harder to sustain if the scheme operator promises investors significantly more than they can earn on their own. However, schemes with arbitrarily high promised returns can be sustained under some conditions.

The Ponzi fraud in our model must collapse in finite time given the amount agents can invest each period is constant while the obligations of the scheme operator keep growing. The reason the scheme doesn't unravel in the final period is that the operator's reputation improves the longer the scheme lasts. Early investors are willing to invest despite their skepticism because they expect to be paid from the funds of new investors if the scheme is indeed a fraud. By contrast, investors in the final period know that there will be no one to bail them out if the scheme is fraud. Nevertheless, they are willing to invest because they reason that if the scheme survived as long as it did, the operator is likely to be genuine.

Beyond showing when a Ponzi fraud can occur in equilibrium, the model offers insights on how such frauds unfold. For example, in our model the fraud is likely to last longer the lower is the promised return, a prediction consistent with what we observe empirically. Our model also suggests which interventions are more likely to deter such frauds. For example, in our model education is less effective than enforcement in ruling out Ponzi equilibria.

The paper is organized as follows. The remainder of this section summarizes the related literature. Section 2 reviews Charles Ponzi's original scheme as a way of motivating our modeling approach. Section 3 describes our model. Section 4 solves for the optimal behavior of agents and introduces the notion of a Ponzi equilibrium. Section 5 shows when Ponzi equilibria can be ruled out. Section 6 establishes when Ponzi equilibria exist. Section 7 derives results on the uniqueness of Ponzi equilibria and reports some comparative statics. Section 8 discusses the welfare implications of our model. We conclude with a discussion of some issues and potential generalizations of our model.

Related Literature

Our work is related to a growing literature on Ponzi schemes. Several papers have similarly studied Ponzi schemes from a theoretical perspective. Early work by O'Connell and Zeldes (1988) considered the possibility of Ponzi schemes in deterministic settings with symmetric information. They showed that infinite horizons and dynamic inefficiency were necessary for Ponzi schemes to arise in such environments. Blanchard and Weil (2001) allowed for uncertainty with symmetric information and showed that Ponzi schemes could also occur in dynamically efficient economies. Abel and Panageas (2022) analyzed a different model of symmetric uncertainty and found a similar result. In all three of these models, Ponzi schemes can either be sustained indefinitely or can be ruled out. Bhattacharya (2003) presented a Ponzi scheme with symmetric information that cannot be sustained indefinitely. He assumed that participants may be able to pressure the government to redistribute resources and make whole those who would otherwise lose from the scheme.

Artzrouni (2009) looked at how the amount left in a Ponzi fraud evolves over time as a function of new investment inflows, withdrawals by previous investors, the promised return on investments, and the actual return the operator earns. His analysis treated the rate of investment and withdrawals as given rather that deriving them from optimal decisions. More generally, he focused on what happens assuming a Ponzi scheme exists without studying whether such a scheme can be an equilibrium. He also abstracted from the outflow of resources stolen by the scheme operator, which figures prominently in our analysis.

On the empirical front, several researchers have studied Ponzi frauds. Frankel (2012) examined why such schemes are so common and identified common characteristics of their perpetrators and victims. Deason et al. (2015) compiled data on 376 Ponzi schemes prosecuted by the SEC between 1988 and 2012 and looked at how the duration, amount invested, and fraction stolen vary with state-level characteristics. Marquet (2011) compiled data on 329 schemes between 2002 and 2011 from various sources, and found that these schemes have become more frequent over time. Springer (2020) constructed a dataset of 1,359 Ponzi schemes between 1960 and 2022 and identified some key trends among these schemes.

More recent work has focused on Ponzi schemes associated with cryptocurrencies. Bartoletti et al. (2020) identify 184 Ponzi schemes coded as smart contracts on the Ethereum platform. Building on their work, Shuang et al. (2023) identify 512 such contracts. The code for these smart contracts is public and in some cases contained explicit comments explaining that the contract was a Ponzi scheme. Participants who entered these contracts could have figured out that they would only be repaid if others entered the contracts after them. This is in contrast to fraudulent schemes in which the fact that repayments come from newcomers remains hidden. Accordingly, the extent of these smart contracts seems more limited than the investment in Ponzi frauds that earlier work documented. A large share of these smart contracts never attracted any users. Most of these smart contracts were created during a three month period in 2016. The contracts that attracted investment involved stakes of a few hundred dollars on average. This suggests such contracts may have been a passing fad written for fun rather than an attempt to commit fraud. That said, cryptographic platforms have also been used to implement Ponzi frauds disguised as legitimate investments. Cong et al. (2023) discuss the $2 billion PlusToken scheme in 2019. Springer (2020) cites several other examples of crypto-related Ponzi schemes in her data.

Other researchers have focused on what we can learn from particular Ponzi schemes. Gurun, Stoffman and Yonker (2018) showed that the collapse of Madoff Investment Securities led to a reduction in assets under management with registered investment advisors in regions that had previously invested more with Madoff. This suggests the performance of schemes affects investment decisions, as is true in our model. Rantala (2019) looked at the Wincapita Ponzi scheme in Finland between 2003 and 2008, focusing on which investors brought in others given the commissions offered for bringing in new investors. Huang et al. (2021) explored related questions on which agents recruited others in large scale Ponzi scheme in China in 2016 that drew in over 4800 investors. Since our model does not allow agents to recruit, we cannot relate our model to these findings.

Finally, our model is related to adverse-selection models of reputation in which a long-lived agent has an incentive to pretend to be a type that is committed to some particular action. See Mailath and Samuelson (2015) for a comprehensive survey. Our particular model is similar to Wiseman (2009) and Hu (2014) in assuming exogenous information that ensures the long-lived agent's type will be revealed asymptotically almost surely. However, agents in those papers would like to maintain a good reputation indefinitely, while agents in our model have no reason to pretend to be the good type after building enough of a reputation to attract the maximal amount of investment. Our model also features a state variable beyond the long-lived agent's reputation, namely the obligation to previous savers. Celentani and Pesendorfer (1996) and Board and Meyer-ter Vehn (2013) also feature non-reputational state variables, although they are qualitatively different from ours.

The closest papers in this literature to ours are Phelan (2006) and Amador and Phelan (2021). These papers considered a long-lived agent who switches exogenously between an opportunistic type (akin to our imposter) and a commitment type (akin to our genuine type). In our model, the long-lived agent's type is fixed. However, our Ponzi equilibrium has a similar structure to the equilibrium in these models: When the long-lived agent has a bad reputation, the opportunistic type will have an incentive to mix between pretending to be the commitment type and revealing it is opportunistic. It keeps doing so until its reputation is high enough, at which point it will act opportunistically and reveal its type. However, the problem faced by the opportunistic type is different in our model. Phelan (2006) studied static decisions, while Amador and Phelan (2021) considered one-period debt where the borrower is always solvent. In our model, by contrast, the borrower must eventually default. While our Ponzi equilibrium has a similar structure to the equilibrium they study, the analysis of whether and when a Ponzi equilibrium exists has no analog in their work.

2 Historical Context: Ponzi's Original Scheme

To motivate our modeling framework, we turn to Charles Ponzi's original scheme that lent these frauds their name. While Ponzi did not originate such frauds, nor was his the largest operation of its kind, Ponzi's scheme is well documented and displays several features common to many of these schemes. Our description is based on Zuckoff (2005).

In 1919, Charles Ponzi – an Italian immigrant living in Boston at the time – stumbled upon a potential arbitrage opportunity involving international reply coupons that people could buy overseas and send to their correspondents in the US to trade for postage. Ponzi realized that purchasing international reply coupons in Italy and exchanging them for stamps in the US was cheaper than buying the same amount of postage in the US. Given his negative previous experience with banks and concerned that bankers might steal his idea, Ponzi decided to raise funds from private investors in order to purchase reply coupons. Ponzi promised investors a fixed 50% return on their investment within 90 days.

While Ponzi was quick to raise funds, he was unable to figure out how to profitably scale his operation. Profiting from discrepancies in postage prices required purchasing coupons in bulk in Italy, bringing them back to the US, exchanging them for postage, and then selling the postage for cash. Ponzi's inquiry with postal officials about exchanging reply coupons directly for cash was rejected out of hand, and he was warned by the US Postmaster's office that it was illegal to use international reply coupons for speculation. Alarmed by Ponzi's operation after they learned about it, postal officials moved to block him by pressuring several countries, including Italy, France, and Romania, to suspend sales of reply coupons in April 1920. In July 1920, the Postmaster further moved to limit the amount of coupons an individual could redeem in the US at one time. Unable to scale up his operation, Ponzi began to pay early investors with funds he raised from new investors.

As Ponzi kept amassing investment and gained notoriety, skeptics began to question his claims. Early on, the state supervisor of small loans, Frank Pope, asked local police to investigate Ponzi. The detectives sent to investigate were sufficiently impressed that they invested with Ponzi and convinced other police officers to do the same. Postal inspectors pressed Ponzi on how he was able to generate profits when foreign countries supposedly stopped issuing coupons, but he managed to evade their questions. Reporters started to investigate Ponzi after one of his early investors sued him for 1 million dollars. By July 1920, the *Boston Post* invited financial journalist Clarence Barron to evaluate Ponzi's operation. Barron pointed out the impossibility of scaling up in a way that would sustain the payoffs Ponzi was promising his investors. But Ponzi was able to continue to attract investment by arguing that bankers were merely trying to avoid having to share their high returns with regular depositors. A New York Times article from July 29, 1920 quotes Ponzi as follows:

> Bankers and business men can easily understand how I could make 100 per cent for myself, but simply because no one ever made an added 50 per cent for the general public they reason that it can't be. You remember the old rube who saw the giraffe for the first time? He stared at it and remarked 'There ain't no such animal.' The truth is, bankers and business men have been doing plenty for themselves under the present banking system, but they have done little for anybody else. I want to change this unfair condition. The depositor in the banks today is not getting a square deal... Yes, I know it is a shock to some of these folks who have been hogging it all, but it is fair and right, and the depositor should get a fair return for his money.[2]

In early August 1920, Ponzi's press agent, William McMasters, contacted the *Boston Post* and offered to sell them information that Ponzi was insolvent. The article based on McMasters' information led to a run on Ponzi's company. At the same time, the Massachusetts Bank Commissioner and Attorney General both launched investigations into Ponzi's company. By mid August, the *Post* reported that Ponzi had been previously arrested for fraud in Canada. With his reputation in tatters and investigators closing in on him, Ponzi surrendered to authorities. By November 1920, he pled guilty to mail fraud in federal court.

The key elements of Ponzi's scheme we wish to emphasize are: (1) Ponzi presented himself to investors as having a legitimate investment opportunity that allowed him to offer them a higher fixed return than they could earn on their own; (2) the fantastic return Ponzi promised his investors generated some skepticism and prompted investigations; (3) early investigations that were unable to establish fraud were followed by even more investment;

[2] Available at https://www.nytimes.com/1920/07/29/archives/exchange-wizard-to-fight-bankers-ponzi-of-boston-promises-new.html. The quote also appears in Zuckoff (2005), page 209.

and (4) a combination of a tarnished reputation and the absence of new investment forced Ponzi to default. We develop a model that aims to capture these features.

3 Model

Consider an economy with one infinitely-lived agent and a succession of overlapping generations of short-lived agents, each of whom live for two periods.

Time is discrete and starts at $t = 0$. Each period, a mass 1 of short-lived agents is born. These agents only care about consumption when old, i.e., the utility of the cohort born at date t over consumption c_t^y when young and consumption c_{t+1}^o when old is given by

$$u(c_t^y, c_{t+1}^o) = c_{t+1}^o \tag{1}$$

Short-lived agents are endowed with y goods when young and nothing when old, so their concern is to save for old age. They have access to a savings technology that yields a gross return of $1 + R_L > 1$. Since the return on investment exceeds the (zero) growth rate of the endowment, we know from O'Connell and Zeldes (1988) that these agents would not be able to sustain an equilibrium Ponzi scheme among themselves.

There is also one long-lived agent who offers to save on behalf of the short-lived agents and pay them a rate $R_H > R_L$. Let x_t denote the amount of funds that (young) short-lived agents invest with the long-lived agent at date t. They will save the remaining amount $y - x_t$ on their own, earning them a return of R_L per unit saved.

The long-lived agent can assume one of two types, genuine and imposter. A genuine long-lived agent has access to a high return technology and an incentive to pay short-lived investors the promised return R_H. We model a genuine agent as a commitment type without explicitly modelling their motive to offer a fixed rate of R_H. One could model this motive, but we prefer to focus on the behavior of an imposter who wants to mimic the genuine type without the additional distraction of why the genuine type behaves as it does.[3]

An imposter type has access to the same return R_L that short-lived agents can earn on their own. While this implies there is no scope for gains from trade between short-lived agents and an imposter, the latter might still want to attract investment from short-lived agents in order to steal it and use it to finance their personal consumption.

[3]One way to endogenize why the genuine type pays a fixed rate R_H is to assume short-lived agents hold beliefs that any agent who offers a return different from R_H must be an imposter. See Amador and Phelan (2021) for a related discussion on endogenizing the behavior of a commitment type.

Formally, let θ denote the long-lived agent's type, where $\theta \in \{\text{genuine, imposter}\}$. The long-lived agent is genuine with probability $\phi \in (0,1)$, i.e.,

$$\Pr(\theta = \text{genuine}) = \phi \tag{2}$$

Since we model the genuine agent as a commitment type, we only need to specify the preferences and choices of the long-lived agent as an imposter.

Denote the imposter's consumption at date t by c_t. The imposter has utility

$$U\left(\{c_t\}_{t=0}^{\infty}\right) = \sum_{t=0}^{\infty} \beta^t c_t \tag{3}$$

where we assume the imposter is relatively impatient, specifically,

$$\beta \leq \frac{1}{1+R_H} \tag{4}$$

Assumption (4) implies that the imposter would not want to keep postponing consumption unless it grew faster than R_H. Assumption (4) also implies $\beta < \frac{1}{1+R_L}$, meaning the imposter will not want to delay consumption for a chance to save at rate R_L.

In terms of resources, let w_t denote the imposter's wealth at the start of date t. We assume the imposter is endowed with no initial wealth, i.e., $w_0 = 0$. We also assume the imposter earns no income and can only obtain resources from short-lived agents.

Finally, we assume that once a positive measure of short-lived agents invest with the long-lived agent, the long-lived agent will be investigated for as long as their type remains uncertain. This is meant to capture how schemes promising high returns attract attention and scrutiny once they take off. Formally, starting right after date $t_0 = \inf\{t : x_t > 0\}$, there is a constant probability $\mu \in (0,1)$ per period that if the long-lived agent is an imposter, their type will be revealed. The investigation process is asymmetric: It can confirm that the long-lived agent is an imposter but it cannot validate that the long-lived agent is genuine.[4]

In what follows, we abstract from the possibility that an agent will be punished if they are revealed to be a fraud. Our results should carry over if we introduce an arbitrarily small penalty (or a penalty that is applied with arbitrarily small probability). In practice, a non-negligible fraction of schemes that were prosecuted have no recorded sentence.

[4]Ponzi equilibria can exist even when $\mu = 0$. The assumption that $\mu > 0$ implies that while the long-lived agent's type remains uncertain, their reputation will improve over time regardless of their default strategy. When $\mu = 0$, their reputation will still improve if the imposter defaults with positive probability every period.

Timing

The timing of the model is as follows. At the start of date 0, young short-lived agents choose the amount x_0 to invest with the long-lived agent. The long-lived agent moves next. A genuine type would invest x_0 in the high-return technology. An imposter must decide how much of x_0 to consume and how much to save at a return R_L.

The timing for any date $t > 0$ is the same if there was no investment prior to date t. That is, if $x_0 = \cdots = x_{t-1} = 0$, young short-lived agents choose an amount x_t to invest with the long-lived agent, after which the long-lived agent moves.

After the first period in which investment is positive, the timing within each period is a bit more elaborate. At the start of the period, the long-lived agent is investigated. If the long-lived agent is an imposter, their type will be revealed with probability μ. Otherwise, no signal will be produced. Short-lived agents observe whether the long-lived agent was exposed as an imposter at the start of the period (or in any previous period). They can also observe whether the long-lived agent defaulted in the past. However, they cannot observe anything else about the long-lived agent, including their investments, consumption, or earnings. At this point, short-lived agents decide how much to invest with the long-lived agent. After short-lived agents invest an amount x_t, the long-lived agent moves. A genuine type would pay previous investors the amount $(1 + R_H)x_{t-1}$ and invest the new amount x_t. An imposter type must decide whether to repay their obligation, knowing that default would reveal that they are an imposter. Since the long-lived agent chooses to default after short-lived agents invest x_t, default at date t can only affect investment $x_{t+1}, x_{t+2}, ...$ after date t. An imposter must also choose how much of the resources they have left at the end of the period to allocate to consumption c_t and how much to save at rate $1 + R_L$.

4 Optimality and Equilibrium

Now that we have described the model, we can discuss what agents should do and define an equilibrium in which all agents choose optimally given what others choose. We start with the decisions of the long-lived agent and then move on to short-lived agents.

