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Adverse Selection, Risk Sharing and Business Cycles

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Abstract: I consider a real business cycle model in which agents have private information about the stochastic realization of their value of leisure. For the case of logarithmic preferences I provide an analytical characterization to the solution of the mechanism design problem for this economy. Moreover, I show a striking irrelevance result: That the stationary behavior of all aggregate variables are exactly the same in the private information economy as in the full information case. I then use a new computational method to show that the irrelevance result holds numerically for more general CRRA preferences.

Keywords: Adverse selection, risk sharing, business cycles, private information, social insurance, optimal contracts, computational methods, heterogeneous agents.

1 Introduction

At least since the seminal paper by Krusell et al. (1998) there has been a long literature analyzing the effects of exogenous borrowing constraints on aggregate fluctuations. The purpose of this paper is to take a more primitive approach by exploring the effects of restrictions to perfect risk sharing but when these restrictions arise from the presence of private information. To this end the paper solves the mechanism design problem of a real business

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cycle (RBC) economy subject to private information and compares its aggregate fluctuations with those of the same economy with full information. The paper is not only interested in evaluating the effects of private information on aggregate fluctuations, but in characterizing the behavior of the optimal contracts and in exploring their implications for the cyclical behavior of consumption and employment inequality.

The model that I use is a simple RBC model with private information. Agents value consumption and leisure and receive idiosyncratic shocks to their value of leisure. These shocks, which are i.i.d. over time and across individuals, are assumed to be private information. The production technology is standard. Output, which can be consumed or invested, is produced with capital and labor using a Cobb-Douglas production function subject to an aggregate productivity shock. The aggregate shock follows an AR(1) process.

Following the literature, a dynamic contract is given a recursive formulation in which its state is given by a promised value to the agent. Given the current state, the contract specifies current consumption, current hours worked and next-period promised values as a function of the value of leisure reported by the agent. Since the model has a large number of agents and the shocks to the value of leisure are idiosyncratic, the social planner needs to keep track as a state variable the whole distribution of promised values across individuals. Given this distribution, the aggregate stock of capital and the current level of aggregate productivity, the social planner seeks to maximize the present discounted utility of agents subject to incentive compatibility, promise keeping and aggregate resource feasibility constraints.

For the case in which the utility of consumption and leisure are logarithmic, the paper provides a sharp analytical characterization of the solution to the mechanism design problem. Consumption, hours worked and next-period promised values are decreasing functions of the reported value of leisure. Moreover, the utility of consumption, utility of leisure and next-period promised values are all linear, strictly increasing functions of the current promised value. The slopes of these functions are all independent of the reported value of leisure, and while the utilities of consumption and leisure have a common slope less than one, the slope of next-period promised values is equal to one. Over the business cycle all of these functions shift vertically while maintaining constant the differences between the high and low values of leisure. In turn, the distribution of promised values shifts horizontally over the business cycle while maintaining its shape. Since promised values increase during an expansion, this

means that the dispersion of the distribution of consumption levels behaves procyclically while the dispersion of the distribution of hours worked behaves countercyclically. In terms of aggregate dynamics I get a strong irrelevance result: That the stationary business cycle fluctuations of all macroeconomic variables (i.e. aggregate output, consumption, investment, hours worked and capital) are exactly the same under private information as under full information. That is, once the information frictions are dealt with in an optimal way they have no implications for the stationary aggregate dynamics of the economy.

For preferences other than the log-log case, analytical results are no longer available and the model must be analyzed numerically. The high dimensionality of the state space, which includes the distribution of promised values across individuals, makes computations difficult. However, an important contribution of the paper is to develop a strategy that makes this problem tractable. In fact, the computational method described here is not only applicable to the model in this paper but to a wide class of economies with heterogeneous agents and aggregate uncertainty.[2] The basic strategy is to parametrize individual decision rules as spline approximations and to keep long histories of the spline coefficients as state variables. Starting from the deterministic steady state distribution, the history of decision rules implied by the spline coefficients is then used to obtain the current distribution of individuals across individual states. This is done performing a large number of Monte Carlo simulations. I then linearize the first order conditions with respect to the coefficients of the spline approximations and solve the resulting linear rational expectations model using standard methods.

Applying this computational method to the economy with logarithmic preferences recovers all of the analytical results proved earlier. Since nothing in the computational method takes advantage of the particular functional form of the utility function, this provides considerable evidence about the accuracy of the method. Having established its accuracy the method is then used to analyze more general preferences. However, for all the CRRA preferences considered the same basic result is obtained: The stationary behavior of all macroe-

[2]The computational method should be applicable to any model in which agents have smooth decision rules, are subject to idiosyncratic uncertainty, and in which the aggregate shocks are small and follow autorregressive processes.

conomic variables in an economy with private information are numerically indistinguishable from the same economy with full information.

Dynamic optimal contracts under private information have been used to study a variety of issues in macroeconomics. For example, they have been used to study optimal consumption inequality (e.g. Atkeson and Lucas 1992, Green 1987, etc.), optimal unemployment insurance (e.g. Hopenhayn and Nicolini 1997, Kocherlakota 2004, etc.), and taxation (e.g. Golosov et al. 2007, Farhi and Werning 2012, etc.). However, any interactions with aggregate fluctuations have been mostly neglected. A notable exception is Phelan (1994) who considered a model in which agents take hidden actions that, together with the realization of a public i.i.d. aggregate shock and an unobservable i.i.d. idiosyncratic shock, determine their observed output levels. Assuming that actions are taken prior to the realization of the aggregate shock, that agents have CARA preferences and that agents have a constant probability of dying, he was able to characterize the model analytically. He found two important results: that the cross-sectional distribution of consumption levels depends on the entire history of aggregate shocks and that there is a well defined long-run distribution over cross-sectional consumption distributions.

