Equilibrium selection in a fundamental model of money

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Motivation

- Models of money always exhibit equilibria where money has no value. This is so because the use of money relies on coordination.

- However, in reality, people seem to coordinate on the use of money. Why is that?

Equilibrium selection in coordination games

- Global Games

  Carlsson and Van Damme (1993)

  Morris and Shin (2003)

  Frankel, Morris and Pauzner (2003)

- Dynamic Games

  Frankel and Pauzner (2000)

  Burdzy, Frankel and Pauzner (2001)

Equilibrium selection in coordination games

	D	C
D	**0 , 0**	**0 , -(c+θ)**
C	**-(c+θ) , 0**	**u-(c+θ) , u-(c+θ)**

- $0 < c < u$ and $u < 2c$.

- θ is a random walk, that starts at $\theta=0$.

- remote regions : $\exists\ \theta_L$ s.t. $c + \theta_L < 0$, and θ_H s.t. $c + \theta_H > u$.

Equilibrium selection in coordination games

- Key results :

- Remote regions + friction → unique equilibrium.

  Friction : incomplete information, asynchronous moves

- As frictions vanish, efficient equilibrium is never selected.

This paper

- equilibrium selection in a model of money.

  KW (1993) + remote regions.

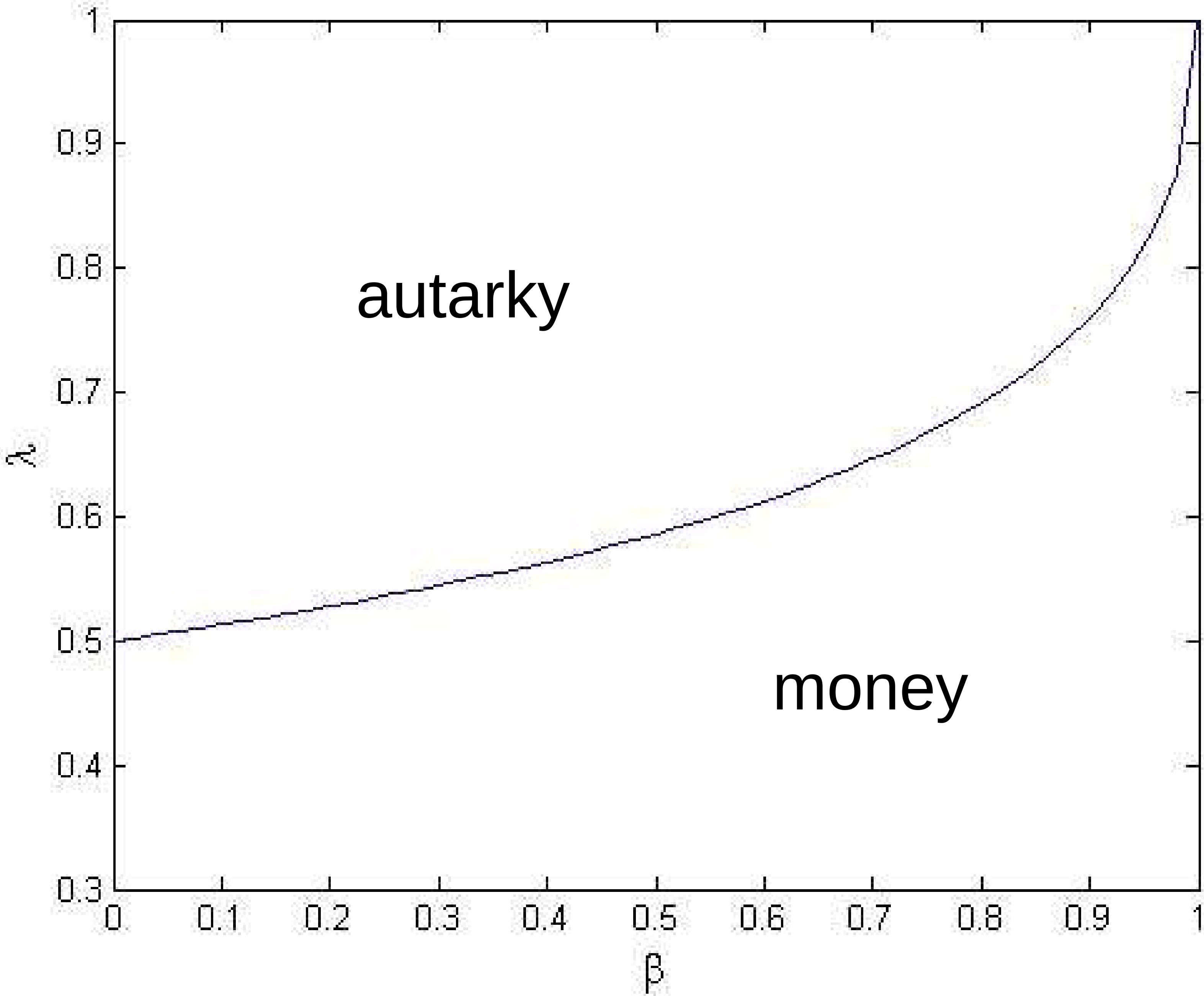
- results:

• remote regions → unique equilibrium.