Optimal Decisions for the Imposter

Since we model the genuine long-lived agent as a commitment type, we only need to solve for the decisions of the imposter. These decisions depend on what short-lived agents know about the long-lived agent's type. Let $\mathbb{I}_t$ denote an indicator that equals 1 if short-lived agents know that the long-lived agent is an imposter when they invest at date t and 0 otherwise. Since there are no gains from trade between short-lived agents and an imposter and the imposter would steal any funds they receive, then $x_{t+s} = 0$ for $s \geq 0$ if $\mathbb{I}_t = 1$. Given $\beta(1+R_L) \leq 1$ from (4), the fact that the imposter will receive no further investments regardless of what they do means they have no reason to delay consuming their wealth if $\mathbb{I}_t = 1$. The imposter's utility in this case equals the wealth w_t they start with in period t.

If $\mathbb{I}_t = 0$ and the imposter has yet to be revealed by date t, they can keep their type hidden by repaying their obligation in the same way a genuine type would. However, they can only do this if their resources $w_t + x_t$ are at least as large as their obligation $(1+R_H)x_{t-1}$. Let S_t be an indicator that is equal to 1 if the long-lived agent is solvent, meaning they can repay their obligation after raising the investment x_t:

$$S_t = \begin{cases} 1 & \text{if } w_t + x_t \geq (1+R_H)x_{t-1} \\ 0 & \text{if } w_t + x_t < (1+R_H)x_{t-1} \end{cases} \tag{5}$$

If $S_t = 0$ and the long-lived agent is insolvent, they have no choice but to reveal their type by the end of date t. The imposter will thus attract no investment beyond date t regardless of what they do, and so should default on their debt and consume any resources they have at the end of date t without delay. Their utility in this case will equal $w_t + x_t$.

This leaves the case where $\mathbb{I}_t = 0$ and $S_t = 1$, meaning the long-lived agent's type has yet to be revealed and the imposter is solvent. If the imposter repays their obligation to the previous cohort $(1+R_H)x_{t-1}$ in full, they would leave short-lived agents unsure if the long-lived agent is an imposter or not. Since they would reveal their type by doing otherwise, their choice amounts to either paying their obligation in full or defaulting and consuming immediately, in which case their utility would be $w_t + x_t$.

Define an indicator d_t where $d_t = 1$ if the solvent imposter defaults at date t and $d_t = 0$ if they repay their obligation in full. We will use c_t to denote the solvent imposter's consumption at date t if $d_t = 0$, i.e., if the solvent imposter chooses not to default.

Let V_t denote the imposter's maximal utility as of date t if $\mathbb{I}_t = 0$ after receiving the investment x_t from short-lived agents. This utility depends on the imposter's wealth w_t as well as whether they are solvent. As noted above, if $S_t = 0$, then $V_t = w_t + x_t$. If $S_t = 1$, the payoff V_t will depend on the investment of short-term investors. This means $V_t = V\left(w_t, S_t; \{x_s\}_{s=0}^{\infty}\right)$. To simplify the notation, we suppress these arguments and use V_t.

Before any investment takes place, i.e., at dates $t < t_0 \equiv \min\{t : x_t > 0\}$, there is nothing for the imposter to do given they have yet to raise any wealth. The non-trivial decisions occur only for $t \geq t_0$. The imposter's decision problem at dates $t \geq t_0$ depends on their obligation x_{t-1} to short-lived agents from the previous period, where we define $x_{-1} \equiv 0$ to capture the fact that there is no outstanding debt at date 0.

If $x_{t-1} = 0$, the imposter has no obligation they can default on. Their only decision is how much to consume and how much to save. Since $t \geq t_0$, the imposter knows that they will be investigated next period. Their maximization problem at date t is given by

$$V_t = \max_{c_t} c_t + \beta \mu w_{t+1} + \beta(1-\mu)V_{t+1} \tag{6}$$

subject to

1. $w_{t+1} = (1+R_L)(w_t + x_t - c_t)$
2. $w_{t+1} \geq 0$ and $c_t \geq 0$

If $x_{t-1} > 0$, the imposter will also have to decide whether to default and reveal themselves or maintain ambiguity about their type. This maximization problem is given by

$$V_t = \max_{c_t, d_t} d_t(w_t + x_t) + (1-d_t)\left[c_t + \beta(\mu w_{t+1} + (1-\mu)V_{t+1})\right] \tag{7}$$

subject to

1. $w_{t+1} = (1+R_L)[w_t + x_t - (1+R_H)x_{t-1} - c_t]$ when $d_t = 0$
2. $w_{t+1} \geq 0$ and $c_t \geq 0$

Inspecting (7), a necessary condition for the imposter to not default is that the continuation value V_{t+1} be sufficiently high. The only reason for an imposter to not default immediately given their high degree of impatience is the prospect of high investment at date $t+1$ or later that would allow them to steal more.

In principle, more than one path for $\{c_t\}_{t=0}^{\infty}$ and $\{d_t\}_{t=1}^{\infty}$ may allow the imposter to attain the maximal possible utility that solves (6) and (7). This will indeed be true in equilibrium. If multiple paths are optimal, the imposter should be willing to play a mixed strategy that randomizes over paths. The imposter's strategy is thus a probability distribution π over paths for $\{c_t\}_{t=0}^{\infty}$ and $\{d_t\}_{t=1}^{\infty}$. Since a mixed strategy assigns probabilities to both paths, the imposter may coordinate between the two decisions when they randomize. For example, the imposter might randomize between a low value for c_t when $d_{t+1} = 0$ and a high value for c_t when $d_{t+1} = 1$. That is, the imposter may choose to save at date t to avoid default in period $t+1$ but would prefer not to delay consumption if they intend to default next period.

Given a strategy profile π, we can compute the conditional probability that the imposter will default in period t if their type remains uncertain and they are solvent, i.e.,

$$\sigma_t = \Pr(d_t = 1 | \mathbb{I}_t = 0, S_t = 1) \tag{8}$$

As we shall now see, the probability σ_t will be a key object that short-lived agents care about.

In short, the optimal decision for an imposter involves when to default and how much to consume or save beforehand. Impatience would dictate that any funds left after paying previous investors should be consumed rather than saved. However, the agent may choose to save to keep the scheme going for longer. The imposter will consume any resources they have at their disposal once they default.

Optimal Decisions for Short-Lived Investors

We now turn to the decisions of short-lived agents. Each cohort must allocate its endowment y when young between saving on their own and investing with the long-lived agent. They make this decision after observing all previous investments, defaults, and investigations of the long-lived agent. We can express this public history at date t as

$$h^t = \{x_0, ..., x_{t-1}, d_1, ..., d_{t-1}, \mathbb{I}_1, ..., \mathbb{I}_t\} \equiv \{\mathbf{x_{t-1}}, \mathbf{d_{t-1}}, \mathbb{I}_\mathbf{t}\} \tag{9}$$

Short-lived agents will choose whichever option offers them the highest expected return. Saving yields a return of $1 + R_L$. The expected return from investing with the long-lived agent depends on the probability that the long-lived agent is an imposter as well as the strategy π that the imposter chooses. Let Φ_t denote the probability that short-lived investors at date t assign to the long-lived agent being genuine given public history and what they believe the imposter's strategy to be, i.e., $\Phi_t = \Pr(\theta = \text{genuine} \mid h^t, \pi)$. In equilibrium, their beliefs about the imposter's strategy coincides with the imposter's actual strategy.

Short-lived investors at date t expect to earn $1 + R_H$ if either the long-lived agent is genuine or if the long-lived agent is an imposter and the imposter (i) is solvent at date t and does not default on investors after raising x_t so that they can go on to raise funds at date $t + 1$; (ii) is not exposed at the start of period $t + 1$; and (iii) is solvent at date $t + 1$ and does not default after raising x_{t+1}. If these conditions are not met, those who invested at date t receive 0. The expected return from investing with the long-lived agent in period t is thus

$$1 + \overline{R}_t = [\Phi_t + (1 - \Phi_t)S_t(1 - \sigma_t)(1 - \mu)S_{t+1}(1 - \sigma_{t+1})](1 + R_H) \tag{10}$$

Short-lived agents in period t are willing to invest with the long-lived agent if

$$1 + \overline{R}_t \geq 1 + R_L$$

If we define $z \equiv \frac{1+R_L}{1+R_H}$, we can rewrite this condition as

$$\Phi_t + (1 - \Phi_t) S_t (1 - \sigma_t)(1 - \mu) S_{t+1} (1 - \sigma_{t+1}) \geq z$$

The total amount invested by short-lived investors will thus be

$$x_t = \begin{cases} y & \text{if } \Phi_t + (1 - \Phi_t) S_t (1 - \sigma_t)(1 - \mu) S_{t+1} (1 - \sigma_{t+1}) > z \\ \text{any } x \in [0, y] & \text{if } \Phi_t + (1 - \Phi_t) S_t (1 - \sigma_t)(1 - \mu) S_{t+1} (1 - \sigma_{t+1}) = z \\ 0 & \text{if } \Phi_t + (1 - \Phi_t) S_t (1 - \sigma_t)(1 - \mu) S_{t+1} (1 - \sigma_{t+1}) < z \end{cases} \tag{11}$$

Next, we specify how short-lived investors update Φ_t. If $t < t_0 = \min\{t : x_t > 0\}$, no investigation will be launched at date $t + 1$. There is also no obligation for the long-lived agent to default on that can reveal it is an imposter. Short-lived agents will then not revise their beliefs between dates t and $t + 1$ if $t < t_0$. If instead $t \geq t_0$, the long-lived agent will be investigated in period $t + 1$ and could have defaulted in period t. If the long-lived agent is revealed as an imposter before short-lived agents make their investment decisions at date $t + 1$, then $\Phi_{t+1} = 0$. Otherwise, short-lived agents should update their beliefs according to Bayes rule. Since an imposter who is insolvent at date t would default at date t and reveal their type, $\mathbb{I}_{t+1} = 0$ implies $S_t = 1$. The law of motion for beliefs is thus given by

$$\Phi_{t+1} = \begin{cases} \Phi_t & \text{if } t < t_0 \\ \dfrac{\Phi_t}{\Phi_t + (1 - \Phi_t)(1 - \sigma_t)(1 - \mu)} & \text{if } t \geq t_0 \text{ and } \mathbb{I}_{t+1} = 0 \\ 0 & \text{if } t \geq t_0 \text{ and } \mathbb{I}_{t+1} = 1 \end{cases} \tag{12}$$

where we adopt the convention that $\Phi_0 = \phi$ and $\sigma_0 = 0$.

Since $\mu > 0$, condition (12) implies that $\Phi_{t+1} > \Phi_t$ if $t \geq t_0$ and $\mathbb{I}_{t+1} = 0$. Each period in which the long-lived agent avoids being exposed as an imposter convinces short-lived agents to favorably revise their likelihood of facing a genuine type. An imposter can thus improve their reputation by keeping the scheme going.

Definition of Equilibrium

An equilibrium consists of a distribution π over paths of functions $\{c_t\}_{t=0}^{\infty}$, $\{d_t\}_{t=1}^{\infty}$ and a path of functions $\{x_t, \Phi_t\}_{t=0}^{\infty}$ that map public history h^t into the relevant strategy space such that all agents choose their actions optimally given the strategy others play and short-lived agents update their beliefs in line with Bayes rule. Formally, for all dates $t \geq 0$,

1. If $x_{t-1} = 0$, any c_t that is assigned positive probability at date t under π solves (6)
2. If $x_{t-1} > 0$, any $\{c_t, d_t\}$ that is assigned positive probability at date t under π solves (7)
3. Investment x_t satisfies (11)
4. Beliefs Φ_t satisfy (12) given the imposter's strategy π

We will refer to an equilibrium as a *Ponzi equilibrium* if after the first date t_0 for which $x_t > 0$, the short-lived agents who invest at date t_0 will be repaid with positive probability at date $t_0 + 1$. That requires (i) $S_{t_0+1} = 1$ so the imposter is solvent at date $t_0 + 1$, and (ii) the probability of default σ_{t_0+1} implied by π is strictly less than 1.

We will say that a Ponzi equilibrium *can last* $T \geq 2$ *periods* if starting from the first date in which short-lived agents first invest, $t_0 = \min\{t : x_t > 0\}$, the agents who invest in periods $t_0, ..., t_0 + T - 2$ will be repaid with positive probability, but the agents who invest in period $t_0 + T - 1$ will for sure not be repaid in period $t_0 + T$. That is,

1. $S_{t_0+j} = 1$ for $j = 1, ..., T - 1$
2. The probability $\prod_{j=1}^{T-1}(1 - \sigma_{t_0+j})$ of no default before T periods is positive
3. Either $S_{t_0+T} = 0$ or $\sigma_{t_0+T} = 1$, so default after T periods is certain

Although the scheme can potentially last T periods, it might end earlier if the imposter is exogenously exposed or if they default before T periods are up.

5 Non-Existence Results

We begin with results on when Ponzi equilibria can be ruled out. We first establish an intermediate result that an imposter will not wait more than one period to default if they expect the amount invested with them to equal y in all periods.

Lemma 1. *Suppose $x_t = y$ for all $t \geq 0$. Then an imposter would default in period 1.*

Intuitively, if agents always invest y, there is no point for the imposter to wait: They will not be able to raise more investment by waiting, and waiting is costly given impatience, the risk of being exposed as an imposter, and the cost of keeping the scheme going.

An implication of Lemma 1 is that Ponzi equilibria can be ruled out whenever $\phi > z$, i.e., when the initial belief that the long-lived agent is genuine is sufficiently high. In period 0, short-lived agents expect to earn at least $\phi(1 + R_H)$ from investing with the long-lived agent. When $\phi > z$, this will exceed the $1 + R_L$ they can earn on their own. Young agents at date 0 will thus invest all of their endowment y with the long-lived agent. Since Φ_t is increasing in t as long as the long-lived agent's type is not revealed, young agents will continue to invest all of their endowment with the long-lived agent until they observe a default. Lemma 1 then implies the imposter will default in period 1. In short, when $\phi > z$, there will not be a Ponzi equilibrium in which the funds raised from new investors are used to pay the original investors. However, the equilibrium will feature fraud. Short-lived agents who invest in periods 0 and 1 will lose all of their investments in period 1 if they face an imposter.

Proposition 1. *Suppose $\phi > z$. Then a Ponzi equilibrium is not possible. The only equilibrium features $x_0 = y$ and $x_1 = y$ if $\mathbb{I}_1 = 0$, i.e., short-lived agents invest all of their wealth with the long-lived agent unless the long-lived agent is exposed. An imposter will default in period* 1.

Next, suppose $\phi = z$. In this case, short-lived agents in period 0 might no longer strictly prefer to invest as they would when $\phi > z$. Given Lemma 1, a Ponzi equilibrium can only occur if the first positive investment x_{t_0} is less than y, i.e., if short-lived agents are exactly indifferent between investing with the long-lived agent and saving on their own when they first invest a positive amount. The expected return to investing in period t_0 is given by $[\phi + (1 - \phi)(1 - \mu)S_{t_0+1}(1 - \sigma_{t_0+1})](1 + R_H)$. For short-lived agents to be indifferent between investing with the infinitely-lived agent and saving on their own in this period, this expression must equal $1 + R_L$. Equating these two and dividing by $1 + R_H$ implies that when $\phi = z$, indifference will only hold if either $S_{t_0+1} = 0$ or $1 - \sigma_{t_0+1} = 0$. There is thus zero probability that short-lived agents who invest in period 0 will be repaid. This again rules out the possibility of Ponzi equilibria while admitting equilibria with fraud.

Proposition 2. *Suppose $\phi = z$. Then a Ponzi equilibrium is not possible. There exist equilibria in which the first positive investment $x_{t_0} < y$ for $t_0 = \min\{t : x_t > 0\}$. However, in these equilibria, the imposter will default in period $t_0 + 1$.*

Finally, suppose $\phi < z$. If μ is large, a long-lived agent who is an imposter is likely to be exposed as an imposter one period after the first positive investment at date t_0, in which case those investors will receive nothing. The expected return from investing with the long-run agent for agents at date t_0 will then be low, and short-lived agents will strictly prefer to save on their own. In this case, there will be no Ponzi equilibria nor fraud.

Proposition 3. *Suppose $\phi < z$. Then a Ponzi equilibrium is not possible if $\mu \geq \frac{1-z}{1-\phi}$.*

To recap, when $\phi \geq z$, a Ponzi scheme cannot occur but outright fraud can: The likelihood that the infinitely-lived agent is genuine is high enough to attract large amounts of investment, and it will not be possible for the infinitely-lived agent to repay them. When $\phi < z$, fraud may be avoided if short-lived agents believe that an imposter is likely to be revealed and will not be able to attract new funds. For a Ponzi equilibrium to be possible requires a mix of initial skepticism about the long-lived investor is genuine ($\phi < z$) and limited investigative capacity ($\mu < \frac{1-z}{1-\phi}$). The first ensures agents do not invest en masse early on, while the latter implies early investors expect that an imposter can still repay them.