My model differs from Phelan (1994), not only because it has hidden types (adverse selection) instead of hidden actions (moral hazard), but because it has a neoclassical production function with persistent aggregate shocks. Besides these differences, an apparent similarity is that even in my model with logarithmic preferences the cross-sectional distributions of consumption and leisure depends on the entire history of aggregate shocks. However, this is only due to the presence of capital. Without it I would get that these cross-sectional distributions only depend on the current realization of aggregate productivity.

In fact the lack of memory in the case of no capital and logarithmic preferences has already been shown by DaCosta and Luz (2013) in a related setting. In that paper Da Costa and Luz consider a finite horizon version of Phelan's economy in which actions are taken after the realization of aggregate productivity, agents have CRRA preferences, and agents live as long as the economy. Contrary to Phelan (1994), their cross-sectional distribution of consumption becomes degenerate as the time horizon of the economy becomes large. Interestingly, Da Costa and Luz find that when log preferences are used that the cross-sectional distribution of consumption does not depend on the entire history of aggregate shocks but on the current

realization. However, when the elasticity of intertemporal substitution is different than one, the cross-sectional distribution of consumption has memory of the past history. A major contribution of this paper over DaCosta and Luz (2013) for the case of logarithmic preferences is that, aside from analyzing an economy with capital and persistent aggregate shocks, I provide a tight analytical characterization of the optimal contracts and an equivalence result with the full information economy. For preferences different from the logarithmic case, I am able to compute solutions for infinite horizon economies instead of two-periods cases.

The equivalence with the full information economy in terms of aggregate variables is related to a result in Farhi and Werning (2012). In that paper Fahri and Werning also consider a Mirlees economy similar to the one in this paper except that it has no aggregate productivity shocks, idiosyncratic shocks are persistent and the social planner is only allowed to optimize with respect to the consumption allocations (labor allocations are taken to be beyond his control). Starting from the steady state of a Bewley economy they perform the dynamic public finance experiment of evaluating the welfare gains of moving to an optimal consumption plan. They show that when preferences are logarithmic, along the transitionary dynamics of the model all aggregate variables behave exactly the same as in the full information case. Interestingly, I obtain a similar equivalence result when optimizing with respect to labor as well as consumption and when the economy is subject to aggregate productivity shocks. However, contrary to Farhi and Werning (2012), my equivalence result only holds for the long-run stationary equilibrium of the model. The transitionary dynamics from an arbitrary initial capital and distribution of promised values will generally differ from the full information case.

The paper is organized as follows. Section 2 describes the economy. Section 3 describes the mechanism design problem. Section 4 characterizes the optimal allocations. Section 5 provides the irrelevance result for the log-log case. Section 6 describes the computational method for solving the mechanism design problem with aggregate fluctuations. Section 7 presents the numerical results. Finally, Section 8 concludes the paper.

2 The economy

The economy is populated by a unit measure of agents subject to stochastic lifetimes. Whenever an agent dies he is immediately replaced by a newborn, leaving the aggregate population level constant. The preferences of an individual born at date T are given by

$$E_T \left\{ \sum_{t=T}^{\infty} \beta^{t-T} \sigma^{t-T} \left[\ln c_t + s_t \ln (1 - h_t) \right] \right\}, \tag{1}$$

where c_t is consumption, h_t is hours worked, s_t is the idiosyncratic value of leisure, σ is the survival probability, and $0 < \beta < 1$ is the discount factor.[3] The idiosyncratic value of leisure s_t takes two possible values: s_L and s_H, with $s_L < s_H$. Realizations of s_t are i.i.d. across time and across individuals and are distributed according to a distribution function $\psi = (\psi_L, \psi_H)$. A key assumption maintained throughout the paper is that s_t is private information of the individual.

Output, which can be consumed or invested, is produced with the following production function:

$$Y_t = e^{z_t} K_{t-1}^{\gamma} H_t^{1-\gamma}$$

where Y_t is output, z_t is aggregate productivity, K_{t-1} is capital and H_t is hours worked. The aggregate productivity level z_t follows a standard AR(1) process given by:

$$z_{t+1} = \rho z_t + \varepsilon_{t+1},$$

where $0 < \rho < 1$ and ε_{t+1} is normally distributed with mean zero and standard deviation σ_ε.

Capital is accumulated using a standard linear technology given by

$$K_t = (1 - \delta) K_{t-1} + I_t,$$

where I_t is gross investment and $0 < \delta < 1$.

3 Mechanism Design

In this section I provide a recursive formulation to the problem of a social planner that seeks to maximize utility subject to incentive compatibility, promise keeping and resource

[3] Later on, preferences will be generalized to be of the CRRA type.

feasibility constraints. In order to do this it will be important to distinguish between two types of agents: young and old. A young agent is one that has been born at the beginning of the current period. An old agent is one that has been born in some previous period.

The social planner decides recursive plans for both types of agents. The state of a recursive plan is the value (i.e. discounted expected utility) that the agent is entitled to at the beginning of the period. Given this promised value, the recursive plan specifies the current utility of consumption, the current utility of leisure and next period promised values as functions of the value of leisure currently reported by the agent.[4] A key goal of the social planner is to structure the recursive plans in such a way that the agents truthfully reveal their private information. Another, conflicting goal, is to structure the plans so that they provide as much insurance as possible. Throughout the paper I will assume that the social planner is fully committed to the recursive plans that he chooses and that the agents have no outside opportunities available.