• efficient equilibrium (i.e., money) is often selected.

• agents coordinate on money due to its intrinsic (durability) and extrinsic (medium of exchange) properties.

• patience matters.



autarky
money
λ
β
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1

Related Literature

- commodity money refinements

  Zhou (2003), Wallace and Zhu (2004), Zhu (2003, 2005)

- Key result: if money has positive intrinsic value, autarky is not a limit of commodity-money equilibria.

- in our case, probability that money ever acquires intrinsic value is arbitrarily small. Besides, results hold if money eventually acquires negative value with probability one.

- our focus is on the relation between properties of money and the coordination value it entails.

Environment

- discrete time, **k** indivisible goods, [0,1] continuum of agents distributed in **k** types.

- type **i** agent obtains utility **u** from good i and produces good **i+1** at cost **c**, **u>c**. Discount factor $\beta \in [0,1]$.

- indivisible money with unit upper bound, distributed to measure **m** of agents.

- agents trade in **k** markets, one for each good. They can identify markets but are randomly paired inside a market.

Environment

- Every period, economy is in some state $\mathbf{z} \in R$. States evolve according to $\mathbf{z_t = z_{t-1} + \Delta z}$.

- $\mathbf{\Delta z}$ follows a continuous probability distribution, that is independent of t, with $E[\Delta z]$, $Var[\Delta z]$.

- There exist $\mathbf{Z}$ such that

  if $\mathbf{z < -Z}$, reject money is strictly dominant

  If $\mathbf{z > Z}$, accept money is strictly dominant

  $\mathbf{Z}$ can be as large as one wants, with no impact on the results. Economy starts at $\mathbf{z = 0}$.

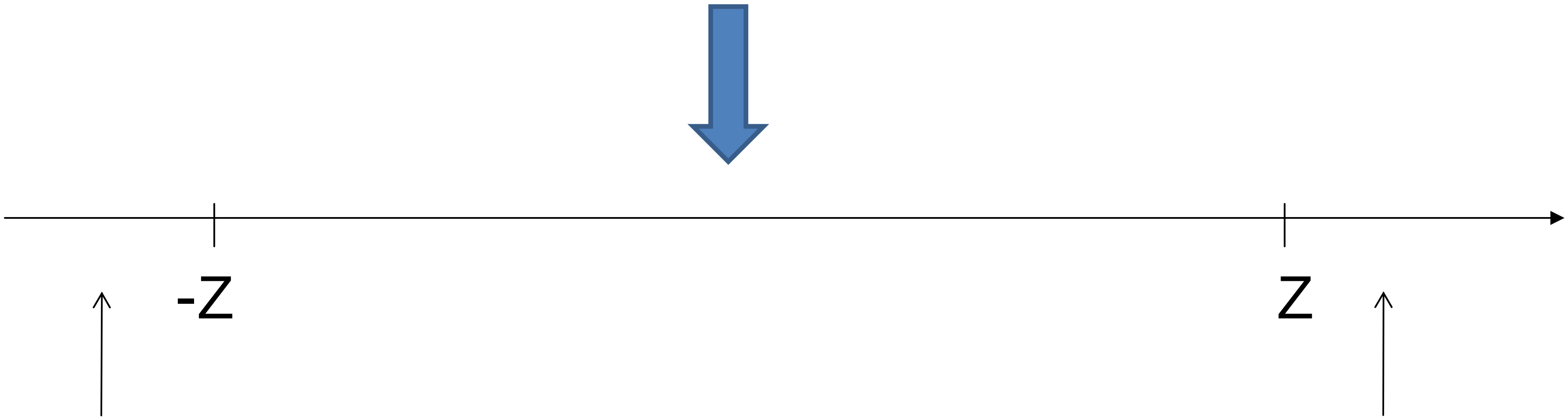
The remote regions

- implication: imposes a condition on beliefs, by ruling out the belief that money is always or is never going to be employed.

- equilibria that depend on such extreme beliefs are tenuous for relying on agents being sure about how they will coordinate their behavior in all states of the world at every point in time, no matter how unlikely the state is or how far away in time it is.

Possible Interpretation of dominant regions : exchange is only viable if there exists a special agent in the economy (say, the government) that provides a safe environment

government provides safe environment for trade



government is unable to provide safe environment

government not only provides safe environment but it has a technology that enforces the use of money in all transactions

Benchmark : var[Δz]=0

- autarky is always an equilibrium.

- money is an equilibrium with enough gains from trade.

$$V_{1,z} = m\beta V_{1,z} + (1-m)(u + \beta V_{0,z})$$

$$V_{0,z} = m(-c + \beta V_{1,z}) + (1-m)\beta V_{0,z}$$

$$-c + \beta V_{1,z} > \beta V_{0,z} \Leftrightarrow \beta\left[(1-m)u + mc\right] > c$$

General case: var[Δz]>0

If economy is in state **z** in period s, let φ(t) be the probability that states larger than **z** are reached for the first time in period t+s (for any **z**).