6 Equilibria when $\phi \leq z$

In the previous section, we established conditions that rule out the possibility of Ponzi equilibria. We now turn to the question of what equilibria are possible in the remaining cases. Proposition 1 establishes that when $\phi > z$, the unique equilibrium has short-lived agents invest all of their endowment with the long-lived agent unless they learn the long-lived agent is an imposter. So the question is what equilibria are possible when $\phi \leq z$.

Our first result concerns the possibility of no-trade equilibria when $\phi \leq z$. Suppose that short-lived agents did not invest with the long-lived agent before period t, and short-lived agents at date t expect $x_{t+1} = 0$. In this case, short-lived agents at date t know that the imposter would not have enough resources to pay them in full in period $t+1$. The expected return from investing with the long-lived agent is thus $\phi(1 + R_H)$. If $\phi \leq z$, this will not exceed $1 + R_L$, so $x_t = 0$ will be optimal. We summarize this in the following proposition:

Proposition 4. *Suppose $\phi \leq z$. Then there exists an equilibrium in which there is no investment at any date. In this equilibrium, $x_0 = 0$ and $x_t(h^t) = 0$ for $h^t = \{\mathbf{x_{t-1}}, \mathbf{d_{t-1}}, \mathbb{I}_\mathbf{t}\} = \{\mathbf{0}, \mathbf{0}, \mathbf{0}\}$ if $t > 0$.*

The equilibrium above relies on the fact that agents are infinitesimal and cannot affect x_t by deviating and investing. That is, if any single agent chose to deviate and invest at date t, x_t would remain equal to 0 and no investigations would be launched in period $t+1$.

Next, we turn to the possibility of Ponzi equilibria. From the previous section, we already know that such equilibria can only arise when $\phi < z$ and $\mu < \frac{1-z}{1-\phi}$. Lemma 1 implies that Ponzi equilibria are not possible if $x_t = y$ for all t, since in that case the imposter would default at date 1. To sustain a Ponzi equilibrium where the imposter does not default immediately thus requires that $x_t < y$ initially.

Our first observation is that when $\phi < z$ and $\mu < \frac{1-z}{1-\phi}$, any equilibrium in which investment $x_t > 0$ at some date $t > 0$ when the long-lived agent's type is uncertain must feature positive investment starting at date 0. That is, in an equilibrium with investment, the date of the first investment $t_0 = \min\{t : x_t > 0\}$ is $t_0 = 0$. Formally:

Lemma 2. *Suppose $\phi < z$ and $\mu < \frac{1-z}{1-\phi}$. Then if $x_t(h^t) > 0$ in equilibrium for some history $h^t = \{\mathbf{x}'_{\mathbf{t-1}}, \mathbf{0}, \mathbf{0}\}$ for some $t > 0$, then investment must be positive all along that history, i.e., each entry in $\mathbf{x}'_{\mathbf{t-1}}$ must be positive.*

In words, if agents expect some investment next period, it cannot be an equilibrium for nobody to invest this period. This is because the long-lived agent will be solvent and have an incentive to not default when investment today is positive but small.

Since Ponzi equilibria are only possible when $\phi < z$ and $\mu < \frac{1-z}{1-\phi}$, Lemma 2 implies that Ponzi equilibria must be associated with positive investment from date 0. We look for Ponzi equilibria in which x_t is strictly between 0 and y until just before the date T at which the Ponzi equilibrium must end in default. Investment x_{T-1} must be positive at date $T-1$ for default to be possible at date T. Lemma 2 then implies that x_t must be strictly positive for $t = 0, ..., T-1$. Imposing that x_t must also be below y makes it easier to sustain a Ponzi equilibrium, since that condition is necessary for $x_{t+1} > x_t$ to help keep the scheme going. Note that this condition is not needed at date $T-1$, since the imposter will default anyway at date T. We will therefore only require that $x_t < y$ for $t = 0, ..., T-2$. We now derive conditions for a Ponzi equilibrium to satisfy this constraint.

If $0 < x_t < y$ at some t, short-lived agents must be indifferent between saving on their own and investing with the long-lived agent. From equation (11), we know that short-lived agents at date t are indifferent between the two if and only if

$$\Phi_t + (1 - \Phi_t) S_t (1 - \sigma_t)(1 - \mu) S_{t+1} (1 - \sigma_{t+1}) = z \tag{13}$$

If a Ponzi equilibrium can last until date T, the imposter must be solvent through date $T-1$ and default with probability less than 1 before date T. That is, $S_t = 1$ and $\sigma_t < 1$ for all $t = 1, ..., T-1$. This means we can replace S_t and S_{t+1} in (13) with 1 for $t \leq T-2$.

At date T, the imposter must either be insolvent ($S_T = 0$) or default with probability 1 despite being solvent ($\sigma_T = 1$) to ensure the scheme cannot last beyond date T. Either way, the cohort that invests with the long-lived agent at date $T-1$ expects to earn a return of

$\Phi_{T-1}(1+R_H)$. This cohort will be willing to invest with the long-lived agent iff

$$\Phi_{T-1} \geq z \tag{14}$$

Turning to the beliefs of agents in a Ponzi equilibrium, we know that short-lived agents at date 0 will rely on their prior for the probability that the long-lived agent is an imposter, i.e.,

$$\Phi_0 = \phi \tag{15}$$

As long as the long-lived agent isn't exposed as a fraud, short-lived agents will update their beliefs according to (12), i.e.,

$$\Phi_t + (1-\Phi_t)(1-\sigma_t)(1-\mu) = \frac{\Phi_t}{\Phi_{t+1}} \quad \text{for } t = 0, ..., T-1 \tag{16}$$

A Ponzi equilibrium that lasts until period T must satisfy condition (13) at each date t in which $0 < x_t < y$, together with conditions (14), (15), and (16). We begin with what these equilibrium conditions imply about beliefs Φ_t and default probabilities σ_t.

Equilibrium Beliefs Φ_t and Default Probabilities σ_t

Consider the infinite system of equations in $\{\Phi_t, \sigma_t\}_{t=0}^{\infty}$ defined by (13) and (16) without imposing any end date T, i.e.,

$$\Phi_t + (1-\Phi_t)(1-\sigma_t)(1-\mu)(1-\sigma_{t+1}) = z \tag{17}$$

$$\Phi_t + (1-\Phi_t)(1-\sigma_t)(1-\mu) = \frac{\Phi_t}{\Phi_{t+1}} \tag{18}$$

for $t = 0, 1, 2, ...,$ together with the initial conditions $\Phi_0 = \phi$ and $\sigma_0 = 0$. We establish the following lemma regarding the solution to this system:

Lemma 3. *Suppose $\phi < z$ and $\mu < \frac{1-z}{1-\phi}$. Then there exists a T^* where $2 \leq T^* < \infty$ such that the unique solution $\{\Phi_t, \sigma_t\}_{t=0}^{\infty}$ to (17) and (18) satisfies*

(i) $\Phi_t \in [\phi, z)$ *for* $t = 0, ..., T^* - 2$

(ii) $\sigma_t \in [0, 1)$ *for* $t = 0, ..., T^* - 1$

(iii) $\Phi_{T^*-1} \geq z$ *and* $\sigma_{T^*} \geq 1$

Also, Φ_t *is increasing in* t *for* $t < T^*$, *i.e.,* $\Phi_{t+1} > \Phi_t$ *for* $t = 0, ..., T^* - 1$.

An implication of Lemma 3 is that a Ponzi equilibrium in which $0 < x_t < y$ until just before the scheme collapses must last until the specific date T^* defined in the lemma.

Proposition 5. *Suppose $\phi < z$ and $\mu < \frac{1-z}{1-\phi}$. Any Ponzi equilibrium that solves (13), (14), (15), and (16) for some T must have $T = T^*$ where T^* is defined in Lemma 3.*

Essentially, if a scheme were to last until some date $T > T^*$, the value of σ_t at $t = T^*$ would have to be at least 1 at $t = T^*$. But then the scheme would end before T. Conversely, if a scheme were to end by some date $T < T^*$, then short-lived agents would assign probability $\Phi_t < z$ at $t = T - 1$ given that $T - 1 \leq T^* - 2$, in violation of (14).

Lemma 3 also implies that the path for beliefs Φ_t and the probability of default σ_t are uniquely determined in any Ponzi equilibrium in which short-lived agents are indifferent about investing with the long-lived agent until just before when the scheme must end. Since this path has $\Phi_{T^*} > \Phi_{T^*-1} \geq z$, we know that short-lived agents will invest their entire endowment y with the long-lived agent at date T^* (and thereafter) as long as the long-lived agent was not revealed to be an imposter.

Lemma 3 further implies that in an equilibrium where short-lived agents are indifferent, σ_t is strictly between 0 and 1 in dates $t = 1, ..., T^* - 1$. Hence, for this type of equilibrium, the imposter must be indifferent between defaulting and repaying the previous period's investors at each of these dates. Whether the imposter should default or maintain the scheme depends on the path of investment x_t over time, since it governs what the imposter will receive if they default and what they will receive if they wait. This leads us to examine what paths for investment will keep the imposter indifferent about when they default.

Equilibrium Paths for Investment $\{x_t\}_{t=0}^{T^*}$

In looking for a Ponzi equilibrium where short-lived agents are indifferent until just before the scheme must collapse, i.e., an equilibrium that solves (13), (14), (15), and (16) for some T, we showed that such an equilibrium will be associated with a unique value T^* for T and a unique path for beliefs Φ_t and default probabilities σ_t for $t = 0, ..., T^*$.

Per Lemma 3, the unique path for beliefs will necessarily have $\Phi_{T^*} > z$. This means investment x_{T^*} must equal y, since short-lived agents would expect to earn a higher return from investing with the long-lived agent even if they expected the imposter to default with certainty. Lemma 3 further implies that Φ_{T^*-1} one period earlier can either equal z or strictly exceed z. If $\Phi_{T^*-1} > z$, investment x_{T^*-1} at date $T^* - 1$ must also equal y. If $\Phi_{T^*-1} = z$, the value of investment x_{T^*-1} is indeterminate and can assume any value between 0 and y. In the latter case where x_{T^*-1} can be flexibly assigned, it is easy to construct a path for $\{x_t\}_{t=0}^{T^*-1}$ that will leave the imposter indifferent about when to default as well as solvent until date T^*. This is summarized in the next Proposition.

Proposition 6. *Suppose $\phi < z$ and $\mu < \frac{1-z}{1-\phi}$. If*

$$x_t = \left[\frac{\beta(1-\mu)}{1+R_H}\right]^{\frac{T^*-t}{2}} y \quad \text{for } t = 0, ..., T^* \tag{19}$$

then the imposter is always solvent before date T^* and would be indifferent about which date t between 1 and T^* to default on. Hence, if the solution to the system of equations (17) and (18) implies $\Phi_{T^*-1} = z$, a Ponzi equilibrium exists.

To construct a Ponzi equilibrium, we set $x_t(h^t)$ equal to the value for x_t in (19) for all histories h^t in which the long-lived agent's type is uncertain, i.e., where $\mathbf{d_{t-1}} = \mathbf{0}$ and $\mathbb{I}_\mathbf{t} = \mathbf{0}$.

The path for $\{x_t\}_{t=0}^{T^*}$ in (19) implies investment grows at a constant rate $\frac{x_{t+1}}{x_t} = \left[\frac{1+R_H}{\beta(1-\mu)}\right]^{1/2}$ until x_t reaches y in period T^*. Since $\beta(1+R_H) \leq 1$, investment grows at rate that exceeds $1+R_H$. New investment x_t thus exceeds the obligation $(1+R_H)x_{t-1}$ to old investors at each t. The imposter will remain solvent even without saving.

We next confirm that the path in (19) leaves the imposter indifferent about when to default. If the imposter intends to default in period t, they should not carry any wealth into date t, i.e., they should set $w_t = 0$. Their utility as of date t would thus be x_t. If they wait to default in period $t+1$, it will be optimal for them to carry over zero wealth into date t once again given they can repay their obligation $(1+R_H)x_{t-1}$ out of x_t. Their utility as of date t would thus be $x_t - (1+R_H)x_{t-1} + \beta(1-\mu)x_{t+1}$. Equating the payoff from defaulting at t and defaulting at $t+1$ implies

$$x_{t-1} = \frac{\beta(1-\mu)}{1+R_H} x_{t+1} \tag{20}$$

This condition ensures that the additional $(1+R_H)x_{t-1}$ an imposter can consume if they default at t exactly offsets the utility from consuming x_{t+1} with probability $1-\mu$ if they default at $t+1$. Since (19) implies $\frac{x_{t+1}}{x_t} = \left[\frac{1+R_H}{\beta(1-\mu)}\right]^{1/2}$ then $\frac{x_{t+1}}{x_{t-1}} = \frac{x_{t+1}}{x_t}\frac{x_t}{x_{t-1}} = \frac{1+R_H}{\beta(1-\mu)}$.

Along this path, the imposter will consume x_0 in date 0, then $x_t - (1+R_H)x_{t-1}$ at dates $t = 1, ..., T-1$, and finally $x_T = y$ at date T if they do not default. If the imposter defaults at date $t < T$, they would consume x_t at that date. If they are exogenously exposed at date $t < T$, they would receive no new investment that period and consume nothing.

The investment path defined by (19) is not the only path that ensures the imposter will be both solvent and indifferent about when they default. Condition (20) defines a second order difference equation in x_t, which requires two boundary conditions to define a unique path. When x_{T^*-1} is indeterminate, the only boundary condition is $x_{T^*} = y$. That means there is a continuum of paths indexed by x_{T^*-1} that all satisfy (20). Suppose we set

$$x_{T^*-1} = \left[\frac{\beta(1-\mu)}{1+R_H}\right]^{1/2} y + \epsilon$$

and then solve backwards x_t at $t = 0, ..., T^* - 2$ using (20). For $\epsilon = 0$, the path for x_t satisfies the strictly inequality $x_t > (1 + R_H)x_{t-1}$. By continuity, as long as $|\epsilon|$ is small, it will still be the case that $x_t \geq (1 + R_H)x_{t-1}$. Thus, we can find additional equilibrium paths $\{x_t\}_{t=0}^{T^*}$ beyond (19) which ensure the imposter does not need to save to remain solvent and which leave the imposter indifferent about when to default.

Can we also find equilibria in which $|\epsilon|$ is large? Suppose we set $x_{T^*-1} = y$, the maximal value for x_{T^*-1}. If we solve backwards using (20), the resulting path will be

$$x_{T^*-3} = \frac{\beta(1-\mu)}{1+R_H} y = x_{T^*-2}$$

The imposter's obligation at date $T^* - 2$ equals $(1 + R_H)x_{T^*-3}$, which exceeds new investment x_{T^*-2}. To avoid default at date $T^* - 2$, the imposter would have to save in period $T^* - 3$. Condition (20) ensures the imposter is indifferent between defaulting at $T^* - 3$ and $T^* - 2$ assuming they do not save in period $T^* - 3$. If there is an equilibrium in which $x_{T^*-1} = y$, it will satisfy a different condition than (20).

The condition for indifference when agents can save, which they must to remain solvent when $x_{T^*-1} = y$, will be particularly important if the solution to (17) and (18) implies $\Phi_{T^*-1} > z$ rather than $\Phi_{T^*-1} = z$ as we have considered so far. In that case, we must have $x_{T^*-1} = y$, and so the question becomes whether a Ponzi equilibrium is possible at all, not just in addition to the equilibrium in (19). We now turn to this scenario.

Equilibrium Paths for Investment with $x_{T^*-1} = y$

Condition (20) dictates when the imposter is indifferent between defaulting in period t and defaulting in period $t + 1$ if $x_t \geq (1 + R_H)x_{t-1}$. Using (20) to substitute in for x_{t-1}, we can be sure that the imposter is both solvent at date t and indifferent between defaulting in period

t and period $t+1$ whenever

$$x_{t-1} = \frac{\beta(1-\mu)}{1+R_H} x_{t+1} \qquad \text{if } x_t \geq \beta(1-\mu)x_{t+1} \tag{21}$$

If $x_t < (1+R_H)x_{t-1}$, the imposter will need positive wealth $w_t > 0$ at date t to delay default until $t+1$. If the imposter intends to default at date t, they should carry no wealth into period t (i.e., they should set $w_t = 0$) and then consume x_t. Let $\overline{w}_{t-1}$ denote the imposter's wealth at the end of period $t-1$ before consuming. Since the imposter will consume $\overline{w}_{t-1}$ at date $t-1$, their utility as of the end of period $t-1$ will be $\overline{w}_{t-1} + \beta(1-\mu)x_t$.