A key difference between the young and the old is in terms of promised values. Since during the previous period the social planner has already decided on some recursive plan for a currently old agent, he is restricted to deliver the corresponding promised value during the current period. On the contrary, the social planner is free to deliver any value to a currently young agent since this is the first period that he is alive. Reflecting this difference, I will specify the individual state of an old agent to be his promised value v and his current value of leisure s. His current utility of consumption, utility of leisure and next-period promised values are denoted by $u_{os}(v)$, $n_{os}(v)$ and $w_{os}(v, z')$, respectively. In turn, the individual state of a young agent is solely given by his current value of leisure s. His current utility of consumption, utility of leisure and next-period promised values are denoted by u_{ys}, n_{ys} and $w_{ys}(z')$, respectively. Observe that next-period promised values of young and old agents are allowed to be contingent on the realization of next-period aggregate productivity z'.

The aggregate state of the economy is given by the triplet (z, K, μ), where z is the aggregate productivity level, K is the stock of capital, and μ is a measure describing the

[4] I formulate the recursive plans in terms of the utility of consumption and leisure (insteady of consumption and leisure levels) in order to obtain a convex feasible set to the social planner's problem. This is crucial for characterizing the solution using first order conditions.

number of old agents across individual promised values v. The social planner seeks to maximize the weighted sum of welfare levels of current and future generations of young agents.[5] In recursive form, the social planner problem is described by the following Bellman equation:

$$V(z,K,\mu)=\max\left\{(1-\sigma)\sum_{s}\left[u_{ys}+sn_{ys}+\beta\sigma E_z\left(w_{ys}\left(z'\right)\right)\right]\psi_s+\theta E_z V\left(z',K',\mu'_{z'}\right)\right\} \quad (2)$$

subject to:

$$(1-\sigma)\sum_{s}e^{u_{ys}}\psi_s+\int\sum_{s}e^{u_{os}(v)}\psi_s d\mu+I\leq e^z K^{\gamma}H^{1-\gamma}, \quad (3)$$

$$H\leq(1-\sigma)\sum_{s}\left(1-e^{n_{ys}}\right)\psi_s+\int\sum_{s}\left(1-e^{n_{os}(v)}\right)\psi_s d\mu, \quad (4)$$

$$u_{ys}+sn_{ys}+\beta\sigma E_z\left[w_{ys}\left(z'\right)\right]\geq u_{y\hat{s}}+sn_{y\hat{s}}+\beta\sigma E_z\left[w_{y\hat{s}}\left(z'\right)\right] \quad (5)$$

$$u_{os}(v)+sn_{os}(v)+\beta\sigma E_z\left[w_{os}(v,z')\right]\geq u_{o\hat{s}}(v)+sn_{o\hat{s}}(v)+\beta\sigma E_z\left[w_{o\hat{s}}(v;z')\right], \quad (6)$$

$$v=\sum_{s}\left\{u_{os}(v)+sn_{os}(v)+\beta\sigma E_z\left[w_{os}(v,z')\right]\right\}\psi_s, \quad (7)$$

$$K'=(1-\delta)K+I, \quad (8)$$

$$\mu'_{z'}(B)=\sigma\sum_{s}\int_{\{(v,s):\ w_{os}(v,z')\in B\}}\psi_s d\mu+(1-\sigma)\sigma\sum_{s:\ w_{ys}(z')\in B}\psi_s, \quad (9)$$

$$z'=\rho z+\varepsilon', \quad (10)$$

where $0<\theta<1$ is the welfare weight of the next-period generation relative to the current-period generation.[6] Equation (3) describes the aggregate feasibility constraint for the consumption good. It states that the total consumption of young and old agents, plus aggregate investment cannot exceed aggregate output.[7] Equation (4) is the aggregate labor feasibility constraint. It states that the input of hours into the production function cannot exceed the total hours worked by young and old agents. Equation (5), which holds for every $(s,\hat{s})$, is

[5]The welfare levels of old agents are predetermined by their promised values at the beginning of the period.

[6]Observe that if u and n are the utility of consumption and leisure, respectively, then consumption and hours worked are given by e^u and $1-e^n$, respectively.

[7]Observe that, given the constant probability of dying $1-\sigma$ and the immediate replacement with newborns, the number of young agents in the economy is always equal to $1-\sigma$.

the incentive compatibility constraint of young agents. It states that the value of truthfully reporting s provides a higher utility level than reporting the alternative $\hat{s}$. Similarly, equation (6) is the incentive compatibility constraint for old agents. Equation (7) is the promise keeping constraint. It states that the recursive plan for an old agent with promised value v must provide him an expected utility equal to that promised value. Equation (8) is the law of motion for the stock of capital. Equation (9) is the law of motion for the measure of old agents across promised values. It states that the number of old agents that at the beginning of the following period will have a promised value in the Borel set B is given by the sum of two terms. The first term sums all currently old agents that receive a next-period promised value in the set B and do not die. The second term does the same for all currently young agents. Observe that since next-period promised values $w_{os}(v, z')$ and $w_{ys}(z')$ are contingent on the realization of next-period aggregate productivity z', that the same is true for the measure $\mu'_{z'}$. Finally, equation (10) describes the stochastic process for aggregate productivity.