Proposition 1: There is a unique equilibrium.

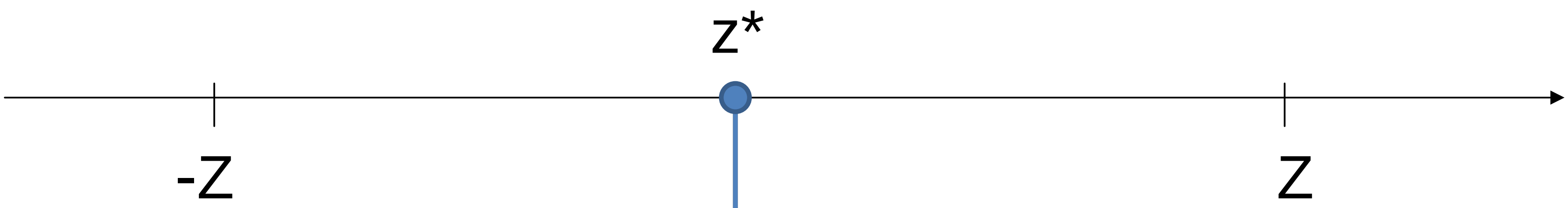
If

$$\sum_{t=1}^{\infty} \beta^t \varphi(t)\big[(1-m)u + mc\big] > c,$$

money is accepted in z $\in$ [-Z,Z]. Otherwise, it is not.