If the imposter intends to default at date $t+1$ instead of t, they should save just enough at date $t-1$ to remain solvent in date t. That is, they should save

$$s_{t-1} = \frac{(1+R_H)x_{t-1} - x_t}{1+R_L}$$

and consume the rest at date $t-1$. At date t, they would consume $(1+R_L)s_{t-1}$ if they are exposed. Otherwise, they would repay their debt and wait to default in period $t+1$. Their utility as of the end of period $t-1$ would be

$$(\overline{w}_{t-1} - s_{t-1}) + \beta\mu(1+R_L)s_{t-1} + \beta(1-\mu)\cdot 0 + \beta^2(1-\mu)^2 x_{t+1}$$

Using the expression $s_{t-1} = \frac{(1+R_H)x_{t-1}-x_t}{1+R_L}$ and equating the two utilities implies

$$\begin{aligned} x_{t-1} &= \frac{1-\beta(1+R_L)}{1-\beta\mu(1+R_L)} \frac{x_t}{1+R_H} + \frac{\beta(1-\mu)(1+R_L)}{1-\beta\mu(1+R_L)} \frac{\beta(1-\mu)}{1+R_H} x_{t+1} \\ &\equiv \alpha \left(\frac{x_t}{1+R_H} \right) + (1-\alpha) \frac{\beta(1-\mu)}{1+R_H} x_{t+1} \end{aligned} \tag{22}$$

where $\alpha = \frac{1-\beta(1+R_L)}{1-\beta\mu(1+R_L)}$ is between 0 and 1 and independent of R_H. Savings s_{t-1} must be positive if $x_t < (1+R_H)x_{t-1}$. Substituting in for x_{t-1} from (22), savings will be positive if

$$(1-\alpha)\left(\beta(1-\mu)x_{t+1} - x_t\right) > 0$$

Combining with (21), the condition that ensures the imposter will be indifferent about defaulting at date t and waiting to default at $t+1$ reduces to

$$x_{t-1} = \begin{cases} \frac{\beta(1-\mu)}{1+R_H} x_{t+1} & \text{if } x_t \geq \beta(1-\mu)x_{t+1} \\ \\ \alpha\left(\frac{x_t}{1+R_H}\right) + (1-\alpha)\frac{\beta(1-\mu)}{1+R_H}x_{t+1} & \text{if } x_t < \beta(1-\mu)x_{t+1} \end{cases} \tag{23}$$

The path that ensures the imposter is indifferent is thus still characterized by a second-order difference equation. Given $x_{T^*} = y$ and a value for x_{T^*-1}, including but not limited to $x_{T^*-1} = y$, condition (23) defines a unique path that ensures the imposter will be indifferent about default even if they have to save at date $t-1$ to delay default.

The path in (23) ensures that the imposter is indifferent about default for all t. An equilibrium requires that the imposter is solvent, i.e., that the imposter's wealth $\overline{w}_{t-1}$ at the end of $t-1$ is enough for them to save the amount $s_{t-1} = \frac{(1+R_H)x_{t-1}-x_t}{1+R_L}$ they need at date t. Our next result provides conditions under which the path (23) with $x_{T^*-1} = y$ is an equilibrium.

Proposition 7. *Suppose $\phi < z$ and $\mu < \frac{1-z}{1-\phi}$. A Ponzi equilibrium with $x_{T^*-1} = y$ exists if $T^* = 2$ or if $z \geq z^*$ for some cutoff $z^* < 1$. The path of investment in this equilibrium is given by (23).*

Recall that when the solution to the system of equations (17) and (18) implies $\Phi_{T^*-1} > z$, investment x_{T^*-1} will have to equal y at date T^*-1. In that case, the only candidate Ponzi equilibrium in which short-lived agents are indifferent until just before date T is the path defined by (23) with terminal conditions $x_{T^*} = x_{T^*-1} = y$. Proposition 7 states that a Ponzi equilibrium will indeed exist if either $T^* = 2$ or if z is sufficiently high and close to 1, i.e., if the promised return $1+R_H$ is not too large relative to $1+R_L$.

To see that a Ponzi equilibrium might not exist for low values of z when $T^* > 2$, consider the case where $T^* = 3$ and $x_2 = x_3 = y$. From (23), the investments that ensure the imposter will be indifferent about when they default are given by

$$\begin{aligned} x_1 &= \frac{\beta(1-\mu)}{1+R_H} y \\ x_0 &= \left[\frac{\alpha}{(1+R_H)^2} + \frac{1-\alpha}{1+R_H}\right]\beta(1-\mu)y \end{aligned}$$

As we let $R_H \to \infty$, the investment x_0 that will keep the imposter indifferent will converge to 0. The amount the imposter must save at date 0 is given by

$$s_0 = \frac{(1+R_H)x_0 - x_1}{1+R_L} = \frac{1-\alpha}{1+R_L}\frac{R_H}{1+R_H}\beta(1-\mu)y$$

This amount converges to $\frac{(1-\alpha)\beta(1-\mu)}{1+R_L}y > 0$ as $R_H \to \infty$. This will inevitably exceed x_0.

Hence, for large values of R_H, the imposter cannot be both solvent and indifferent.

Intuitively, the tension for the existence of Ponzi equilibria is that x_0 must satisfy two conditions. For the imposter to be indifferent between defaulting at date 1 and defaulting at date 2, the value of x_0 can't be too small: Waiting to default at date 2 is very tempting given how large x_2 is, so getting to consume x_0 instead of saving it must be sufficiently attractive. At the same time, x_0 can't be too large for the imposter to be able to cover an obligation of $(1+R_H)x_0$ by saving and using the new inflows x_1. For these forces not to be in tension requires that $1+R_H$ be close to $1+R_L$, implying z will be close to 1.

This example does not mean Ponzi schemes are never possible for low values of z. Our next proposition reinforces this point by showing that for any $z \in (0,1)$, there are always parameters that ensure $\Phi_{T^*-1} = z$ and hence a Ponzi equilibrium exists per Proposition 6.

Proposition 8. *For any $z \in (0,1)$, there exist values of ϕ and μ that ensure $\Phi_{T^*-1} = z$ and that a Ponzi equilibrium exists.*

To summarize, when the long-lived agent has a low initial reputation ($\phi < z$) and there is limited enforced ($\mu < \frac{1-z}{1-\phi}$), Ponzi equilibria in which an imposter keeps repaying earlier investors with the funds raised from new investors may be possible. Such Ponzi equilibria are harder to sustain when the promised repayment is generous (i.e., when z is close to 0), but Ponzi equilibria can occur even when the promised return is arbitrarily large. Stronger enforcement that raises μ can eliminate Ponzi equilibria, while educating people to be more skeptical of such schemes cannot eliminate such equilibria so long as ϕ remains positive.

The reason Ponzi schemes end in finite time without unraveling is that the reputation of the long-lived agent rises over time time if the scheme keeps going. Early on, agents are willing to invest despite the belief that the long-lived agent is likely an imposter since they still expect to be repaid from the funds of new investors. The last cohort that invests with an imposter knows there will no funds to repay them in case of fraud. However, they reckon the long-lived agent is likely to be genuine given how long the scheme has lasted.

7 Uniqueness and Comparative Statics

In this section, we examine whether Ponzi equilibria, when they exist, can be uniquely characterized. Proposition 4 tells us that when a Ponzi equilibrium exists, a no-trade equilibrium must exist as well. A Ponzi equilibrium thus cannot be the unique equilibrium outcome. However, there may still be a unique Ponzi equilibrium outcome. In that case, we can carry out comparative statics conditional on being in a Ponzi equilibrium. That can tell us what variation to expect in data on a variety of Ponzi schemes such as in the data sets compiled in some of the empirical work that we discussed in the Introduction.

Proposition 5 establishes that any Ponzi equilibrium that solves (13), (14), (15), and (16) is associated with a unique terminal date T^* at which an imposter must default. Such a Ponzi equilibrium is also associated with a unique path for the probability of default σ_t at each date t and a unique path for the beliefs Φ_t of short-lived agents over the likelihood of facing a genuine investor is also uniquely determined. The path of investments $\{x_t\}_{t=0}^{T^*}$ is uniquely determined if the latter path implies $\Phi_{T^*-1} > z$, since in this case $x_{T^*} = x_{T^*-1} = y$ and $\{x_t\}_{t=0}^{T^*}$ can be solved backwards using (23). However, if $\Phi_{T^*-1} = z$, the path of investments will not be uniquely determined even though T^*, σ_t, and Φ_t are. In that case, there will be a continuum of different equilibrium paths for how investment evolves over time.

That leaves the question of whether there exist additional Ponzi equilibria that do not solve (13), (14), (15), and (16). Our next result shows that if the imposter is sufficiently impatient, there will be no such Ponzi equilibria.

Proposition 9. *There exists a $\overline{\beta} \in \left(0, \frac{1}{1+R_H}\right]$ such that if $0 < \beta < \overline{\beta}$ and there exists a Ponzi equilibrium that lasts until date T, then $0 < x_t < y$ for $t = 0, ..., T-2$ in that equilibrium.*

In essence, a highly impatient agent will want to consume if investment attains its maximal possible value. However, even a highly impatient agent will not default when $x_t < y$ if they expect much larger investment in the future. High impatience does not rule out Ponzi equilibria, but it will limit which Ponzi equilibria are possible.

Imposing $\beta < \overline{\beta}$, we can describe how the maximal date T in the unique Ponzi equilibrium varies with ϕ and z, at least for values of μ that are sufficiently close to 0.

Proposition 10. *Suppose $\phi < z$. If μ is sufficiently small, then T^* in Lemma 3 is decreasing in ϕ and increasing in z. That is, with minimal oversight, a Ponzi scheme can last longer if the long-lived agent has a worse initial reputation (lower ϕ) or offers a lower relative return (higher z).*

Intuitively, the maximal duration of the Ponzi scheme corresponds to the time it takes beliefs to rise from their initial value of ϕ to the value z that draws in all short-lived agents to invest and induces the long-lived agent to default. Reducing the starting point ϕ or increasing the end point z requires beliefs to grow more before the scheme must collapse. However, the time it takes to travel from ϕ to z depends on the endogenous variable σ_t which governs the rate at which short-lived agents revised their beliefs. In principle, a lower reputation or a less generous return should make investing with the long-lived agent less attractive, requiring the long-lived agent to default with lower probability to keep short-lived agents indifferent. In that case, making it another period would not be as informative, and so short-lived agents would not revise their beliefs upwards as much. It is easy to show that the probability of default σ_1 in the first period falls when we lower ϕ or increase z. But because the expected payoff from investing with the long-lived agent at each t depends on both σ_t and σ_{t+1}, we cannot prove this for the entire path of σ_t except in the special case where $\mu = 0$. Although our analytical result only holds in a neighborhood of $\mu = 0$, numerically we confirmed the result for all values $\mu < \frac{1-z}{1-\phi}$ we considered.

Since the proof of Proposition 10 uses the fact that a lower ϕ or higher z is associated with (weakly) lower σ_t, it follows that the Ponzi scheme not only can last longer but is also less likely to end each period. A higher z or lower ϕ will therefore be associated with a longer lasting scheme on average. This prediction accords with data from prosecution records reported in Marquet (2011) that confirms schemes promising higher returns are shorter on average. This pattern remains even after excluding the Madoff scheme, which did not involve a particularly high return and lasted an exceptionally long period of multiple decades, in contrast to most schemes that last a few years at most.[5] Marquet argues the reason for this pattern is that higher promised payments are more difficult to sustain. Our model offers a more nuanced take. In principle, we can sustain a scheme for as long as we want by starting at a smaller initial investment that allows the scheme room to grow. However, imposters who promise high returns have a greater incentive to steal, which means they build more reputation when they survive. Such schemes draw large-scale investment earlier.

8 Welfare

Our results imply that when a Ponzi equilibrium exists, no trade is also an equilibrium. A natural question to ask when there are multiple equilibria is whether they can be Pareto ranked. Is society worse off under a Ponzi equilibrium?

Answering this question requires taking a stand on the welfare of the genuine type. We assume that this type is weakly better off if they receive funding than if they do not. Presumably, the reason a genuine type is willing to offer a return of $1 + R_H$ is that it has access to a technology that yields a return of at least $1 + R_H$ and which they can scale beyond their own endowment. If that were the case, the long-lived agent would weakly benefit from trade with short-lived agents.[6] Under this assumption, we get the following result.

[5] Madoff promised clients a stable return rather than a high return. Since agents in our model are risk neutral, it is hard to use our model to analyze that particular Ponzi operation.

[6] A separate question is why the commitment type has to pay savers more than what these agents can earn on their own. One possibility we already alluded to above is that savers cannot distinguish the commitment

Proposition 11. *If there exists both a Ponzi equilibrium and a no-trade equilibrium for the same parameter values, then the Ponzi equilibrium Pareto dominates the no-trade equilibrium ex ante.*

The argument for this result is as follows. By assumption, the genuine type is weakly better off with trade. The imposter type benefits, since they would consume nothing without trade but can consume at least $x_0 > 0$ in the Ponzi equilibrium. As for short-lived agents, they earn a return of $1 + R_L$ in the no-trade equilibrium. Given that they always have the option to save on their own, they must be weakly better under the Ponzi equilibrium. No agent is ex-ante worse off in the Ponzi equilibrium, while the imposter is strictly better off.

The reason that the Ponzi equilibrium is superior to the no-trade equilibrium is not because Ponzi schemes are welfare improving. Rather, it is because trade with the commitment type creates surplus. A long-lived imposter can cut into some of this surplus by passing themselves off as genuine. However, a Ponzi scheme is not necessary to achieve gains from trade. A severe penalty for imposters will drive out imposters and allow agents to gain from trading with a commitment type without requiring a Ponzi scheme.

Note that our model also abstracts from various features that would imply that a Ponzi fraud can destroy the surplus. With those features, it would no longer be the case that Ponzi equilibria are better than no trade. For example, investigations in our model do not consume resources. The fact that a Ponzi equilibrium triggers investigations would therefore not cut into the gains from trade with genuine long-lived agents. We also do not model entry for commitment types. As such, imposters in our model do not drive out productive agents that can create surplus by trading with short-lived agents.

The key takeaway from Proposition 11 is not that Ponzi schemes should be encouraged. Rather, it illustrates that Ponzi schemes require not only a low initial reputation and low enforcement, but also an inefficient initial allocation that lets agents gain from trade. This is what allows an imposter to take advantage and profit at the expense of savers.

9 Discussion

We conclude with several observations about how our model relates both to real-world Ponzi schemes and to the theoretical literature on bubbles and pyramid scams.

type and has to worry about the possibility of an imposter. Outsiders might be skeptical that the long-lived agent is genuine if they offer less than $1 + R_H$. This microfoundation, also raised by Amador and Phelan (2021), does not explain why the relevant rate is $1 + R_H$ as opposed to some other value.

Theft versus Excessive Risk-Taking

While Charles Ponzi never actually invested in the postage arbitrage he told his investors he was undertaking, other Ponzi schemes began as a a genuine attempt to undertake an investment that failed to pay off, and the scheme operator borrowed to pay previous debt holders while trying to get the investment to pan out. Our model does not capture this scenario, although a modified version of it can.[7]

To explore this possibility, we consider an extension of our model in Appendix B that allows an imposter to either steal funds or invest them in a risky technology. In this case, borrowing until an investment pays off can still be understood as a Ponzi fraud: An imposter pools with a commitment type that is safe to invest with and does not disclose to its lenders that they are in fact undertaking a risky investment. If lenders knew that the investment was risky, they would not fund it.

The detailed analysis of this case is in Appendix B. Here, we sketch out the main ideas. Suppose that each period, the imposter can consume, save at the risk-free rate $1 + R_L$, or undertake a risky investment that yields a return of $1 + R$ with probability λ and 0 with probability $1 - \lambda$. Short-lived agents cannot observe what the imposter does with their funds. We impose parameter restrictions such that if agents knew they were facing the imposter, they would refuse to trade. The interest rate they would require to let the long-lived agent invest in the risky technology would be high given the risk of getting nothing. At such a high interest rate, the imposter would prefer to steal rather than invest in the risky technology. Thus, short-lived agents would refuse to trade with a long-lived agent if they reveal that they are an imposter. If the imposter instead pools with the commitment type and offers to pay $1 + R_H$, our parametric assumptions ensure that they will have an incentive to undertake the risky investment even if they pay investors $1 + R_H$ if they succeed.

We present a numerical example of an equilibrium in which an imposter uses the funds they receive to operate a risky technology. If the technology pays off, short-lived agents observe the success and can force repayment of $1 + R_H$. The scheme then ends. If the risky investment pays nothing, the imposter borrows to pay off previous investors and uses the rest to operate the risky technology. In contrast to our benchmark model, where the imposter intends to steal and will default on some agent, here there is a possibility that all agents will be repaid in full. Nevertheless, the imposter hides the fact that they are undertaking risky

[7] Springer (2020) distinguishes between *intentional* Ponzi schemes that are frauds from the start and *unintentional* schemes where genuine investors hit by a shock borrow to cover their shortfalls until they recover. She finds that among the 1,359 schemes in her dataset based on prosecuted cases, 1,225 were intentional and did not involve actual investment. This may reflect that unintentional schemes are less likely to be prosecuted. However, outright fraud with no actual investment as in our model appears quite common.

investments, since short-lived agents would refuse to invest with them if they knew. The fact that repayments come from new investment is concealed from agents in equilibrium.