Since the objective function in equation (2) is linear and increasing and equations (3)-(9) define a convex feasible set, the solution to the social planning problem is unique.[8]

4 Characterization of optimal allocations

In this section I characterize the solution to the mechanism design problem as the solution to two simpler planning problems and some side conditions. The two planning problems solve the allocations of old and young agents, respectively, while the side conditions represent aggregate feasibility constraints.

4.1 Planning problem for old agents

Consider the problem of maximizing the expected discounted "social profits" of providing a recursive plan to an old agent, subject to incentive compatibility and promise keeping constraints. Given a promised value to the old agent v, this planning problem is described

[8]For a proof, see Section 1 in the Technical Appendix.

as follows:

$$P(v,z,K,\mu) \tag{11}$$
$$= \max \sum_s \psi_s \left\{ q(z,K,\mu)(1-e^{n_{os}}) - e^{u_{os}} + \theta\sigma E_z \left[\frac{\lambda(z',K',\mu'_{z'})}{\lambda(z,K,\mu)} P\left(w_{os}(z'),z',K',\mu'_{z'}\right) \right] \right\}$$

subject to

$$u_{os}(v) + sn_{os}(v) + \beta\sigma E_z[w_{os}(v,z')] \geq u_{o\hat{s}}(v) + sn_{o\hat{s}}(v) + \beta\sigma E_z[w_{o\hat{s}}(v;z')], \tag{12}$$

$$v = \sum_s \{u_{os}(v) + s_s n_{os}(v) + \beta\sigma E_z[w_{os}(v,z')]\}\psi_s, \tag{13}$$

where q is the social value of labor and λ is the social value of consumption. Observe that the "social profits" are given by the social value of the hours worked by the old agent, net of the consumption goods that are transferred to him. Also observe that the planner discounts future social profits using the social discount rate θ and the survival probability σ. The planner takes the functions λ, q, and the law of motion for $\left(z',K',\mu'_{z'}\right)$ as given.

It is possible to show that P is strictly decreasing, strictly concave, and differentiable in v. These properties allow me to establish the following lemmas.[9]

Lemma 1 *At the optimal allocation,*

$$u_{oH}(v) + s_H n_{oH}(v) + \beta\sigma E_z[w_{oH}(v,z')] > u_{oL}(v) + s_H n_{oL}(v) + \beta\sigma E_z[w_{oL}(v,z')] \tag{14}$$

$$u_{oL}(v) + s_L n_{oL}(v) + \beta\sigma E_z[w_{oL}(v,z')] = u_{oH}(v) + s_L n_{oH}(v) + \beta\sigma E_z[w_{oH}(v,z')]. \tag{15}$$

Proof: It follows from an analysis of the first order conditions and from the fact that P is strictly concave in v.[10]■

This lemma is quite intuitive. It states that at the optimal plan it is easy to convince an old agent with a high value of leisure not to report the low value of leisure, but that it is hard to convince an old agent with a low value of leisure not to report the high one.

Using Lemma 1 the first order conditions to the planning problem for old agents are simplified to the following:

$$0 = -e^{u_{oLt}(v)}\psi_L + \xi_{ot}(v) + \eta_t(v)\psi_L, \tag{16}$$

[9]In what follows, a variable x_s will be denoted x_L when $s = s_L$ and x_H when $s = s_H$.

[10]See Section 2 in the Technical Appendix for the details.

$$0 = -e^{u_{oHt}(v)}\psi_H - \xi_{ot}(v) + \eta_t(v)\psi_H, \tag{17}$$

$$0 = -q_t e^{n_{oLt}(v)}\psi_L + s_L\xi_{ot}(v) + \eta_t(v)s_L\psi_L, \tag{18}$$

$$0 = -q_t e^{n_{oHt}(v)}\psi_H - s_L\xi_{ot}(v) + \eta_t(v)s_H\psi_H, \tag{19}$$

$$0 = \lambda_t\beta\sigma\xi_{ot}(v) + \lambda_t\eta_t(v)\beta\sigma\psi_L - \theta\lambda_{t+1}\sigma\psi_L\eta_{t+1}[w_{oL,t+1}(v)], \tag{20}$$

$$0 = -\lambda_t\beta\sigma\xi_{ot}(v) + \lambda_t\eta_t(v)\beta\sigma\psi_H - \theta\lambda_{t+1}\sigma\psi_H\eta_{t+1}[w_{oH,t+1}(v)], \tag{21}$$

$$u_{oLt}(v) + s_L n_{oLt}(v) + \beta\sigma E_t[w_{oL,t+1}(v)] = u_{oHt}(v) + s_L n_{oHt}(v) + \beta\sigma E_t[w_{oH,t+1}(v)], \tag{22}$$

$$\begin{aligned} v &= \{u_{oLt}(v) + s_L n_{oLt}(v) + \beta\sigma E_t[w_{oL,t+1}(v)]\}\psi_L \\ &\quad + \{u_{oHt}(v) + s_H n_{oHt}(v) + \beta\sigma E_t[w_{oH,t+1}(v)]\}\psi_H, \end{aligned} \tag{23}$$

where, for simplicity, I have switched from state-dependence notation to time-dependence notation under the convention that a variable is dated t if it becomes known at date t. In equations (16)-(21) $\xi_{ot}(v)$ and $\eta_t(v)$ are the Lagrange multipliers to equations (22) and (23), respectively. Since, $\eta_t(v) = -dP(v, z_t, K_{t-1}, \mu_t)/dv$ and P is strictly concave in v it follows that $\eta_t(v)$ is strictly increasing in v. This property allows me to provide the following partial characterization:

Lemma 2 *At the optimal plan,*

$$u_{oHt}(v) < u_{oLt}(v),$$

$$n_{oHt}(v) > n_{oLt}(v),$$

$$w_{oH,t+1}(v) < w_{oL,t+1}(v), \text{ almost surely.}$$

Proof: It follows from a simple analysis of the first order conditions (16)-(21) and the fact that η_{t+1} is a strictly increasing function.[11]■

This lemma is also quite intuitive. It says that when an old agent reports a high value of leisure, the planner allows him to enjoy more leisure but, in compensation, he receives less consumption and is promised a worse treatment in the future.