Proof

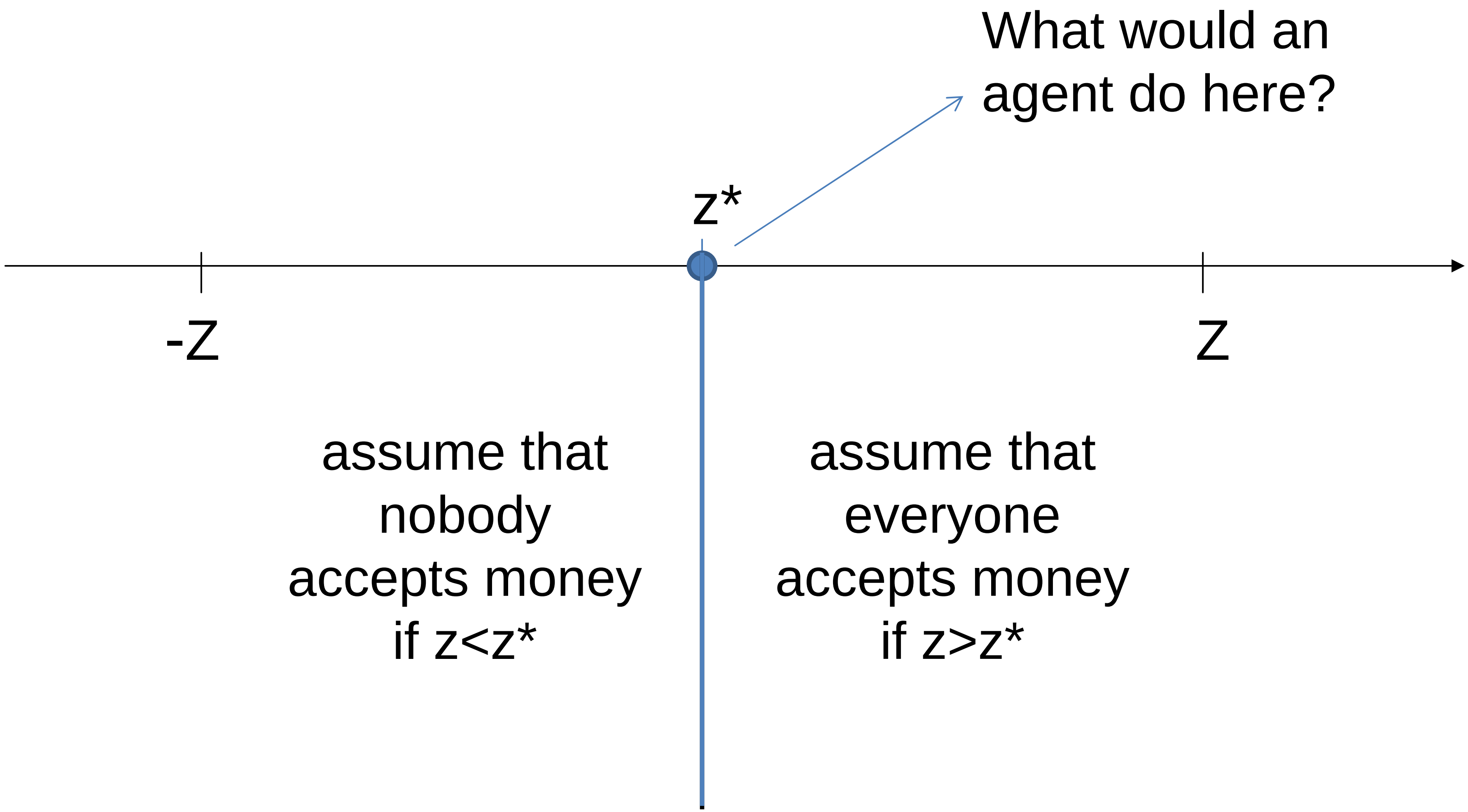
pick a point $z^* \in [-Z,Z]$



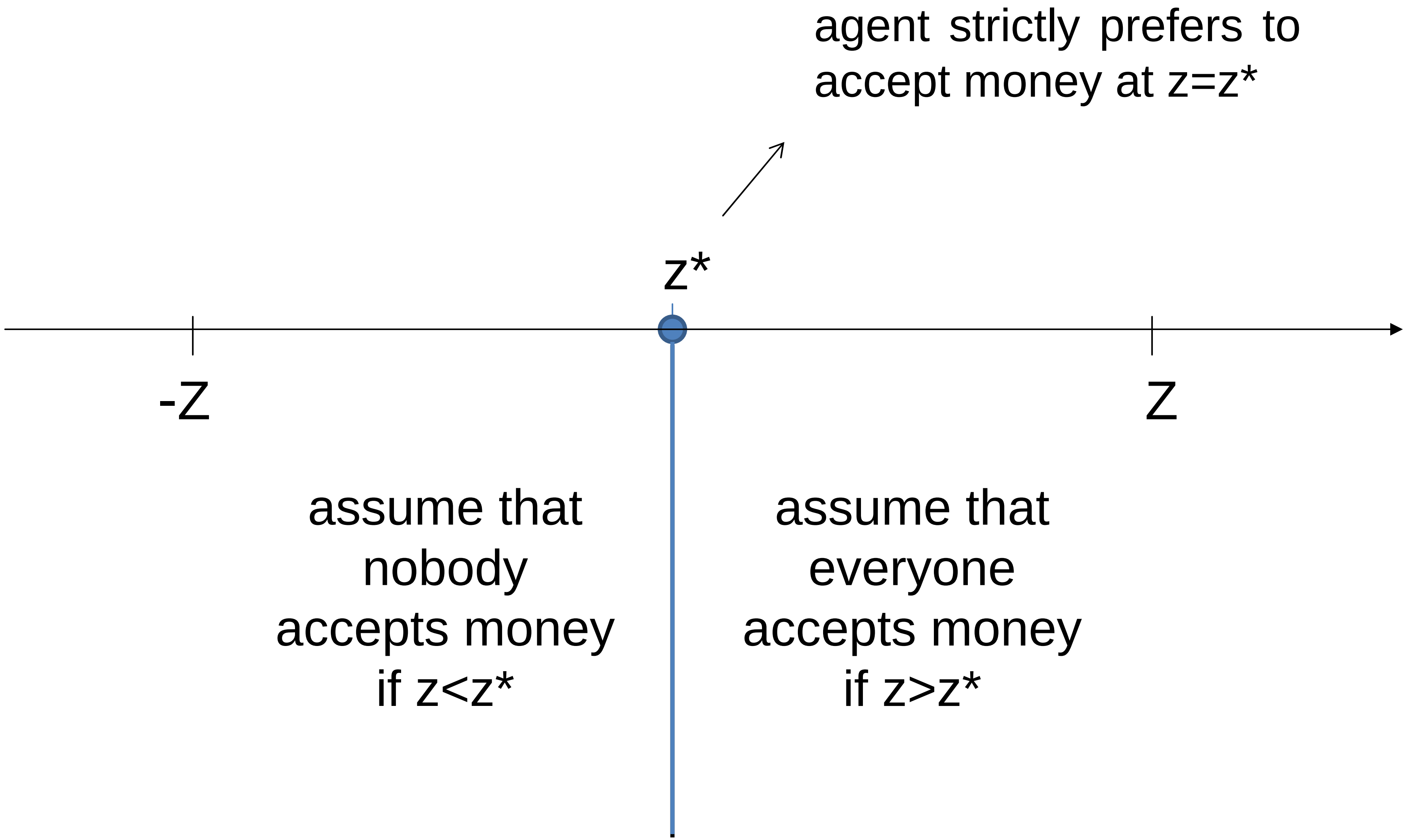
assume that nobody accepts money if $z<z^*$

assume that everyone accepts money if $z>z^*$

Proof


What would an agent do here?
z*
-Z
Z
assume that nobody accepts money if z<z*
assume that everyone accepts money if z>z*