How Ponzi Schemes End

Charles Ponzi's scheme collapsed when too many of his investors demanded repayment. No such run occurs in our model. In the Ponzi equilibrium of our model, the investor is willing to wait to default only because they expect more investment in the future. Equilibrium investment must thus increase over time. Default occurs when investment peaks, not when it falls. What forces the scheme in our model to collapse is that investment stops growing rather than that investment starts to fall.

The notion that a Ponzi scheme must collapse when investment stops growing as opposed to when it starts to decline is consistent with some historical episodes. For example, Madoff's scheme collapsed when insufficient growth in new investment forced Madoff to admit his transgressions to his sons, who then turned him in to authorities. Before Madoff's two sons turned him in, Madoff tried to give out some of the funds that remained to his friends and family, consistent with the dynamics in our model.

The reason our model cannot generate a run is that it lacks two elements that seem to be essential for a run to occur. First, the imposter must have a reason not to default immediately when investment declines. The scenario in Appendix B, where the agent is waiting for investment to pan out, may create such an incentive. Alternatively, the endowment y might be random, and the imposter might prefer to wait to see if it rises again. Second, there has to be a reason for investment to decline gradually rather stop all at once. One possibility is the same temporary fall in the endowment y that may lead to lower investment. Alternatively, short-lived agents may receive a less stark signal than we assume, causing them to lower Φ_t but not to stop investing altogether. We leave such modelling for future work.

Asymmetric Information versus Naivety

Another feature of Ponzi frauds that is arguably missing in our model is the gullibility and naivety of some investors. The agents who invest with the long-lived agent in our model are fully aware of the risks they face. Indeed, short-lived agents are essentially co-conspirators, willing to invest with an agent they believe is quite likely to be an imposter because they know that in this case they might still be paid by the investment of subsequent cohorts.

The evidence suggests that both types of forces play a role. Frankel (2012) surveys the evidence on victims of Ponzi schemes and concludes

> This research on the attitudes of Ponzi scheme victims suggests that they are driven by two strong tendencies, which render them more vulnerable to the lure of the schemes. One powerful tendency is the drive to trust; it is a tendency that borders on gullibility. The other tendency is 'heightened risk tolerance.' " (p137)

She cites studies which find that victims of these schemes tend to more educated than non-victims on average, although this may be due to selection if wealthier targets are more attractive to scammers and also tend to be more educated. She also argues that investors in such schemes often demonstrate greed, a lack of empathy, and a willingness to invest in schemes they find suspicious. A nice illustration of this comes from work by Baker and Faulkner (2003) who surveyed participants in a scheme involving the California-based Fountain Oil and Gas Company in the late 1980s. This scheme involved separate joint-ventures in specific wells. An assistant district attorney involved in the case explains the Ponzi nature of the scheme in the article: "[M]oney had come in for investors to be used for a specific well. In some cases it was diverted to other wells, or other expenses of Fountain, or to specific personal purchases." While Fountain pressured its investors to refer their venture to others, only 24% of respondents did so, and each only made a single referral. 31% of respondents explained this reluctance as due to the investment being too risky. 23% of respondents said they didn't trust Fountain. One respondent explained that "I was always a little suspicious all the way along. These guys seemed a little slimy... My greed and their smooth superficial successful exterior overcame my suspicions." (p1194)

Leuz et al. (2023) found similar results for pump-and-dump schemes in Germany that involved touting a stock in order to sell to eager investors at a high price. Specifically, they found that there appears to be an investor type that invests in these schemes early on and tends to earn higher returns than other types. They interpret this pattern as evidence that some agents understand the risk of these schemes and invest in them despite the potential losses inherent to such a scheme.

An important feature of our model in which agents are skeptical rather than gullible is its implication that agents would adjust their investment in response to information. Here, there is broad consensus in the literature that positive actions by Ponzi scheme operators are key to attracting additional investors. Frankel (2012) writes

> Timely payments at short intervals help establish a reputation for trustworthiness. Each payment brings added proof of the con artist's credibility. (p39)

Victims of particular Ponzi schemes report in surveys that the fact an operation had been running for a while helped them overcome their concens and invest.

Comparisons with Bubbles and Pyramid Scams

Finally, we turn to how the Ponzi schemes we study are related to phenomena such as bubbles and pyramid schemes. We define a bubble as an asset whose price exceeds the expected present discounted value of the dividends it pays out over its lifetime. Since it is not optimal to buy an asset at this price and hold it indefinitely, the existence of a bubble typically requires that agents who hold the asset can find someone else to sell it to, just as a lender in a Ponzi scheme must rely on subsequent lenders to be repaid. A pyramid scheme is a setup in which agents pay for the right to recruit new participants into the scheme. An agent who buys into the scheme can only profit if they can find others to also buy into the scheme.

While economists sometimes use the terms Ponzi schemes, bubbles, and pyramid scams to describe the same phenomena, our model reveals some distinctions between the three. In symmetric full information settings, the concepts are closely related. Specifically, the existence of a Ponzi equilibrium implies that a bubble and a pyramid scheme can also exist. For suppose there exists an equilibrium in which a borrower keeps paying their old debt with new debt. In symmetric full information settings, the scheme is transparent to all agents. We can therefore eliminate the borrower and let lenders interact directly with one another. In particular, lenders can spend the amount they would have lent out to buy an intrinsically worthless asset and then sell to the lenders whose funds they would have received under the Ponzi scheme. This corresponds to a bubble. Likewise, we can get agents to spend the amount they would have lent out to join a pyramid scheme and then recruit those whose funds they would have received under the Ponzi scheme into the scheme.

With private information, this equivalence can break down. In particular, lenders in our model are unsure whether their returns are coming from new investors or from an actual technology that the scheme operator has access to. The fact that the proceeds investors earn may come from a productive technology rather than from other investors means that we cannot just eliminate the scheme operator and construct equivalent equilibria with bubbles or pyramid schemes. While there are private information models of bubbles and pyramid schemes, those models are qualitatively different from our model.

Examples of private information models of bubbles include Allen, Morris and Postlewaite (1993), Conlon (2004), Doblas-Madrid (2012), and Awaya, Iwasaki and Watanabe (2022). In these models, if a bubble occurs, it will collapse by a finite date, similarly to how a Ponzi scheme collapses by a finite date in our model if the long-lived agent is an imposter. In these models, all agents know that the asset is overvalued. However, the fact that the asset is overvalued is not common knowledge. Agents are willing to buy an asset they know is overvalued in the hope of selling it to an agent who does not know the asset is overvalued, or who does not know that all other agents know the asset is overvalued, and so on. Agents in these models know that they will profit at the expense of others. By contrast, the agents in our model do not know until the long-lived agent's type is revealed whether their profits come from others or from some real underlying investment.

Our model is arguably more similar to dynamic private information models of asset trade than to models of bubbles. For example, Awaya and Krishna (2022) develop a dynamic model in which an informed seller sells an asset they know is worthless to agents who are unsure whether the asset is valuable. The price of the asset rises over time as agents become more convinced that the asset is valuable given the absence of negative news. The price crashes once the asset is publicly revealed to be worthless. However, the price of the asset in their model is always equal to what uninformed buyers expect the asset will pay out and so is not a true bubble. Their model also does not allow agents to resell an asset after buying it, and so does not involve Ponzi-like transfers.

Turning to pyramid schemes, Stivers, Smith and Jin (2019) and Antler (2023) develop models of multilevel marketing in which there is a scheme operator similar to the long-lived agent in our model. The operator offers to sell distribution rights for a good they produce, as well as bonuses to distributors both for selling the good and for recruiting new distributors. A pure pyramid scheme corresponds to the case where the good is intrinsically worthless and agents buy the distribution rights only to earn recruiting bonuses. The key issue in these models is how to sustain a pure pyramid scheme if it is common knowledge that there are finitely many agents. Stivers, Smith and Jin (2019) assume agents incorrectly estimate the probability they can recruit new buyers, while Antler (2023) assumes agents are boundedly rational and hold beliefs that are correct on average across agents but not necessarily true for any given agent. Similar to private information models of bubbles, all agents understand that their profit comes from the funds of others, in contrast to our model. Agents gamble that they can successfully recruit enough new agents to make participation profitable.

A version of these models that would be closer in spirit to ours is one in which agents are uncertain whether the good produced by the scheme operator is valuable. They could then be unsure whether their profits come from selling the good or whether they would need to recruit new agents to profit. While this is similar to our model, some aspects would likely be different. In particular, a key question in our Ponzi scheme model is whether the long-lived agent is solvent and can keep the scheme going. Uncertainty about the quality of the good does not involve the scheme operator but the demand of future buyers.

One feature that our model of Ponzi schemes shares with models of bubbles and pyramid schemes is that when all agents are rational, gains from trade can be a precondition for these schemes to exist. Barlevy (2025) argues that in models of bubbles where agents are rational, an inefficient initial allocation is typically necessary for a bubble to occur, whether it is because trading the asset creates private value (as in symmetric information models) or because trade between speculators cuts into the surplus that agents can create when they trade (as in asymmetric information models). A model of pyramid schemes in which agents are unsure if the good in question is valuable would also involve gains from trade in certain states of the world. Proposition 11 shows that the same feature plays a crucial role in our model as well: Ponzi equilibria exist only when the no-trade outcome is inefficient.

10 Conclusion

This paper developed an asymmetric information model of Ponzi schemes that allows us to incorporate an element of misrepresentation, a key feature of Ponzi frauds like the one Charles Ponzi originally perpetrated. Our framework allows us to examine questions such as when a Ponzi scheme can be an equilibrium, how the scheme evolves over time, what features can potentially prevent it, and what these schemes tell us about welfare.

There are various elements that our analysis overlooks. For example, our benchmark model does not incorporate uncertainty in either earnings y or the return $1 + R_L$. As such, we cannot use our model to make inference on when and why Ponzi schemes fail. Empirically, Ponzi schemes appear to be more likely to be exposed during recessions. Presumably, this is due to the decline in inflows into these schemes during recessions as well as lower returns that make it difficult for imposters to remain solvent. Uncertain endowments may also allow for schemes that end in a run. Capturing this formally is beyond the scope of our current setup. We also assumed short-lived agents who withdraw all of their savings after one period. In practice, agents often reinvest some of their earnings rather than withdraw it all. Ignoring this simplifies our analysis but may overlook an important facet of such schemes. We similarly ignore recruiting fees, which recent empirical work has suggested is an important feature of many real-world Ponzi frauds. Finally, our model ignores the possibility of heterogeneity across investors. For example, agents might have access to different rates of return R_L or might differ in their initial beliefs about whether the long-lived agent is genuine. That would replace the indifference condition with conditions that govern the marginal investor. We leave these issues for future work.

Appendix A: Proofs

Proof of Lemma 1: The statement is necessarily true if $(1+R_L)y+y<(1+R_H)y$, since then the imposter would be insolvent even if they saved all of the resources x_0 they received. The non-trivial argument is for the case where this condition does not hold.

We first show that there exists a finite date at which the imposter must default. Let w_t^* denote the maximal wealth the imposter can have at the end of date t, i.e., if the imposter consumes nothing. Since $x_0 = y$, then $w_0^* = y$. Next, suppose $w_t^* \leq y$ for some date t. Then

$$
\begin{aligned}
w_{t+1}^* &= (1+R_L)w_t^* + y - (1+R_H)y \\
&= w_t^* - R_H y + R_L w_t^* \\
&\leq w_t^* - (R_H - R_L)y
\end{aligned}
$$

Since w_{t+1}^* falls below w_t^* by a discrete amount, the maximum wealth the imposter can have will cease to be positive in finite time. In that case, the imposter will have less than $(1+R_H)y$ and will be forced to default.

Next, let T denote the earliest date at which default occurs with probability 1, i.e., either $\sigma_t = 1$ or $S_T = 0$. Suppose $T \geq 2$. In this case, $T-1$ and $T-2$ are both nonnegative. Since T is the earliest date at which default occurs with probability 1, the imposter must be solvent at date $T-1$, i.e., $S_{T-1} = 1$. Given the definition of T, the imposter must weakly prefer defaulting at date T over defaulting at date $T-1$. If the imposter defaults at date $T-1$, their continuation payoff would be

$$w_{T-1} + y$$

If they wait to default at date T, the best they can do is immediately consume any resources they have left over at $T-1$ and then eat any resources they receive at date T and default. This gives them a utility of

$$w_{T-1} + y - (1+R_H)y + \beta(1-\mu)y$$

where we use the fact that investment at date $T-2$ is y. This expression is strictly below $w_{T-1} + y$. The imposter should therefore default at date $T-1$, which contradicts the assumption that T is the earliest date at which default occurs with probability 1, and so waiting until date T to default with certainty could not be optimal. ■

Proof of Proposition 1: Short-lived agents receive at least $\phi(1+R_H)$ if they invest with long-term agents at date 0. If $\phi > z$, this would exceed the return $1+R_L$ on savings, and so $x_0 = y$. Since Φ_t is increasing over time as long as the long-lived agent does not default, the same logic implies that x_t equals y for $t > 0$ until there is a default. By Lemma 1, the imposter must default in period 1. ■

Proof of Proposition 2: Suppose a Ponzi equilibrium existed. Given Lemma 1, we would need $x_{t_0} < y$ for a Ponzi equilibrium to exist. Short-lived agents at date t_0 must therefore be indifferent between investing with the long-lived agent and saving on their own. Short-lived agents at date t_0 are indifferent if $\Phi_{t_0} + (1 - \Phi_{t_0})(1 - \mu)(1 - \sigma_{t_0+1}) = z$. Since $\Phi_{t_0} = \phi = z$, this condition requires that $\sigma_{t_0+1} = 1$. That is, the probability that the agents who invest in period t_0 are repaid if the long-lived agent is an imposter must be 0. But then this cannot be a Ponzi equilibrium, which requires $\sigma_{t_0+1} < 1$. ■

Proof of Proposition 3: Suppose a Ponzi equilibrium existed. For short-lived agents to be willing to invest at date t_0, it must be the case that $\Phi_{t_0} + (1 - \Phi_{t_0})(1 - \mu)(1 - \sigma_{t_0+1}) \geq z$. For $\Phi_{t_0} = \phi$, we have $(1 - \mu)(1 - \sigma_{t_0+1}) \geq \frac{z-\phi}{1-\phi}$. If $\mu > \frac{1-z}{1-\phi}$, then $1 - \mu < 1 - \frac{1-z}{1-\phi} = \frac{z-\phi}{1-\phi}$. In that case, we would need $1 - \sigma_{t_0+1} > 1$ to ensure short-lived agents are willing to invest, which means $\sigma_{t_0+1} < 0$. Since $\sigma_{t_0+1} \geq 0$, it follows that short-lived agents at date t_0 would be better off saving on their own. But this contradicts our assumption that there exists a Ponzi equilibrium. ■

Proof of Proposition 4: To confirm that no trade is an equilibrium, we need to verify that each cohort of is willing not to invest given the strategies of all other agents.

We start at date 0 and proceed inductively. Agents who save on their own at date 0 earn a return of $1 + R_L$. If they expect that all other agents will save in period 0, which implies $x_0 = 0$, then they know that there will be no investigation regardless of what they do since each agent is infinitesimal. If they further that expect $x_1(h^1) = 0$ for $h^1 = \{0, 0\}$, then they know that a long-lived agent who is an imposter will be insolvent in period 1. The expected return from investing with the long-lived agent will then equal to $\phi(1 + R_H)$. When $\phi \leq z$, this expression will not exceed $(1 + R_L)$. Short-lived agents at date 0 would therefore be willing to save given their expectations of the strategies of other agents.

Next, suppose there is no investment before date t. Since $x_0 = \cdots = x_{t-1} = 0$, the imposter will not have any resources from past investments. If an agent expects all other short-lived agents at date t to not invest, meaning $x_t(h^t) = 0$ for $h^t = \{\mathbf{0}, \mathbf{0}, \mathbf{0}\}$, and they expect no short-lived agent will invest in period $t + 1$, meaning $x_{t+1}(h^{t+1}) = 0$ for $h^{t+1} = \{\mathbf{0}, \mathbf{0}, \mathbf{0}\}$, then by the same argument as for period 0, they would be willing to save.

Hence, when there is no trade at any date, all short-lived agents behave optimally, confirming that this allocation is indeed an equilibrium. ■

Proof of Lemma 2: Suppose $x_t(h^t) > 0$ for $h^t = \{\mathbf{x'_{t-1}}, \mathbf{0}, \mathbf{0}\}$. The proof is by contradiction.