[11]See Section 2 in the Technical Appendix for the details.

4.2 Planning problem for young agents

Now consider the problem of maximizing the expected discounted "social surplus" of providing a recursive plan to a young agent, subject to incentive compatibility constraints:

$$\max \sum_{s} \left\{ \frac{u_{ys} + sn_{ys} + \beta\sigma w_{ys}}{\lambda(z, K, \mu)} + q(z, K, \mu)\left(1 - e^{n_{ys}}\right) - e^{u_{ys}} \right.$$

$$\left. + \theta\sigma E_z \left[\frac{\lambda\left(z', K', \mu'_{z'}\right)}{\lambda(z, K, \mu)} P\left(w_{ys}(z'), z', K', \mu'_{z'}\right) \right] \psi_s \right\}$$

subject to

$$u_{ys} + sn_{ys} + \beta\sigma E_z\left[w_{ys}(z')\right] \geq u_{y\hat{s}} + sn^{y}_{\hat{s}} + \beta\sigma E_z\left[w_{y\hat{s}}(z')\right],$$

where the planner takes not only the functions λ, q, and the law of motion for $\left(z', K', \mu'_{z'}\right)$ as given, but the value function P that solves the planning problem for the old agents. Observe that the social surplus is the lifetime utility level of the young agent (in current consumption units), plus the expected discounted social value of the hours that will be worked by the agent, net of the consumption goods that will be transferred to him.

Using the strict concavity of P with respect to v, it is possible to show a similar result as in Lemma 1 but for the young agents. The first order conditions to the planning problem for young agents then become the following:

$$0 = \psi_L - \lambda_t e^{u_{yLt}} \psi_L + \lambda_t \xi_{yt}, \tag{24}$$

$$0 = \psi_H - \lambda_t e^{u_{yHt}} \psi_H - \lambda_t \xi_{yt}, \tag{25}$$

$$0 = s_L \psi_L - \lambda_t q_t e^{n_{yLt}} \psi_L + \lambda_t s_L \xi_{yt}, \tag{26}$$

$$0 = s_H \psi_H - \lambda_t q_t e^{n_{yHt}} \psi_H - \lambda_t s_L \xi_{yt}, \tag{27}$$

$$0 = \beta\sigma\psi_L + \lambda_t \beta\sigma\xi_{yt} - \theta\lambda_{t+1}\sigma\psi_L \eta_{t+1}\left(w_{yL,t+1}\right), \tag{28}$$

$$0 = \beta\sigma\psi_H - \lambda_t \beta\sigma\xi_{yt} - \theta\lambda_{t+1}\sigma\psi_H \eta_{t+1}\left(w_{yH,t+1}\right), \tag{29}$$

$$u_{yLt} + s_L n_{yLt} + \beta\sigma E_t\left[w_{yL,t+1}\right] = u_{yHt} + s_L n_{yHt} + \beta\sigma E_t\left[w_{yH,t+1}\right], \tag{30}$$

where ξ_{yt} is the Lagrange multiplier to equation (30).

I can now provide a result analogous to Lemma 2.

Lemma 3 *At the optimal plan,*

$$u_{yHt} < u_{yLt},$$

$$n_{yHt} > n_{yLt},$$

$$w_{yH,t+1} < w_{yL,t+1}, \text{ almost surely.}$$

Proof: It follows from a simple analysis of the first order conditions (24)-(29) and the fact that η_{t+1} is a strictly increasing function.[12]■

4.3 Side conditions

The following lemma states conditions under which the solutions to the planning problems for the old agents and the young agents solve the economy-wide mechanism design problem.

Lemma 4 *Suppose that* $\{u_{ost}(v), n_{ost}(v), w_{os,t+1}(v), \xi_{ot}(v), \eta_t(v)\}_{t=0}^{\infty}$ *solve equations (16)-(23) and that* $\{u_{yst}, n_{yst}, w_{ys,t+1}, \xi_{yt}, \eta_t(v)\}_{t=0}^{\infty}$ *solve equations (24)-(30) for some stochastic process* $\{q_t, \lambda_t\}_{t=0}^{\infty}$.

Additionally, suppose that there exists a stochastic process $\{K_{t-1}, \mu_t, I_t, H_t\}_{t=0}^{\infty}$ *such that* $\{q_t, \lambda_t, K_{t-1}, \mu_t, I_t, H_t\}_{t=0}^{\infty}$ *satisfies the following equations (almost surely):*

$$0 = q_t - e^{z_t} K_{t-1}^{\gamma} (1-\gamma) H_t^{-\gamma}, \tag{31}$$

$$0 = -\lambda_t + \theta E_t \left\{ \lambda_{t+1} \left[e^{z_{t+1}} \gamma K_t^{\gamma-1} H_{t+1}^{1-\gamma} + 1 - \delta \right] \right\}, \tag{32}$$

$$0 = K_t - (1-\delta) K_{t-1} - I_t \tag{33}$$

$$(1-\sigma) \sum_s e^{u_{yst}} \psi_s + \int \sum_s e^{u_{ost}(v)} \psi_s d\mu_t + I_t = e^{z_t} K_{t-1}^{\gamma} H_t^{1-\gamma}, \tag{34}$$