Proof

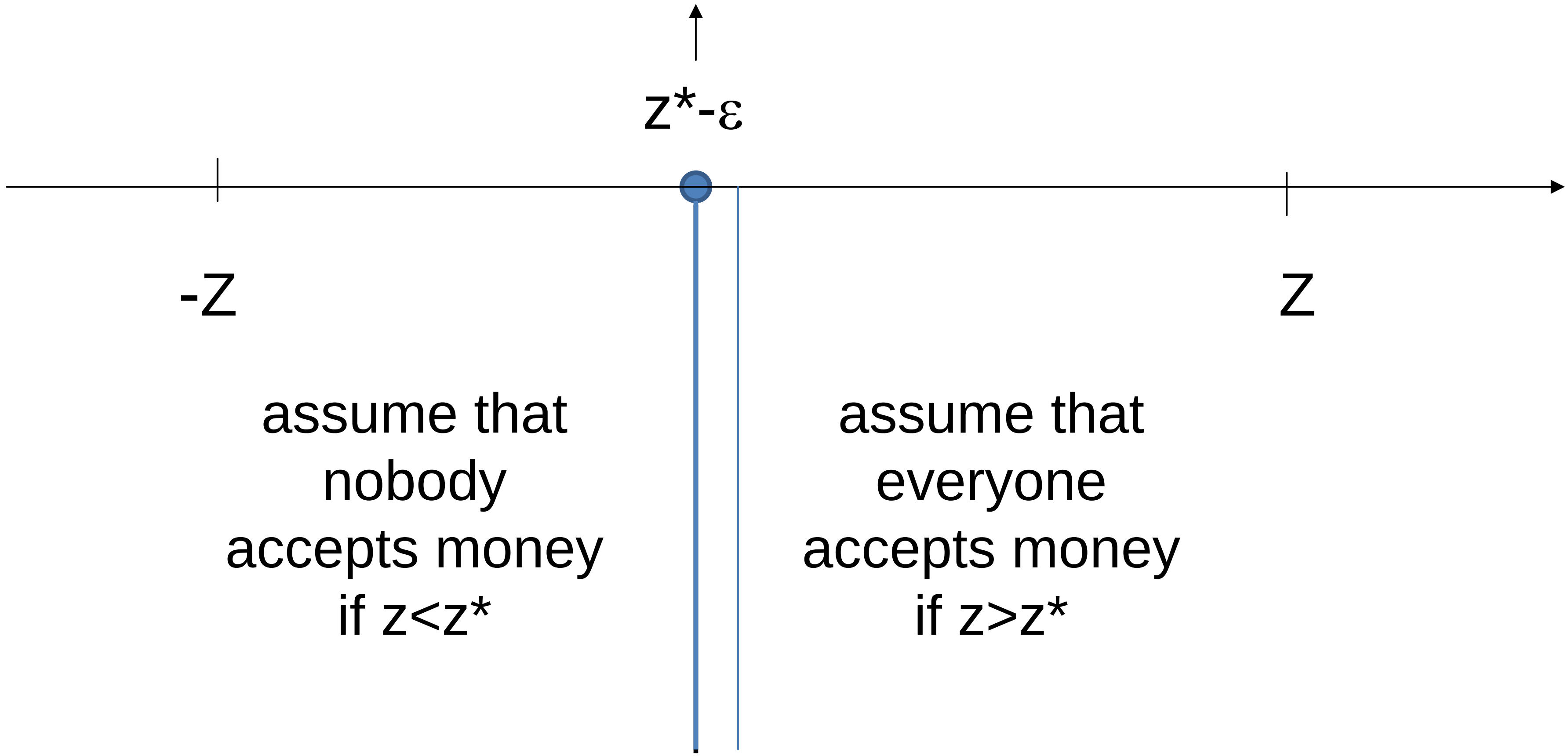

agent strictly prefers to accept money at z=z*
z*
-Z
Z
assume that nobody accepts money if z<z*
assume that everyone accepts money if z>z*

Proof

by continuity on z, agent strictly prefers to accept money at $z = z^* - \varepsilon$