Suppose that $x'_{t-1} = 0$. Since $x_t(h^t) > 0$, short-lived agents know that an imposter would be solvent and able to repay in full if only they deviated and invested. Moreover, the imposter would strictly prefer to repay: Defaulting would yield infinitesimal benefits, but revealing their type would prevent them from stealing a positive measure of investment in the future, which there must be given Φ_t increases over time and must exceed z in finite time. The expected return from investing with the long-lived agent at date t is thus

$$[\Phi_t + (1-\Phi_t)(1-\mu)](1+R_H)$$

Since Φ_t is increasing with t, this return is bounded below by

$$[\phi + (1-\phi)(1-\mu)](1+R_H)$$

The latter expression exceeds $1+R_L$ if $\phi + (1-\phi)(1-\mu) > z$. The latter condition is directly implied by $\mu < \frac{1-z}{1-\phi}$. If we start with an equilibrium where $x'_{t-1} = 0$, agents will have an incentive to unilaterally deviate. Applying the same argument by induction, we have that $x'_j > 0$ for all $j = 0, ..., t-1$. ■

Proof of Lemma 3: Suppose that $\Phi_t \in [\phi, z)$ and $\sigma_t \in [0,1)$ for $t = 0, ..., T-2$ for some $T \geq 2$. We want to show that $\Phi_{t+1} > \Phi_t$ for all $t = 0, ..., T-2$ and that $0 < \sigma_{T-1} < 1$.

First, equation (18) implies that

$$\Phi_{t+1} = \frac{\Phi_t}{\Phi_t + (1-\Phi_t)(1-\sigma_t)(1-\mu)}$$

Since $\mu > 0$, the denominator is a weighted average of Φ_t times 1 and $1-\Phi_t$ times an expression strictly below 1. As long as $\Phi_t \in [0,1)$, then $\Phi_{t+1} > \Phi_t$. Since $\Phi_t < z < 1$ for $t = 0, ..., T-2$, we have that $\Phi_{t+1} > \Phi_t$ for $t = 0, ..., T-2$.

To show that $\sigma_{T-1} \in [0,1)$, we proceed by induction. By definition, $\sigma_0 = 0$. Evaluating (17) for $t = 0$ and using the fact that $\Phi_0 = \phi$ implies

$$1 - \sigma_1 = \left(\frac{1}{1-\mu}\right)\frac{z-\phi}{1-\phi}$$

Rearranging the inequality $\mu < \frac{1-z}{1-\phi}$ yields $\frac{1}{1-\mu} < \frac{1-\phi}{z-\phi}$. This implies

$$1 - \sigma_1 = \left(\frac{1}{1-\mu}\right) \frac{z-\phi}{1-\phi} \in (0,1)$$

which implies $\sigma_1 \in (0,1)$.

Next, suppose that $\sigma_t \in [0,1)$ and $\Phi_t \in [\phi, z)$ for $t = 0, ..., t^*$. Let us divide (17) for $t = t^*$ by (17) for $t = t^* - 1$. This yields

$$\frac{1-\sigma_{t^*+1}}{1-\sigma_{t^*-1}} = \frac{(z-\Phi_{t^*})/(1-\Phi_{t^*})}{(z-\Phi_{t^*-1})/(1-\Phi_{t^*-1})} \tag{24}$$

Since Φ_{t^*-1} and Φ_{t^*} are both below z, the RHS of (24) is positive. Since $\sigma_{t^*-1} \in [0,1)$, it follows that $\sigma_{t^*+1} < 1$.

Next, the expression $\frac{z-\Phi_t}{1-\Phi_t}$ is decreasing in Φ_t, since the derivative of this expression with respect to Φ_t is equal to $-\frac{1-z}{(1-\Phi_t)^2} < 0$. Since $\Phi_{t^*} > \Phi_{t^*-1}$, equation (24) implies

$$\frac{1-\sigma_{t^*+1}}{1-\sigma_{t^*-1}} < 1$$

or, upon rearranging, $\sigma_{t^*+1} > \sigma_{t^*-1}$. Since $\sigma_{t^*-1} \geq 0$, then $\sigma_{t^*+1} > 0$.

The final step is to show that there exists a T such that $\Phi_{T-1} \geq z$. Suppose to the contrary that $\Phi_t < z$ for all t. In that case, $\Phi_t + (1-\Phi_t)(1-\mu) \leq z + (1-z)(1-\mu)$ which is bounded away from 1. This would mean $\frac{\Phi_{t+1}}{\Phi_t} > \frac{1}{z+(1-z)(1-\mu)}$ and so $\lim_{t\to\infty} \Phi_t = \infty$, which is a contradiction. Hence, T must be finite. Define T^* as the value of T to establish the lemma. ■

Proof of Proposition 5: First, suppose there was an equilibrium that solves (13), (14), (15), and (16) for some T where $T > T^*$. From Lemma 1, we know that $\sigma_t \geq 1$ for $t = T^* < T$. Since probabilties are less than 1, this must mean $\sigma_{T^*} = 1$. But then the scheme must end at date T^* which is before date T. The Ponzi equilibrium cannot last until date T.

Next, suppose there was an equilibrium that solves (13), (14), (15), and (16) for some T where $T < T^*$. For this to be an equilibrium that can last until date T, we must have either $\sigma_T = 1$ or $S_T = 0$ for some $T \leq T^* - 1$. Either way, short-lived agents at date $T - 1 \leq T^* - 2$ know they will not be repaid if the long-lived agent is an imposter. The return they expect to receive is thus $\Phi_{T-1}(1 + R_L)$. But Lemma 1 tells us that $\Phi_t < z$ for $t \leq T^* - 2$. This means these agents at date $T - 1$ would not invest with the long-lived asset. But then there would be no obligation to default on at date T. ■

Proof of Proposition 6: To confirm that the path for $\{x_t\}_{t=0}^{T^*}$ is an equilibrium, we first need to verify that investors are indifferent between saving on their own and investing with the long-lived agent in periods $t = 0, ..., T^* - 1$. This follows from Lemma 3 and the fact that $\Phi_{T^*-1} = z$. Second, we need to verify the long-lived agent is indifferent between defaulting in any period $t = 1, ..., T^* - 1$ and prefers to default at date T^*. The indifference before date T^* follows from the discussion in the text that before date T^*, the imposter will be solvent without having to save, and that when there is no need to save, the imposter will be indifferent about defaulting at date t iff $x_{t-1} = \frac{\beta(1-\mu)}{1+R_H} x_{t+1}$. By the same argument, the imposter will strictly prefer to default in period T^* than wait to default in period $t + 1$. ■

Proof of Proposition 7: We conjecture a particular investment path $\{x_t\}_{t=0}^{T^*}$ and confirm that there exists a cutoff z^* such that our conjecture is indeed an equilibrium for $z \geq z^*$.

We conjecture that the path $\{x_t\}_{t=0}^{T^*-2}$ implied by (23) satisfies the following properties:

(i) If $T^* - t$ is even: $x_t = \frac{\beta(1-\mu)}{1+R_H} x_{t+2}$

(ii) If $T^* - t$ is odd and $x_{t+1} \geq \beta(1-\mu)x_{t+2}$: $x_t = \frac{\beta(1-\mu)}{1+R_H} x_{t+2}$

(iii) If $T^* - t$ is odd and $x_{t+1} < \beta(1-\mu)x_{t+2}$: $x_t = \alpha \left(\frac{x_{t+1}}{1+R_H}\right) + (1-\alpha)\frac{\beta(1-\mu)}{1+R_H} x_{t+2}$

This path coincides with the path (19) from Proposition 6 for periods t where $T^* - t$ is even, but allows for different values of x_t in periods t where $T^* - t$ is odd.

When $T^* = 2$, this path implies $x_0 = \frac{\beta(1-\mu)}{1+R_H} y$, $x_1 = x_2 = y$. Since $\frac{x_1}{x_0} > 1 + R_H$, the imposter will be solvent at date 1. When $T^* = 2$, this is the only date where solvency matters (there is no debt to default on in period 0 and the imposter will default with certainty in period 2). So a Ponzi equilibrium exists in this case.

Consider a date t where $T^* - t$ is even (and $T^* - t \geq 2$). From (ii) and (iii), we know x_{t-1} satisfies (23). That ensures that if $\overline{w}_{t-1} \geq s_{t-1} = \frac{(1+R_H)x_t - x_{t-1}}{1+R_L}$, the imposter will be indifferent between defaulting in period t and period $t + 1$.

Next, consider a date t where $T^* - t$ is odd (and $T^* - t \geq 3$). From (i), we know that $x_{t-1} = \frac{\beta(1-\mu)}{1+R_H} x_{t+1}$. The imposter will be indifferent between defaulting in period t and period $t + 1$ if they do not need to save to avoid default at date t, i.e., if $x_t \geq (1 + R_H)x_{t-1}$. The argument that they will not need to save to avoid default at dates t where $T^* - t$ is odd is by induction over the odd whole numbers.

First, when $T^* - t = 1$, we have $x_{T^*-1} = y > \beta(1-\mu)y = (1+R_H)x_{T^*-2}$. So the imposter does not need to save in order to avoid default at date $T^* - 1$.

Next, suppose the for some odd number k, we have $x_{T^*-k} \geq (1+R_H)x_{T^*-k-1}$. We need to show that $x_{T^*-k-2} \geq (1+R_H)x_{T^*-k-3}$. There are two options for x_{T^*-k-2}:

- $x_{T^*-k-2} = \alpha\left(\frac{x_{T^*-k-1}}{1+R_H}\right) + (1-\alpha)\frac{\beta(1-\mu)}{1+R_H}x_{T^*-k}$. In this case, (iii) implies that $x_{t-k-1} < \beta(1-\mu)x_{t-k}$. Combining the two conditions implies $x_{t-k-2} > \frac{x_{t-k-1}}{1+R_H} = \frac{x_{t-k-3}}{\beta(1-\mu)} \geq (1+R_H)x_{T^*-k-3}$.
- $x_{T^*-k-2} = \frac{\beta(1-\mu)}{1+R_H}x_{T^*-k}$. In this case, then since $T^* - k - 3$ must be even, (i) implies that $x_{T^*-k-3} = \frac{\beta(1-\mu)}{1+R_H}x_{T^*-k-1}$. Since $x_{T^*-k} \geq (1+R_H)x_{T^*-k-1}$ is true by assumption we can multiply both sides by $\frac{\beta(1-\mu)}{1+R_H}$ to confirm that $x_{T^*-k-2} \geq (1+R_H)x_{T^*-k-3}$.

To recap, if $\overline{w}_{t-1}$ exceeds the amount the imposter needs to save, the conjectured path leaves the imposter indifferent between defaulting at date t and date $t+1$ at each date t.

To confirm that the imposer can afford to save when $T^* - t$ is odd, it will suffice to show that $x_{t-1} \geq s_{t-1}$ in those dates where $s_{t-1} > 0$. In that case, we have

$$s_{t-1} = \frac{(1+R_H)x_{t-1} - x_t}{1+R_L} = \frac{1-\alpha}{1+R_L}\left(\beta(1-\mu)x_{t+1} - x_t\right)$$

Evaluating $x_{t-1} - s_{t-1}$, we have

$$\begin{aligned} x_{t-1} - s_{t-1} &= \left[\frac{\alpha}{1+R_H} + \frac{1-\alpha}{1+R_L}\right]x_t + \left[\frac{1-\alpha}{1+R_H} - \frac{1-\alpha}{1+R_L}\right]\beta(1-\mu)x_{t+1} \qquad (25) \\ &\geq \frac{x_t}{1+R_H} + \left[1 - \frac{1}{z}\right](1-\alpha)\frac{\beta(1-\mu)}{1+R_H}x_{t+1} \qquad (26) \end{aligned}$$

This expression will turn positive as $z \to 1$. For any finite sequence, we can find a minimum value of z that ensures the imposter remains solvent at all dates. Denote this value by z^*. This proves the claim. ■

Proof of Proposition 8: Here, we build on Proposition 10 which shows that T^* is decreasing in ϕ holding other parameters fixed. As $\phi \to z$, the value of T^* converges to 2: It will take less time to reach z if we start close to z, and the probability σ_t only increases in ϕ. Likewise, the value of T^* must tend to ∞ as $\phi \to 0$: It will take more time to reach z if we start closer to 0, and the the probability σ_t only decreases as ϕ becomes smaller. For any $2 < \mathcal{T} < \infty$, this means that $\Phi_{\mathcal{T}-1}$ must transition from $\Phi_{\mathcal{T}-1} > z$ to $\Phi_{\mathcal{T}-1} < z$ as ϕ increases. By continuity, there must exist a ϕ such that $\Phi_{\mathcal{T}-1} = z$. ■

Proof of Proposition 9: From Lemma 2, we know that $x_t > 0$ for $t = 0, ..., T-1$.

Next, we argue $x_t < y$ for $t = 0, ..., T-2$ if $\beta < \overline{\beta}$ for some $\overline{\beta} \in (0,1)$. For suppose there was a date $t \in \{0, ..., T-2\}$ for which $x_t = y$. After raising y in period t, the imposter must choose whether to default in period t or not. They have three options:

(i) default on their obligation of $(1+R_H)x_t$, which would yield a continuation utility of

$$w_t + y$$

(ii) Wait one period and then default, which would yield a continuation utility of

$$w_t + y - (1+R_H)x_{t-1} + \beta(1-\mu)x_{t+1}$$

(iii) Wait to default after date $t+1$, which would yield a continuation utility of at most

$$w_t + y - (1+R_H)x_{t-1} + \frac{\beta(1-\mu)}{1-\beta(1-\mu)}y$$

The latter expression is due to the fact that there are y resources available each period, so the best the agent can do after not defaulting is consuming y as long as they are not exposed.

We first claim that $w_t \leq (1+R_L)x_{t-1}$ for all t. The argument is by induction. At date 1, the imposter's wealth $w_1 \leq (1+R_L)x_0$, since $w_0 = 0$ and the most they can save is the amount they receive at date 0 at a return of $1+R_L$.

Next, suppose the imposter's wealth $w_t \leq (1+R_L)x_{t-1}$ at date t. At date $t+1$, the imposter's wealth must satisfy

$$\begin{aligned} w_{t+1} &\leq (1+R_L)(w_t + x_t - (1+R_H)x_{t-1}) \\ &\leq (1+R_L)(x_t - (R_H - R_L)x_{t-1}) \\ &\leq (1+R_L)x_t \end{aligned}$$

Since $x_t > 0$ in any Ponzi equilibrium, we knowt hat $(1+R_H)x_{t-1} > (1+R_L)x_{t-1}$ for any date $t = 1, ..., T-1$. Combining this with the fact that $w_t \leq (1+R_L)x_t$, we have $w_t \leq (1+R_L)x_{t-1}$ for all t, we have

$$w_t + y - (1+R_H)x_{t-1} < y$$

Denote $w_t + y - (1+R_H)x_{t-1}$ at date t by $y - \epsilon_t$. In the limit as $\beta \to 0$, the imposter will prefer to default immediately to waiting to default until after date $t+1$. Hence, for sufficiently small β, the imposter would prefer to either default at date t or $t+1$ than to wait beyond date $t+1$, which is inconsistent with a Ponzi equilibrium that lasts until period T.

Note that the imposter will not necessarily default for any value of x_t, since for x_t small we can always have $\frac{x_{t+1}}{x_t} \geq \frac{1}{\beta(1-\mu)}$. But it will not be possible for investment to grow enough to keep the imposter interested when x_t is already large.

The value of β that ensures the imposter would default at date t if $x_t = y$ depends on ϵ_t. To obtain a single value $\overline{\beta} > 0$, we need to make sure that $\inf\{\epsilon_t\}_{t=0}^{\infty} > 0$ for all Ponzi equilibria. Here, we use the fact that all Ponzi equilibria end by some finite date $\overline{T}$. To see this, note that $\Phi_{t+1} \geq \frac{\Phi_t}{\Phi_t+(1-\Phi_t)(1-\mu)} > \Phi_t$ where the last inequality uses the fact that $\mu > 0$. Hence, there exists some finite T such that $\Phi_t \geq z$ for $t \geq T$. From that date on, short-lived agents invest y in every period. But the imposter will not be able to sustain this scheme: Each period, he will have to finance a growing shortfall given it adds an additional obligation of at least $(R_H - R_L)y$ each period, until eventually the shortfall will exceed y and the imposter will be insolvent. Hence, a Ponzi scheme must end by a finite date $\overline{T}$. Define $\overline{\beta}$ as the smallest value of β that ensures the imposter would rather get y now than wait and earn $y - \epsilon_t$. ■

Proof of Proposition 10: We start with a lemma for the special case where $\mu = 0$.

Intermediate Lemma: If $\mu = 0$, the solution to (17) and (18) features $\sigma_{2k} = 0$ and $\Phi_{2k+1} = \Phi_{2k}$ for $k = 0, 1, 2, ...$

Proof of Intermediate Lemma: The proof is by induction. Since there is nothing to default on at date 0, we have $\sigma_0 = 0$. The belief Φ_0 is equal to the prior probability ϕ. From (18), we can solve for $\Phi_1 = \frac{\phi}{\phi+(1-\phi)} = \phi = \Phi_0$. So the statement holds for $k = 0$.