$$H_t = (1-\sigma) \sum_s \left(1 - e^{n_{yst}}\right) \psi_s + \int \sum_s \left(1 - e^{n_{ost}(v)}\right) \psi_s d\mu_t, \tag{35}$$

$$\mu_{t+1}(B) = \sigma \sum_s \int_{\{v:\ w_{os,t+1}(v) \in B\}} \psi_s d\mu_t + (1-\sigma) \sigma \sum_{s:\ w_{ys,t+1} \in B} \psi_s, \tag{36}$$

$$z_{t+1} = \rho z_t + \varepsilon_{t+1}, \tag{37}$$

with (z_0, K_{-1}, μ_0) *given.*

[12] See Section 2 in the Technical Appendix for the details.

Then, $\{u_{ost}(v), n_{ost}(v), w_{os,t+1}(v), u_{yst}, n_{yst}, w_{ys,t+1}, K_{t-1}, I_t, \mu_t\}_{t=0}^{\infty}$ *is the optimal plan generated by the solution to the economy-wide mechanism design problem (equations 2-10) and the initial condition* (z_0, K_{-1}, μ_0).

Proof: It follows from verifying that the first order conditions to the sequential formulation of problem (2)-(10) are given by equations (16)-(23), (24)-(30) and (31)-(37).[13]■

Given the equivalence of first order conditions mentioned in the proof to the above lemma, it follows that the converse is also true: A solution to the economy-wide mechanism design problem solves the planning problems for the old and young agents and the side conditions (31)-(37).

5 An irrelevance result

In this section I provide a striking result: Under the optimal plan, the stationary behavior of all aggregate variables (i.e. aggregate consumption, capital, investment and hours worked) is exactly the same as in the case of public information. In particular, the stationary behavior of all aggregate variables is the same as in a representative agent economy with identical preference and technology parameters (but where the value of leisure is public information). This establishes that, at least for the functional forms for preferences and technology considered so far, that the information frictions introduced play no role on aggregate fluctuations.

5.1 Linear allocation rules

In this section I characterize the functional forms for the allocation rules of old agents.

[13]See Sections 1 and 2 in the Technical Appendix for the details.

Lemma 5 *The allocation rules for old agents have the following functional forms:*

$$u_{ost}(v) = u_{ost} + bv, \tag{38}$$

$$n_{ost}(v) = n_{ost} + bv, \tag{39}$$

$$w_{os,t+1}(v) = w_{os,t+1} + v, \tag{40}$$

$$\ln \eta_t(v) = f_t + bv, \tag{41}$$

$$\ln \xi_{ot}(v) = g_{ot} + bv, \tag{42}$$

where $0 < b = \frac{1-\beta\sigma}{1+\bar{s}} < 1$ *and* $\bar{s} = s_H\psi_H + s_L\psi_L$.

Proof: It is straightforward to verify that these functional forms satisfy equations (16)-(23), (24)-(30) and (31)-(37), and that these equations become the following:

$$0 = -e^{u_{oLt}}\psi_L + e^{g_{ot}} + e^{f_t}\psi_L, \tag{43}$$

$$0 = -e^{u_{oHt}}\psi_H - e^{g_{ot}} + e^{f_t}\psi_H, \tag{44}$$

$$0 = -q_t e^{n_{oLt}}\psi_L + s_L e^{g_{ot}} + e^{f_t} s_L\psi_L, \tag{45}$$

$$0 = -q_t e^{n_{oHt}}\psi_H - s_L e^{g_{ot}} + e^{f_t} s_H\psi_H, \tag{46}$$

$$0 = \lambda_t\beta\sigma e^{g_{ot}} + \lambda_t e^{f_t}\beta\sigma\psi_L - \theta\lambda_{t+1}\sigma\psi_L e^{f_{t+1}+bw_{oL,t+1}}, \tag{47}$$

$$0 = -\lambda_t\beta\sigma e^{g_{ot}} + \lambda_t e^{f_t}\beta\sigma\psi_H - \theta\lambda_{t+1}\sigma\psi_H e^{f_{t+1}+bw_{oH,t+1}}, \tag{48}$$

$$u_{oLt} + s_L n_{oLt} + \beta\sigma E_t\left[w_{oL,t+1}\right] = u_{oHt} + s_L n_{oHt} + \beta\sigma E_t\left[w_{oH,t+1}\right], \tag{49}$$

$$0 = \left\{u_{oLt} + s_L n_{oLt} + \beta\sigma E_t\left[w_{oL,t+1}\right]\right\}\psi_L + \left\{u_{oHt} + s_H n_{oHt} + \beta\sigma E_t\left[w_{oH,t+1}\right]\right\}\psi_H, \tag{50}$$

$$0 = \psi_L - \lambda_t e^{u_{yLt}}\psi_L + \lambda_t\xi_{yt}, \tag{51}$$

$$0 = \psi_H - \lambda_t e^{u_{yHt}}\psi_H - \lambda_t\xi_{yt}, \tag{52}$$

$$0 = s_L\psi_L - \lambda_t q_t e^{n_{yLt}}\psi_L + \lambda_t s_L\xi_{yt}, \tag{53}$$

$$0 = s_H\psi_H - \lambda_t q_t e^{n_{yHt}}\psi_H - \lambda_t s_L\xi_{yt}, \tag{54}$$