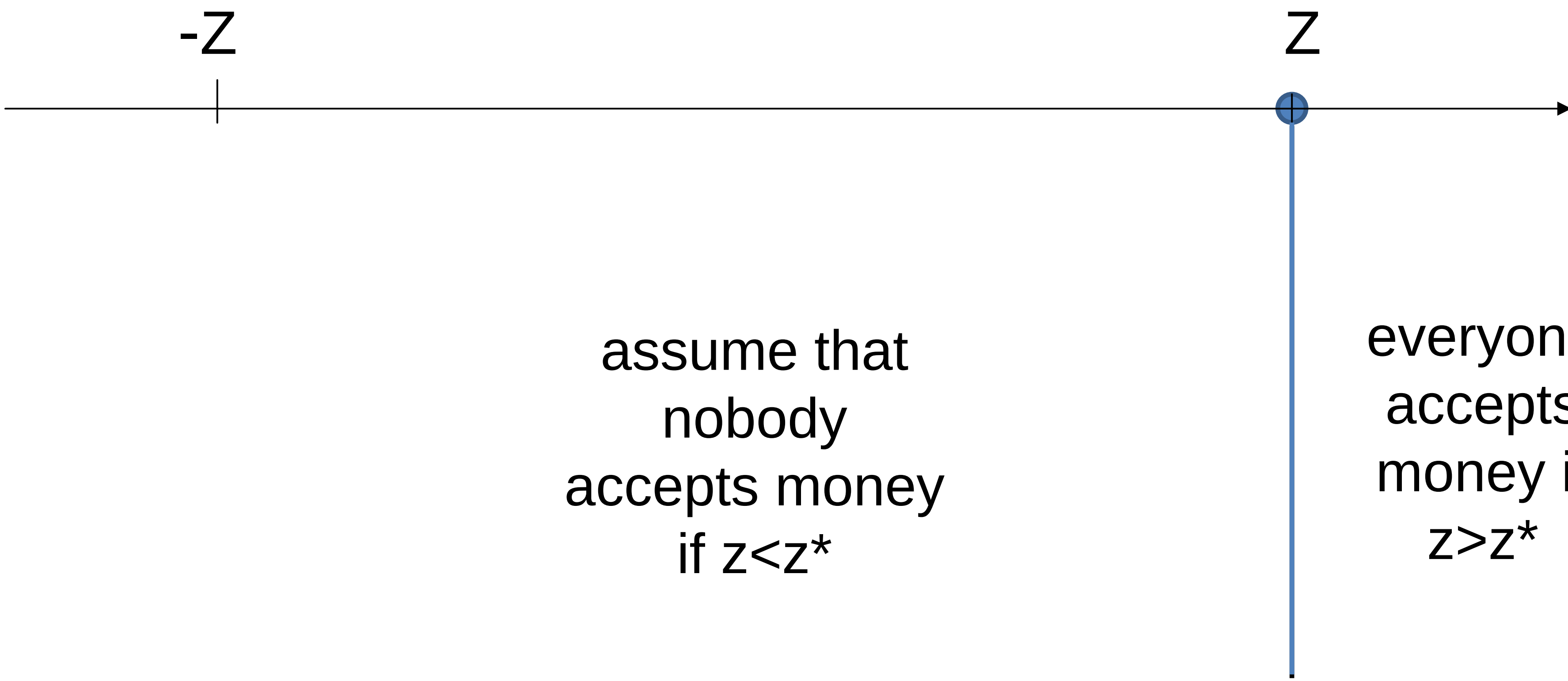
since same reasoning applies to all other agents, z^* can be replaced with $z^*-\varepsilon$

incentives to accept money increase if we relax assumption in the region $z<z^*$

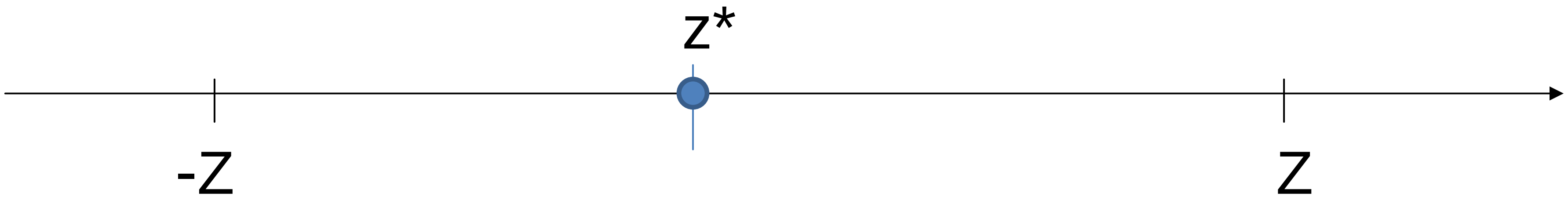


Proof

we start the process by letting z*=Z



Proof



we need thus to compare value functions of accepting money and not accepting money at a generic point z*

Proof

If the agent accepts money, he obtains

$$-c+\sum_{t=1}^{\infty}\beta^{t}\varphi(t)\left(\int_{0}^{\infty} f(z^{*}+s \mid t_{\varphi}=t)V_{1,z^{*}+s}ds\right)$$

If the agent does not accept money, he obtains

$$\sum_{t=1}^{\infty}\beta^{t}\varphi(t)\left(\int_{0}^{\infty} f(z^{*}+s \mid t_{\varphi}=t)V_{0,z^{*}+s}ds\right)$$

Proof

Thus, he accepts money as long as

$$\sum_{t=1}^{\infty} \beta^t \varphi(t) \left(\int_0^{\infty} f(z^* + s | t_\varphi = t)(V_{1,z^*+s} - V_{0,z^*+s}) ds \right) > c$$

Now, for any z $\geq$ z*, we have

$$V_{1,z} = m\beta E_z V_{1,z} + (1-m)(u + \beta E_z V_{0,z})$$

$$V_{0,z} = m(-c + \beta E_z V_{1,z}) + (1-m)\beta E_z V_{0,z}$$

$$V_{1,z} - V_{0,z} = (1-m)u + mc$$

$$\sum_{t=1}^{\infty} \beta^t \varphi(t) \left[(1-m)u + mc \right] > c$$

Comment

durability

medium of exchange

$$V_{1,z} = m\beta E_z V_{1,z} + (1-m)(u + \beta E_z V_{0,z})$$

$$V_{0,z} = m(-c + \beta E_z V_{1,z}) + (1-m)\beta E_z V_{0,z}$$

$$V_{1,z} - V_{0,z} = (1-m)u + mc$$

What if we drop the properties of money?