Next, suppose the statement holds for k. We need to show it also holds for $k+1$. Evaluating (17) when $t = 2k$ and $t = 2k+1$ and combining them, we have

$$\frac{(1-\sigma_{2k+1})(1-\sigma_{2k+2})}{(1-\sigma_{2k})(1-\sigma_{2k+1})} = \frac{z-\Phi_{2k+1}}{1-\Phi_{2k+1}} \cdot \frac{1-\Phi_{2k}}{z-\Phi_{2k}} \tag{27}$$

Since $\Phi_{2k} = \Phi_{2k+1}$, the RHS of (27) reduces to 1. From this, it follows that $\sigma_{2k+2} = \sigma_{2k}$. But the latter is equal to 0. Hence, $\sigma_{2(k+1)} = 0$.

Using (18) evaluated at $t = 2k+2$, we have

$$\Phi_{2k+3} = \frac{\Phi_{2k+2}}{\Phi_{2k+2} + (1-\Phi_{2k+2})(1-\sigma_{2k+2})} \tag{28}$$

Since $\sigma_{2k+2} = 0$, it follows that $\Phi_{2k+3} = \Phi_{2k+2}$, i.e., $\Phi_{2(k+1)+1} = \Phi_{2(k+1)}$. ■

Using the lemma, we can reduce the system of equations given by (17) and (18) so that it is easier to work with. Setting $t = 2k$ in (17) and using the fact that $\sigma_{2k} = 0$, we have

$$\Phi_{2k} + (1-\Phi_{2k})(1-\sigma_{2k+1}) = z \tag{29}$$

Next, setting $t = 2k+1$ in (18) and using the fact that $\Phi_{2k} = \Phi_{2k+1}$, we have

$$\Phi_{2k} + (1-\Phi_{2k})(1-\sigma_{2k+1}) = \frac{\Phi_{2k}}{\Phi_{2k+2}} \tag{30}$$

Since $\sigma_{2k} = 0$ and $\Phi_{2k+1} = \Phi_{2k}$, we can solve for the relevant equilibrium objects using the system of equations defined over the variables $\{\Phi_{2k}, \sigma_{2k+1}\}_{k=0}^{\infty}$.

Let $\Phi_{2k}(z)$ and $\sigma_{2k+1}(z)$ denote the solution to (29) and (30) given a value for z. We pick two values $z'' > z'$ that satisfy $\mu < \frac{1-z''}{1-\phi} < \frac{1-z'}{1-\phi}$.

For $k = 0$, we have $\Phi_0(z) = \phi$ regardless of z. Trivially, then, $\Phi_0(z'') \leq \Phi_0(z')$. Turning to σ_1, we can use (29) to solve for σ_1, i.e.,

$$\sigma_1 = 1 - \frac{z-\phi}{1-\phi} \tag{31}$$

This expression is decreasing in z. Since $z'' > z'$, we have $\sigma_1(z'') < \sigma_1(z')$. Since $\mu < \frac{1-z''}{1-\phi} < \frac{1-z'}{1-\phi}$, it follows that both $\sigma_1(z')$ and $\sigma_1(z'')$ are between 0 and 1.

We now proceed by induction. Suppose that for some integer k, we have

(i) $\phi \leq \Phi_{2k}(z'') \leq \Phi_{2k}(z') < z' < z''$

(ii) $0 < \sigma_{2k+1}(z'') < \sigma_{2k+1}(z') < 1$

We want to show that these conditions also hold for the integer $k+1$, i.e., that the same conditions hold for $\Phi_{2k+2}(z)$ and $\sigma_{2k+3}(z)$.

We begin with $\Phi_{2k+2}(z)$. From (30) to get

$$\begin{aligned}
\Phi_{2k+2}(z') &= \frac{\Phi_{2k}(z')}{\Phi_{2k}(z') + [1-\Phi_{2k}(z')][1-\sigma_{2k+1}(z')]} \\
&= \frac{1}{1+\left(\frac{1}{\Phi_{2k}(z')}-1\right)[1-\sigma_{2k+1}(z')]}
\end{aligned} \tag{32}$$

Since $0 < \Phi_{2k}(z'') \leq \Phi_{2k}(z') < 1$ and $0 < \sigma_{2k+1}(z'') < \sigma_{2k+1}(z') < 1$, we have

$$\left(\frac{1}{\Phi_{2k}(z')}-1\right)[1-\sigma_{2k+1}(z')] < \left(\frac{1}{\Phi_{2k}(z'')}-1\right)[1-\sigma_{2k+1}(z'')] \tag{33}$$

which implies that $\Phi_{2k+2}(z'') \leq \Phi_{2k+2}(z')$ as claimed.

Next, (29) implies

$$\begin{aligned}
1-\sigma_{2k+3}(z') &= \frac{z'-\Phi_{2k+2}(z')}{1-\Phi_{2k+2}(z')} \\
1-\sigma_{2k+3}(z'') &= \frac{z''-\Phi_{2k+2}(z'')}{1-\Phi_{2k+2}(z'')}
\end{aligned}$$

Since $z'' > z'$, we have

$$\frac{z''-\Phi_{2k+2}(z'')}{1-\Phi_{2k+2}(z'')} \geq \frac{z'-\Phi_{2k+2}(z'')}{1-\Phi_{2k+2}(z'')}$$

Since we just showed that $\Phi_{2k+2}(z') \geq \Phi_{2k+2}(z'')$ and the expression $\frac{z-\phi}{1-\phi}$ is decreasing in ϕ for $\phi < z$, we have

$$\frac{z'-\Phi_{2k+2}(z'')}{1-\Phi_{2k+2}(z'')} \geq \frac{z'-\Phi_{2k+2}(z')}{1-\Phi_{2k+2}(z')}$$

Combining inequalities yields

$$\sigma_{2k+3}(z'') < \sigma_{2k+3}(z') \tag{34}$$

From Lemma 3, we have that $T^* - 2 = \sup\{t : \Phi_t(z) < z\}$. Since we just showed that $\Phi_t(z)$ is decreasing in z from date 2 on, it follows that T^* is weakly increasing in z when $\mu = 0$. By continuity, the claim should hold for μ close to 0 as well.

We can use the same argument to show that for $\phi \in (0, z)$, if $0 < \phi' < \phi'' < z$, then $T^*(\phi'') > T^*(\phi')$. ■

Appendix B: Ponzi Schemes with Risky Investment

In this Appendix, we consider a variation of the model in the text in which the imposter can undertake a risky but profitable investment. In contrast to our benchmark model in which the long-lived agent only benefits from stealing and will necessarily default on some cohort, adding investment will make it possible for the long-lived agent to avoid default. Nevertheless, the long-lived agent still preys on short-lived agents by pooling with a commitment type. If short-lived agents knew that the agent they invest with can only invest in a risky technology, they would refuse to invest with them.

Investment Technology

Formally, we modify the model to allow the imposter to invest in a risky technology. This is in addition to the options of consuming and saving at the same rate of return $1 + R_L$ that short-lived agents can achieve on their own. The assumptions that characterize the risky technology are as follows:

- The return on a risky investment initiated at date t is realized in period $t + 1$. It equals $1 + R$ with probability λ and 0 with probability $1 - \lambda$, where $\lambda < z$ and $R > R_H$.
- If the long-lived agent invests in the risky technology and it yields a positive payoff, short-lived agents observe that the payoff was positive and that the investment was risky. A successful risky investment thus reveals the long-lived agent's type.
- There is a court that can verify whether the long-lived agent defaulted and which can seize the proceeds from any successful investment of the long-lived agent and use them to pay short-lived investors or those who inherit the unpaid obligations of the previous generations.
- The court cannot prevent the long-lived agent from stealing to consume. It can only seize the proceeds from investment. A long-lived agent who defaults will therefore not benefit from investing, but they can benefit from stealing funds and consuming them.
- The court cannot identify whether the technology that long-lived agent use is risky. It can punish the long-lived agent for defaulting but not for operating a risky technology.

Under these assumptions, the long-lived agent will repay short-lived agents from the previous period if their investment succeeds. In addition, if they default, they will not invest in the risky technology given they expect the court to punish them for defaulting.

Timing

We integrate the risky technology into the timeline of our model as follows.

In each period $t \geq 0$, the imposter chooses at the end of each period whether to consume the resources they have at the end of the period, save them at rate $1 + R_L$, or invest them in the risky technology.

In each period $t \geq 1$, if the imposter previously undertook the risky investment, its payoff is revealed at the start of the period, before the imposter might be exogenously exposed.

Equilibrium when Long-Lived Agent type Revealed

We now introduce a parametric assumption that ensures that if the long-run agent is exposed as an imposter, they will be unable to profitably trade with short-lived agents.

Assumption 1: $\lambda\beta\left(R - \frac{R_L}{\lambda}\right) < 1$

The next argument establishes that under this condition, there will be no incentive for short-lived agents to trade with the long-lived agent if they are revealed to only have access to the risky technology.

Claim 1: There are no gains from trade between short-lived agents and a known imposter. If the imposter is exposed at date t, then $x_{t+s} = 0$ for $s \geq 0$ without loss of generality.

Proof: Let $1 + r_{t+1}$ denote the return on investment to short-lived investors from date t if the long-lived agent undertook the risky investment at date t and it pays out at $t + 1$.

Define $r^* = R - \frac{1}{\lambda\beta}$ as the cutoff rate at which the expected return after paying investors, $\lambda\beta(R - r^*)$, is equal to 1. If $r_{t+1} > r^*$, the long-lived agent will not initiate the risky investment. To see this, define V_{t+1} as the continuation value per unit invested in the risky technology for the exposed imposter at date $t + 1$ when there is no evidence of a risky investment that paid off. Since the long-lived agent always has the option of doing nothing, $V_{t+1} \geq 0$. If the exposed imposter chooses to undertake the risky investment, their expected utility per unit invested in the risky technology will be

$$\begin{aligned} \lambda\beta\left(R - r_{t+1}\right) + (1-\lambda)\beta V_{t+1} &< \lambda\beta\left(R - r^*\right) + (1-\lambda)\beta V_{t+1} \\ &\leq 1 + (1-\lambda)\beta V_{t+1} \\ &\leq 1 + \beta V_{t+1} \end{aligned}$$

Hence, if $r_{t+1} > r^*$, the long-lived agent will prefer to steal the funds they receive to investing them in the risky technology.

Assumption 1 implies that $r^* < \frac{R_L}{\lambda}$. Let $1 + r_t^0$ denote the return on investment to short-lived investors from date t if there is no payoff to a risky investment at date $t+1$. To attract short-lived investment at date t, the long-lived agent must offer an expected return of at least $1 + R_L$. Hence, if the long-lived agent invests in the risky technology, we must have

$$\lambda r_{t+1} + (1-\lambda) r_{t+1}^0 \geq R_L \tag{35}$$

Rearranging, we have

$$r_{t+1}^0 \geq \frac{R_L - \lambda r_{t+1}}{1-\lambda} \tag{36}$$

For the long-lived agent to be willing to operate the risky technology, we need $r_{t+1} \leq r^*$. Hence, if the long-lived agent invests, the return when the investment does not pay must be a least $\frac{R_L - \lambda r^*}{1-\lambda}$, which under Assumption 1 must be strictly positive.

Suppose the long-lived agent borrows to invest in the risky technology at date t. If the payoff on the risky investment was zero, the long-lived agent would have to borrow from new investors to pay off their promised return of r_{t+1}^0. Thereafter, their debt would grow at a rate bounded away from zero unless they invested in the risky technology and it paid off. Otherwise, the long-lived agent would have to borrow at a rate of $1 + R_L$ if they chose not to invest given short-lived agents would demand that as the safe return, or they would borrow at a rate that exceeds $\frac{R_L - \lambda r^*}{1-\lambda} > 0$ if they invested in the risky asset and it paid zero. As long as the long-lived agent failed to make a successful risky investment, their debt obligation would grow without bound.

Since the debt obligation grows without bound, there exists some finite date t^* in which the long-lived agent will not be able to borrow enough from new investors to pay investors from date $t^* - 1$ an expected return of $1 + R_L$. Knowing this, short-lived agents will not agree to invest in period $t^* - 1$ if the investment was unsuccessful. That means the long-lived agent cannot invest in the risky technology in period $t^* - 2$, since we know they need to borrow resources in period $t^* - 1$ if their investment is unsuccessful to offer investors in period $t^* - 2$ an expected return of $1 + R_L$. Repeating the same argument, the long-lived agent cannot borrow to invest in the risky technology at date t, which is a contradiction. ■

Essentially, even when the long-lived agent's type is known, short-lived agents still cannot monitor the long-lived agent. If the high return on the project fails to materialize, they cannot verify whether this is because the investment failed or because the imposter stole the funds and didn't invest anything. This implies that the long-lived agent cannot promise too high of a return in case the investment is successful: If they did, they would have an incentive to not invest at all. A low promised return if the risky investment is successful requires a high return if it is unsuccessful. But that could create a situation where the long-lived agent may have to keep rolling over debt for arbitrarily long periods, which cannot happen when the endowment is constant. That is, a strategy of rolling over debt until an investment succeeds and can be used to repay debt is not sustainable.

Optimal Behavior when Investment Stops Growing

We now proceed to look for a Ponzi equilibrium while the imposter's type is uncertain. We begin with two additional parametric assumptions. We then characterize the imposter's optimal strategy if x_t is equal to y for all t.

Our first assumption is that the long-lived agent is impatient:

Assumption 2: $\beta < \frac{1}{1+R_H}$

As before, this assumption implies that $\beta < \frac{1}{1+R_L}$ so the long-lived agent would prefer consuming immediately to saving and consuming after one period.

Our next assumption rules out the case where the long-lived agent prefers stealing to investing in the risky technology.

Assumption 3: $\beta\lambda(R - R_H) > 1$

If the inequality above were reversed and $\beta\lambda(R - R_H)$ were less than 1, the long-lived agent would strictly prefer to consume resources immediately to investing them in the risky technology: The marginal utility from consuming exceeds the marginal utility from investing, and the long-lived agent can do anything after consuming that it could after investing, while if they invest successfully their options to continue raising funds would end. Assumption 3 is necessary but not sufficient for the imposter to invest in the risky technology.

Before we turn to the question of whether the imposter chooses to undertake the risky investment, we obtain a result about what the imposter would do once beliefs were sufficiently optimistic to ensure short-lived agents would keep investing y in every period.

Claim 2: Suppose $x_{t+s} = y$ for all $s \geq 0$ and the long-lived agent's type is not revealed by date t. The imposter will default by date $t + 1$.

Proof: When $x_{t+s} = y$, the imposter would increase their debt obligation by at least the amount $(R_H - R_L)y$ each period in which they fail to successfully invest and do not default.

Their debt obligation would eventually exceed y, and the imposter would have to default if they were unsuccessful in investing. Let $t + S$ denote the earliest date of default for an imposter whose type remains uncertain, meaning they neither defaulted nor successfully invested in the past.

Suppose $S > 1$. What could the imposter have done in period $t + S - 1$ if it was behaving optimally? They would have started period $t + S - 1$ with wealth w_{t+S-1}. If they consumed the resources they had access to in date $t + S - 1$ after defaulting, their utility would be

$$w_{t+S-1} + y$$

Waiting to default in period $t + S$ without exposing their type would require them to pay their obligation of $(1 + R_H)x_{t+S-2} = (1 + R_H)y$. If they consumed the amount $w_{t+S-1} + y - (1 + R_H)y$, their utility would be

$$w_{t+S-1} + y - (1 + R_H)y + \beta(1 - \mu)y$$

This is strictly less than defaulting and consuming everything at date $t + S - 1$. Hence, if the imposter chose to wait until date $t + S$ to default for $S > 1$, they would not have chosen to avoid default and consume the resources they had left at the end of period $t + S - 1$. Avoiding default and saving the resources left at the end of period $t + S - 1$ would yield even lower utility. It follows that if the imposter first defaults in date $t + S$ for $S > 1$, it must be because they chose to invest their wealth in the risky project at date $t + S - 1$.

For the imposter to have resources to invest in the risky project at date $t + S - 1$ without revealing their type, it must be the case that $w_{t+S-1} + y - (1 + R_H)y > 0$. This means that $w_{t+S-1} > 0$. For the agent to have started with this wealth required them to save $w^*_{t+S-2} = \frac{w_{t+S-1}}{1+R_L}$. But the imposter would have been better off investing w^*_{t+S-2} at the end of period $t + S - 2$ given that $\beta(1 + R_L) < 1$. Hence, the imposter would not wait to invest beyond date t and would not default beyond date $t + 1$. ■

Constructing a Ponzi Equilibrium

In a Ponzi equilibrium where the imposter uses new funds to cover their obligation to previous investors, short-lived agents would revise their beliefs Φ_t upwards as long as the long-lived agent does not default. Eventually, Φ_t would be high enough that all short-lived agents would choose to invest, implying $x_t = y$ for all subsequent t. From Claim 2, we know that the imposter would default within one period. We now look for an equilibrium in which ϕ is low enough so that x_0 can fall below y and the long-lived agent is willing to postpone default for several periods and use new funds to cover their obligations.

Once again, we look for a Ponzi equilibrium in which $x_t \in (0, y)$ until just before the imposter defaults. This requires that short-lived agents be indifferent between saving on their own and investing with the long-lived agent until just before the imposter defaults.