$$0 = \beta\sigma\psi_L + \lambda_t\beta\sigma\xi_{yt} - \theta\lambda_{t+1}\sigma\psi_L e^{f_{t+1}+bw_{yL,t+1}}, \tag{55}$$

$$0 = \beta\sigma\psi_H - \lambda_t\beta\sigma\xi_{yt} - \theta\lambda_{t+1}\sigma\psi_H e^{f_{t+1}+bw_{yH,t+1}}, \tag{56}$$

$$u_{yLt} + s_L n_{yLt} + \beta\sigma E_t\left[w_{yL,t+1}\right] = u_{yHt} + s_L n_{yHt} + \beta\sigma E_t\left[w_{yH,t+1}\right], \tag{57}$$

$$0 = q_t - e^{z_t} K_{t-1}^{\gamma} (1-\gamma) H_t^{-\gamma} \tag{58}$$

$$0 = -\lambda_t + \theta E_t \left\{ \lambda_{t+1} \left[e^{z_{t+1}} \gamma K_t^{\gamma-1} H_{t+1}^{1-\gamma} + 1 - \delta \right] \right\} \tag{59}$$

$$0 = K_t - (1-\delta) K_{t-1} - I_t \tag{60}$$

$$(1-\sigma) \sum_s e^{u_{yst}} \psi_s + V_t \sum_s e^{u_{ost}} \psi_s + I_t = e^{z_t} K_{t-1}^{\gamma} H_t^{1-\gamma}, \tag{61}$$

$$H_t = (1-\sigma) \sum_s (1 - e^{n_{yst}}) \psi_s + \sigma - V_t \sum_s e^{n_{ost}} \psi_s, \tag{62}$$

$$V_{t+1} = \sigma V_t \sum_s e^{bw_{os,t+1}} \psi_s + (1-\sigma) \sigma \sum_s e^{bw_{ys,t+1}} \psi_s \tag{63}$$

$$z_{t+1} = \rho z_t + \varepsilon_{t+1}, \tag{64}$$

with (z_0, K_{-1}, V_0) given, and where

$$V_t = \int e^{bv} d\mu_t. \blacksquare \tag{65}$$

This establishes not only that the functional forms given by equations (38)-(42) are satisfied but that the dependence of the solution on promised values is completely summarized by the moment V_t in equation (65).

Observe that the deterministic steady state conditions can be obtained from equations (43)-(63) by setting the aggregate productivity level z_t to zero and imposing that all variables are constant over time. The appendix provides such conditions.

5.2 Fluctuations of optimal allocation rules

This section provides tight cross-restrictions on the stationary fluctuations of key variables of the model. To this end, for any variable x_t I define

$$\Delta x_t = x_t - x, \tag{66}$$

where x is the deterministic steady state value of variable x_t.

Lemma 6 *At the stationary optimal plan,*

$$\Delta u_{yt} \equiv \Delta u_{yHt} = \Delta u_{yLt}, \tag{67}$$

$$\Delta n_{yt} \equiv \Delta n_{yHt} = \Delta n_{yLt}, \tag{68}$$

$$\Delta w_{y,t+1} \equiv \Delta w_{yH,t+1} = \Delta w_{yL,t+1}, \tag{69}$$

$$\Delta u_{ot} \equiv \Delta u_{oHt} = \Delta u_{oLt}, \tag{70}$$

$$\Delta n_{ot} \equiv \Delta n_{oHt} = \Delta n_{oLt}, \tag{71}$$

$$\Delta w_{o,t+1} \equiv \Delta w_{oH,t+1} = \Delta w_{oL,t+1}. \tag{72}$$

Moreover,

$$\Delta \ln \xi_{yt} = -\Delta \ln \lambda_t \tag{73}$$

$$\Delta u_{yt} = -\Delta \ln \lambda_t \tag{74}$$

$$\Delta n_{yt} = \Delta u_{yt} - \Delta \ln q_t \tag{75}$$

$$\Delta \ln \lambda_{t+1} + \Delta f_{t+1} + b\Delta w_{y,t+1} = 0 \tag{76}$$

$$\Delta u_{ot} = \Delta f_t = \Delta g_{ot} \tag{77}$$

$$\Delta n_{ot} = \Delta u_{ot} - \Delta \ln q_t \tag{78}$$

$$\Delta \ln \lambda_t + \Delta u_{ot} = \Delta \ln \lambda_{t+1} + \Delta u_{o,t+1} + b\Delta w_{o,t+1} \tag{79}$$

$$\Delta u_{ot} + \bar{s}\Delta n_{ot} + \beta\sigma E_t \left[\Delta w_{o,t+1}\right] = 0 \tag{80}$$

$$\Delta \ln V_t = -\Delta \ln \lambda_t - \Delta u_{ot} \tag{81}$$

Proof: *Using equations (66), (67)-(81) and (141)-(161) it is straightforward to verify that equations (43)-(57) and equation (63) are satisfied.*[14] *Also, equations (58)-(62) become the following:*

$$0 = e^{\Delta \ln q_t + \ln q} - e^{z_t} K_{t-1}^{\gamma} (1-\gamma) {H_t}^{-\gamma} \tag{82}$$

$$0 = -e^{\Delta \ln \lambda_t} + \theta E_t \left\{ e^{\Delta \ln \lambda_{t+1}} \left[e^{z_{t+1}} \gamma K_t^{\gamma-1} {H_{t+1}}^{1-\gamma} + 1 - \delta \right] \right\} \tag{83}$$

$$0 = K_t - (1-\delta) K_{t-1} - I_t \tag{84}$$

[14]For instance, using equation (66), equation (48) can be rewritten as follows:

$$0 = -e^{\Delta \ln \lambda_t + \ln \lambda} \beta\sigma e^{\Delta g_{ot} + g_o} + e^{\Delta \ln \lambda_t + \ln \lambda} e^{\Delta f_t + f} \beta\sigma\psi_H - \theta e^{\Delta \ln \lambda_{t+1} + \ln \lambda} \sigma\psi_H e^{\Delta f_{t+1} + f + b(\Delta w_{o,t+1} + w_{oH})}$$