Comment

durability allows to postpone benefit of past effort

medium of exchange implies that the cycle production consumption is continuously repeated

$$V_{1,z} = m\beta E_z V_{1,z} + (1-m)(u + \beta E_z V_{0,z})$$

$$V_{0,z} = m(-c + \beta E_z V_{1,z}) + (1-m)\beta E_z V_{0,z}$$

Comment

$$V_{1,z} = (1-m)u$$

$$V_{0,z} = m(-c + \beta E_z V_{1,z}) + (1-m)\beta E_z V_{0,z}$$

$$V_{1,z} - V_{0,z} = (1-m)u + mc - \beta[mE_z V_{1,z} + (1-m)E_z V_{0,z}]$$

$$\sum_{t=1}^{\infty} \beta^t \varphi(t)\{(1-m)u + mc - \beta[mE_z V_{1,z} + (1-m)E_z V_{0,z}]\} > c$$

Condition for uniqueness of money equilibrium:

$$\sum_{t=1}^{\infty} \beta^t \varphi(t)\big[(1-m)u + mc\big] > c,$$

Condition for existence of money equilibrium

$$\beta\big[(1-m)u + mc\big] > c$$

Condition for uniqueness can be written as:

$$\lambda\beta\big[(1-m)u + mc\big] > c$$

where

$$\lambda = \sum_{t=1}^{\infty} \beta^{t-1} \varphi(t),$$

$$\lambda\beta\left[(1-m)u + mc\right] > c \qquad \lambda = \sum_{t=1}^{\infty} \beta^{t-1}\varphi(t),$$

- λ = 0: autarky is always the unique equilibrium.

- λ = 1: money is the unique equilibrium

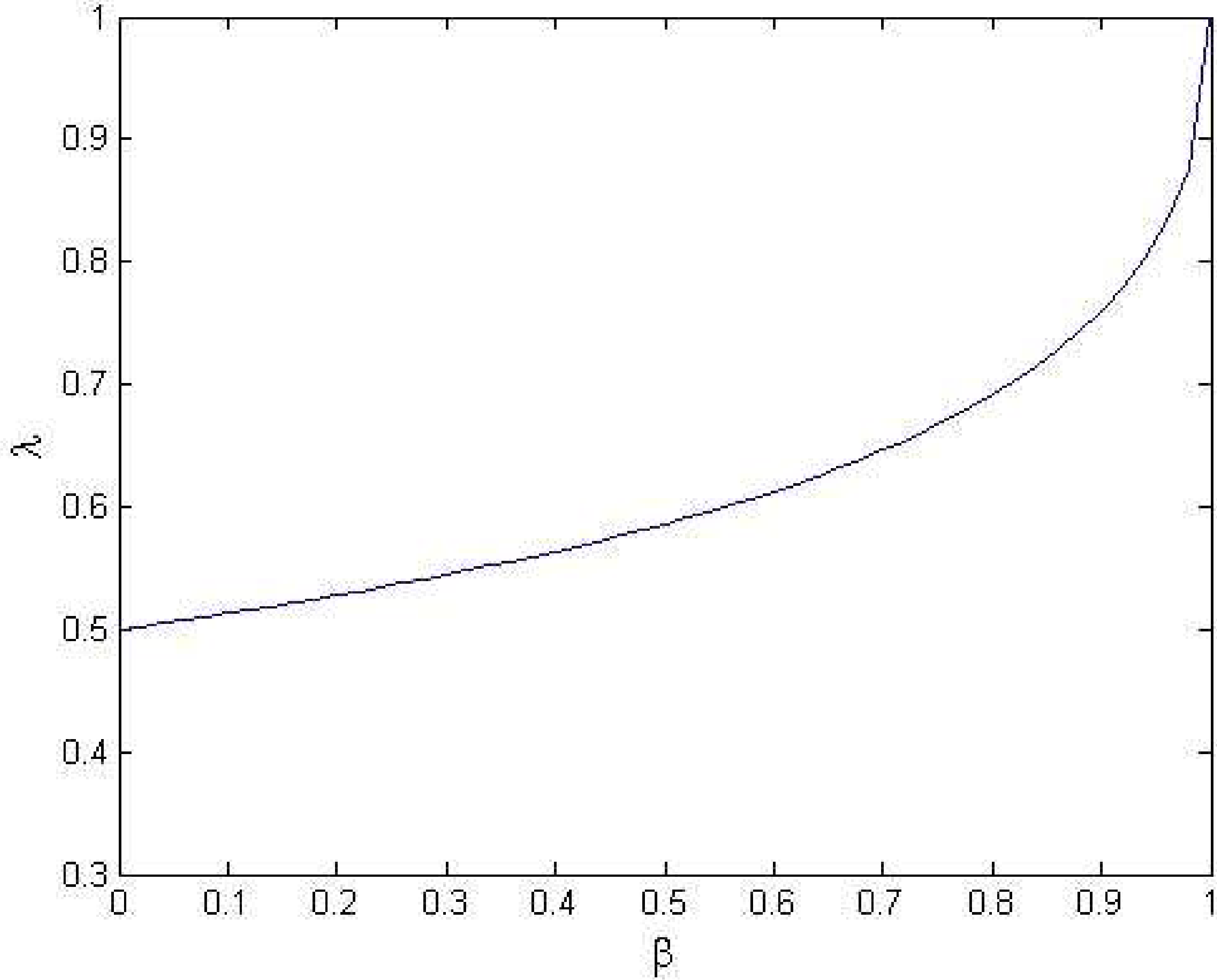
The effect of β

- λ is increasing in β.