To characterize such an equilibrium, we introduce some notation. Let Φ_t denote the belief of the short-lived agent at the time they invest at date t that the long-lived agent is the genuine type. Let σ_t denote the probability that a solvent imposter defaults at date t. It will also be useful to define several indicator variables. Let $\mathbb{I}_t$ denote a dummy variable that is equal to 1 if short-lived agents know at the time they invest at date t that the long-lived agent is an imposter and 0 otherwise. $\mathbb{I}_t$ can equal 1 if the long-lived agent defaulted in the past, was exogenously exposed before short-lived agents invest at date t, or if the long-lived agent successfully invested in the risky technology in the past and thereby revealed its type. Let $\mathbb{U}_t$ denote an indicator variable that equals 1 if the long-lived investor chooses to undertake the risky project at the end of date t. Finally, let S_t be an indicator variable of whether the imposter is solvent at date t. That is, $S_t = 1$ if $w_t + x_t \geq (1 + R_H)x_{t-1}$ and 0 otherwise.

If short-lived agents save on their own at any date, they will earn $1 + R_L$.

If they invest with the long-lived agent at date t and the long-lived agent is genuine, which occurs with probability Φ_t, then the long-lived agent will pay $1 + R_H$ in full.

If the long-lived agent is an imposter, which occurs with probability $1 - \Phi_t$, and does not undertake the risky project at date t, i.e., if $\mathbb{U}_t = 0$, then the long-lived agent will repay if they are not exogenously exposed next period, if they are solvent next period, and if they do not default. The probability of repayment in this case will be $(1 - \mu)S_{t+1}(1 - \sigma_{t+1})$.

If the long-lived agent is an imposter and does undertake the risky project at date t, i.e., if $\mathbb{U}_t = 1$, then the long-lived agent will repay if either their investment is successful, which occurs with probability λ, or if they are not exposed next period, if they are are solvent next period, and if they do not default. The probability of repayment in this case will be $\lambda + (1 - \lambda)S_{t+1}(1 - \mu)(1 - \sigma_{t+1})$.

The condition that leaves short-lived agents indifferent between the two is given by

$$z = \begin{cases} \Phi_t + (1 - \Phi_t)[(1 - \mu)(1 - \sigma_{t+1})S_{t+1}] & \text{if } \mathbb{U}_t = 0 \\ \\ \Phi_t + (1 - \Phi_t)[\lambda + (1 - \lambda)(1 - \mu)(1 - \sigma_{t+1})S_{t+1}] & \text{if } \mathbb{U}_t = 1 \end{cases} \tag{37}$$

Turning to the beliefs of the agent, Short-lived agents will update their beliefs as follows:

$$\Phi_{t+1} = \begin{cases} \dfrac{\Phi_t}{\Phi_t + (1-\Phi_t)(1-\sigma_t)(1-\mu)} & \text{if } \mathbb{I}_{t+1} = 0 \text{ and } \mathbb{U}_t = 0 \\[2ex] \dfrac{\Phi_t}{\Phi_t + (1-\Phi_t)(1-\sigma_t)(1-\lambda)(1-\mu)} & \text{if } \mathbb{I}_{t+1} = 0 \text{ and } \mathbb{U}_t = 1 \\[2ex] 0 & \text{if } \mathbb{I}_{t+1} = 1 \end{cases} \tag{38}$$

The system of difference equations has the boundary condition $\Phi_0 = \phi$ and $\sigma_0 = 0$.

As in the benchmark model, the probability Φ_t will grow while $\mathbb{I}_t = 0$, i.e., while the long-lived agent's type remains uncertain. If $\phi < \frac{z-\lambda}{1-\lambda}$, the unique path $\{\Phi_t, \sigma_t\}$ that solves (37) and (38) will satisfy the following properties:

1. There exists a finite $T \geq 2$ such that $\sigma_t < 1$ for all $t < T$ and $\sigma_T \geq 1$
2. $\Phi_t < \frac{z-\lambda}{1-\lambda}$ for $t = 0, 1, ...T-1$ and $\Phi_{T-1} \geq \frac{z-\lambda}{1-\lambda}$
3. Φ_t and σ_t are both increasing in t for $t = 1, ..., T$

In contrast to our benchmark model, there is no guarantee that the value of σ_1 that solves this system of equations will be positive whenever $\phi > \frac{z-\lambda}{1-\lambda}$. For parameters that imply $\sigma_1 > 0$, we know that σ_t will be between 0 and 1 for $t = 1, ..., T-1$. An equilibrium with these values for σ_t requires the imposter to be indifferent between defaulting and not defaulting.

We thus need to check that there exists a path $\{x_t\}_{t=0}^{\infty}$ that leaves the imposter indifferent about defaulting in every period. We now turn to the imposter's decision.

A Ponzi Equilibrium with $T = 2$

For simplicity, we focus on the case where $T = 2$. Given remaining parameter values, we can choose ϕ to ensure that $\sigma_1 \in (0, 1)$ and $\sigma_2 \geq 1$ by setting ϕ just below $\frac{z-\lambda}{1-\lambda}$. We focus on the case in which the value of σ_2 that solves the system of equations defined by (37) and (38) is strictly greater than 1. That is the generic case; when $T = 2$, then $\sigma_2 \geq 1$ and is exactly equal to 1 for exactly one value of ϕ. If $\sigma_2 > 1$, then in equilibrium we will have $x_1 = x_2 = y$. The only equilibrium value we would need to solve for is x_0.

From Claim 2, we know that at date $t = 2$, the imposter should default if their type is not revealed. In that case, they will raise x_2 in new funds and should immediately consume them together with any wealth w_2 they have at the start of the period.

Next, at date $t = 1$, the imposter must choose whether to default. If they default, they should consume the amount $w_1 + x_1$: This is better than saving given their impatience, and operating the risky technology is unprofitable since their proceeds will be seized if the investment is successful. Their payoff as of date 1 in this case will thus be

$$w_1 + x_1 \tag{39}$$

If the imposter repays their obligation $(1 + R_H)x_0$, they must decide between consuming, saving, and investing the $w_1 + x_1 - (1 + R_H)x_0$ they have access to. Given they intend to default in period 2, consuming dominates saving. If they consume their available resources at date 1, their expected payoff will be

$$w_1 + x_1 - (1 + R_H)x_0 + \beta(1 - \mu)x_2 \tag{40}$$

If they invest in the risky technology, their expected payoff will be

$$\beta\lambda[(R - R_H)x_1 + (1 + R)(w_1 - (1 + R_H)x_0)] + \beta(1 - \lambda)(1 - \mu)x_2 \tag{41}$$

Finally, at date $t = 0$ there is no default decision. The imposter must choose between consuming x_0, saving x_0 and earning the riskless return $1 + R_L$, and investing x_0 in the risky technology whose return is stochastic.

For $0 < \sigma_1 < 1$ to be optimal, we need the imposter to be indifferent between defaulting in period 1 and choosing whatever action is optimal when not defaulting.

If they intend to default in period 1, the imposter should consume x_0 and default in period 1 if they are not exposed, yielding an expected payoff of

$$x_0 + \beta(1 - \mu)x_1 \tag{42}$$

If they do not intend to default in period 1, they must choose whether to save, consume, or invest in period 0. If they consume, they will receive x_0 in utility in period 0 and start period 1 with $w_1 = 0$ in wealth. If they save, they will receive 0 in utility in period 0 and start period 1 with $w_1 = (1 + R_L)x_0$ in wealth. If they invest, they will receive 0 in utility in period 0 and start period 1 with either $(R - R_H)x_0$ if their investment is successful and $w_1 = 0$ if their investment is unsuccessful. The continuation utility would be β times the expressions in (40) or (41), depending on what the imposter chooses.

To ensure that the imposter is indifferent between defaulting and not defaulting in period 1, the expression in (42) must equal the maximal value of consuming, investing, or saving in period 0 and then choosing either (40) or (41).

As an example, suppose we look for the value of x_0 that leaves the imposter indifferent between defaulting and investing in period 1 after investing in period 0 (and being unsuccessful so that another choice can be made). Then x_0 will solve

$$y = \beta\lambda[(R - R_H)y - (1+R)(1+R_H)x_0] + \beta(1-\lambda)(1-\mu)y \tag{43}$$

Numerical Example

Consider the following parameter values:

$$\begin{array}{rcl@{\qquad}rcl} R_H & = & 0.10 & \phi & = & 0.99 \\ R_L & = & 0.09 & \beta & = & 0.80 \\ R & = & 65.0 & \mu & = & 0.40 \\ \lambda & = & 0.02 & & & \end{array}$$

We can verify that these parameters satisfy Assumptions 1, 2, and 3.

Given these values, the value of x_0 that solves (43) is given by $x_0 = 0.438y$. At this value, if the imposter has no wealth at date 1, they will be indifferent between defaulting at date 1 on their investors from period 0 and consuming the new inflow of funds $x_1 = y$ on the one hand and paying the investors from date 0 the amount $(1 + R_H)x_0 = 0.482y$ that is owed to them at date 1 and then investing the remaining $0.518y$ in the risky technology. The expected ex-ante payoff to the imposter from following either of these strategies after undertaking the risky investment in period 0 is $0.925y$.

We can verify that the expected payoff to other strategies is lower than $0.925y$. To do this, we need to identify the strategies the imposter can follow. Since there is no obligation in 0, default is not a consideration at date 0. Instead, the imposter must choose between investing, consuming, or saving. Since we know from Claim 2 that they will default in period 2, there is no benefit to saving in period 1. That means that in period 1, the imposter will either invest, consume, or default. At date 2, the imposter will default if they haven't already. There are thus nine strategies to consider.[8] The strategy with the second highest ex-ante payoff is for the imposter to invest in period 0 and then consume $x_1 - (1 + R_H)x_0$ if

[8]The imposter can in principle mix between actions, but they would do this only if they are indifferent, in which case the payoff will be the same as to the pure strategy.

they fail, which yields an ex-ante expected payoff of $0.924y$. The next best strategies involve consuming x_0 in period 0 and then either investing or defaulting in period 1 since both give the same payoff. This strategy yields an expected payoff of $0.918y$. The payoff to saving in period 0 is lower still.

We can further verify that there exists no other Ponzi equilibrium in which an optimizing imposter is indifferent between defaulting and not defaulting in period 1. That is, we can look at the value of x_0 that leaves the imposter indifferent between defaulting in period 1 and some strategy that involves not defaulting in period 1. In all of these cases, defaulting in period 1 is not optimal.

Intuitively, the parameters we chose involve a relatively high value to μ, the probability of being exposed. That makes the strategy of avoiding default in order to wait for a higher investment x_t at some future t and then stealing it less attractive: The imposter is likely to be exposed with high probability before they can steal. By contrast, a higher value of μ does not make the investment less valuable, since a successful investment already exposed the long-lived agent as an imposter. High values of μ thus encourage the imposter to invest in the risky technology.

In short, when we modify the model to allow for risky investments, we can construct a Ponzi equilibrium in which the imposter invests the x_0 funds they raise in period 0 and then uses new funds to pay old investors while investing any remaining proceeds. In this case, the imposter can avoid defaulting on any agents if their investment happens to be successful. But the imposter would still be hiding the fact that they are undertaking risky investments as opposed to the commitment type who can guarantee a safe return of R_H every period.

References

Abel, Andrew, and Stavros Panageas. 2022. "Running Primary Deficits Forever in a Dynamically Efficient Economy: Feasibility and Optimality." NBER working paper 30554.

Allen, Franklin, Stephen Morris, and Andrew Postlewaite. 1993. "Finite Bubbles with Short Sale Constraints and Asymmetric Information." *Journal of Economic Theory*, 61: 206–229.

Amador, Manuel, and Christopher Phelan. 2021. "Reputation and Sovereign Default." *Econometrica*, 89(4): 1979–2010.

Antler, Yair. 2023. "Multilevel marketing: pyramid-shaped schemes or exploitative scams?" *Theoretical Economics*, 18(2): 633–668.

Artzrouni, Marc. 2009. "The Mathematics of Ponzi Schemes." *Mathematical Social Sciences*, 58(2): 190–201.

Awaya, Yu, and Vijay Krishna. 2022. "Rational Exuberance and Bubbles." University of Rochester.

Awaya, Yu, Kohei Iwasaki, and Makoto Watanabe. 2022. "Rational Bubbles and Middlemen." *Theoretical Economics*, 17(4): 1559–1587.

Baker, Wayne, and Robert Faulkner. 2003. "Diffusion of Fraud: Intermediate Economic Crime and Investor Dynamics." *Criminology*, 41(4): 1173–1206.

Barlevy, Gadi. 2025. *Asset Bubbles and Macroeconomic Policy.* MIT Press.

Bartoletti, Massimo, Salvatore Carta, Tiziana Cimoli, and Roberto Saia. 2020. "Dissecting Ponzi schemes on Ethereum: Identification, analysis, and impact." *Future Generation Computer Systems*, 102: 259–277.

Bhattacharya, Utpal. 2003. "The Optimal Design of Ponzi Schemes in Finite Economies." *Journal of Financial Intermediation*, 12(1): 2–24.

Blanchard, Olivier, and Philippe Weil. 2001. "Dynamic Efficiency, the Riskless Rate, and Debt Ponzi Games under Uncertainty." *Advances in Macroeconomics*, 1(2): 1–23.

Board, Simon, and Moritz Meyer-ter Vehn. 2013. "Reputation for Quality." *Econometrica*, 81(6): 2381–2462.

Celentani, Marco, and Wolfgang Pesendorfer. 1996. "Reputation in Dynamic Games." *Journal of Economic Theory*, 70: 109–132.

Cong, Lin William, Kim Grauer, Daniel Rabetti, and Henry Updegrave. 2023. "The Dark Side of Crypto and Web3: Crypto-Related Scams." Cornell University and Chainalysis.

Conlon, John. 2004. "Simple Finite Horizon Bubbles Robust to Higher Order Knowledge." *Econometrica*, 72(3): 927–36.

Deason, Stephen, Shivaram Rajgopal, Gregory Waymire, and Roger White. 2015. "Who Gets Swindled in Ponzi Schemes?" Emory University.

Diamond, Peter. 1965. "National Debt in a Neoclassical Growth Model." *American Economic Review*, 55(5, Part 1): 1126–1150.

Doblas-Madrid, Antonio. 2012. "A Robust Model of Bubbles with Multidimensional Uncertainty." *Econometrica*, 80(5): 1845–1893.

Frankel, Tamar. 2012. *The Ponzi Scheme Puzzle: A History and Analysis of Con Artists and Victims.* Oxford University Press.

Gurun, Umit, Noah Stoffman, and Scott Yonker. 2018. "Trust Busting: The Effect of Fraud on Investor Behavior." *Review of Financial Studies*, 31(4): 1341–1376.

Huang, Li, Oliver Zhen Li, Yupeng Lin, Chao Xu, and Haoran Xu. 2021. "Gender and Age-based Investor Affinities in a Ponzi Scheme." *Humanities and Social Science Communications*, 8(60).

Hu, Ju. 2014. "Reputation in the Presence of Noisy Exogenous Learning." *Journal of Economic Theory*, 153(1): 64–73.

Jarvis, Christopher. 2000. "The Rise and Fall of Albania's Pyramid Schemes." *Finance and Development*, 37(1).

Leuz, Christian, Steffen Meyer, Maximilian Muhn, Eugene Soltes, and Andreas Hackethal. 2023. "Who Falls Prey to the Wolf of Wall Street? Investor Participation in Market Manipulation." NBER working paper 24083.

Mailath, George, and Larry Samuelson. 2015. "Reputations in Repeated Games." In *Handbook of Game Theory*. 165–238. Elsevier.

Marquet, Christopher. 2011. "The Marquet Report on Ponzi Schemes." Marquet International.

O'Connell, Stephen, and Stephen Zeldes. 1988. "Rational Ponzi Games." *International Economic Review*, 29(3): 431–450.

Phelan, Christopher. 2006. "Public Trust and Government Betrayal." *Journal of Economic Theory*, 130(1): 27–43.

Rantala, Ville. 2019. "How Do Investment Ideas Spread through Social Interaction? Evidence from a Ponzi Scheme." *Journal of Finance*, 74(5): 2349–2389.

Shuang, Yifei, Ke Tang, Yang You, and Xi Zhao. 2023. "Ponzi Schemes on Blockchain." Tsinghua University, Xi'an Jiatong University, and University of Hong Kong.

Springer, Marie. 2020. *The Politics of Ponzi Schemes: History, Theory, and Policy.* Taylor-Francis-Routledge Press.

Stivers, Andrew, Douglas Smith, and Ginger Jin. 2019. "The Alchemy of a Pyramid: Transmutating Business Opportunity into a Negative Sum Wealth Transfer." University of Maryland and Federal Trade Commission.

Wiseman, Thomas. 2009. "Reputation and Exogenous Private Learning." *Journal of Economic Theory*, 144: 1352–7.

Zuckoff, Mitchell. 2005. *Ponzi's Scheme: The True Story of a Financial Legend.* Random House.