- As $\beta \to 0$, $\lambda \to \varphi(1)$
  - If $E(\Delta z) = 0$, $\varphi(1) = 1/2$.
  - Risk dominant strategy in a 1-shot game.

- As $\beta \to 1$, $\lambda \to 1$

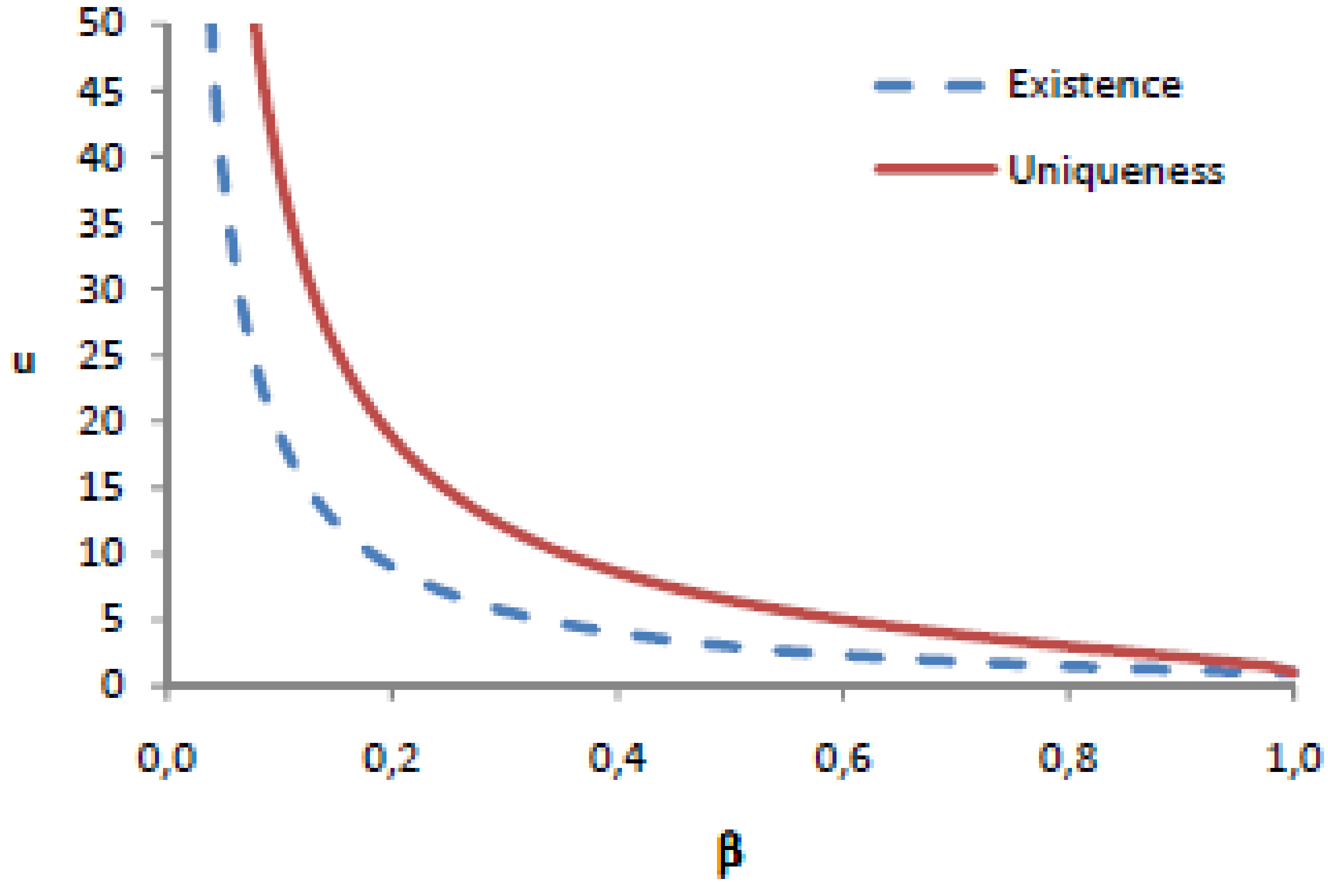
Example: Δz normal



Comparing our results with Kiyotaki and Wright (1993)

c=1 m=0.5

ΔZ normal



Summing up results

1. money is the unique equilibrium if there are large gains from trade. Autarky is the unique equilibrium if there are small gains from trade.

2. if there is any gain from trade, as agent's discount factor approaches one, money becomes the unique equilibrium.

Conclusion

- people coordinate on the use of money due to its intrinsic and extrinsic properties
- coordination on the use of money increases with patience. This is true even if in the long run the economy will most likely reach states where money has negative intrinsic value.

- conjecture: prevalence of money may have more to do with coordination and less to do with essentiality.