Using equations (77) and (79), this equation becomes

$$0 = e^{\Delta \ln \lambda_t + \Delta f_t} e^{\ln \lambda} \left[-\beta\sigma e^{g_o} + e^f \beta\sigma\psi_H - \theta\sigma\psi_H e^{f + b w_{oH}} \right],$$

which is satisfied because of the steady state condition (146).

$$e^{-\Delta \ln \lambda_t} (1-\sigma) \frac{1}{\lambda} \frac{\theta}{\theta - \sigma\beta} + I_t = e^{z_t} K_{t-1}^{\gamma} H_t^{1-\gamma}, \tag{85}$$

$$H_t = 1 - e^{-\Delta \ln \lambda_t - \Delta \ln q_t} (1-\sigma) \frac{\bar{s}}{\lambda q} \frac{\theta}{\theta - \sigma\beta} \tag{86}$$

$$z_{t+1} = \rho z_t + \varepsilon_{t+1} \tag{87}$$

A proof that a stationary solution to equations (82)-(87) exists will be provided in Section 5.4. A stationary process for λ_t and q_t uniquely determines a stationary process for Δu_{ot}, Δn_{ot} and $\Delta w_{o,t+1}$ from equations (78)-(80). In particular, they are given by

$$\Delta u_{ot} = -\beta\sigma \Delta \ln \lambda_t + (1-\beta\sigma) \sum_{j=1}^{\infty} (\beta\sigma)^j E_t \left[\Delta \ln \lambda_{t+j}\right] + b\bar{s} \sum_{j=1}^{\infty} (\beta\sigma)^j E_t \left[\Delta \ln q_{t+j}\right],$$

$$\Delta w_{o,t+1} = \frac{\Delta \ln \lambda_t + \Delta u_{ot} - \Delta \ln \lambda_{t+1} - \Delta u_{o,t+1}}{b},$$

$$\Delta n_{ot} = \Delta u_{ot} - \Delta \ln q_t.$$

Corresponding realizations for $\Delta \ln \xi_{yt}$, Δu_{yt}, Δn_{yt}, $\Delta w_{y,t+1}$, Δf_t, Δg_{ot} and $\Delta \ln V_t$ are then determined from equations (73)-(77) and (81).[15] ■

As the following Corollary states, Lemma 6 provides a greatly simplified method for solving a stationary solution to the original mechanism design problem.

Corollary 7 *Finding a stationary solution to equations (16)-(23), (24)-(30) and (31)-(37) is equivalent to finding a stationary solution to equations (82)-(87).*

For future reference, the following summarizes results in (38)-(42), (66) and (70)-(72):

$$u_{oLt}(v) = u_{oL} + bv + \Delta u_{ot}, \tag{88}$$

$$u_{oHt}(v) = u_{oH} + bv + \Delta u_{ot}, \tag{89}$$

$$n_{oLt}(v) = n_{oL} + bv + \Delta n_{ot}, \tag{90}$$

$$n_{oHt}(v) = n_{oH} + bv + \Delta n_{ot}, \tag{91}$$

$$w_{oL,t+1}(v) = w_{oL} + v + \Delta w_{o,t+1}, \tag{92}$$

$$w_{oH,t+1}(v) = w_{oH} + v + \Delta w_{o,t+1}. \tag{93}$$

[15] See Section 3 in the Technical Appendix for the details.

Equations (88)-(89) indicate that $u_{oLt}(v)$ and $u_{oHt}(v)$ are linear parallel functions with slope less than one that shift vertically over the business cycle by exactly the same amounts. Equations (90)-(91) indicate that the same is true for $n_{oLt}(v)$ and $n_{oHt}(v)$. Equations (92)-(93) indicate that $w_{oL,t+1}(v)$ and $w_{oH,t+1}(v)$ also are linear parallel functions that shift vertically over the business cycle by exactly the same amounts. However, the slope of these functions is equal to one. Thus, promised values follow a random walk process with innovations that depend on the realization of the idiosyncratic and aggregate shocks.[16]

5.3 Fluctuations in the optimal amount of inequality

In this section I characterize the fluctuations in the distributions of promised values, consumption levels and hours worked implied by the fluctuations in the optimal allocation rules.

Observe, from equations (36) and (40) that for every interval (v_1, v_2) the steady state distribution μ satisfies that:

$$\mu\left[(v_1, v_2)\right] = \sigma \sum_s \psi_s \mu\left[(v_1 - w_{os}, v_2 - w_{os})\right] + (1-\sigma)\,\sigma \sum_{s:\ w_{ys} \in (v_1, v_2)} \psi_s. \tag{94}$$

Define

$$\Delta_t = \frac{\Delta \ln \lambda_t + \Delta f_t}{b}. \tag{95}$$

From equations (36), (40), (76) and (79) we then have that for every $(v_1 - \Delta_{t+1}, v_2 - \Delta_{t+1})$:

$$\begin{aligned} \mu_{t+1}\left[(v_1 - \Delta_{t+1}, v_2 - \Delta_{t+1})\right] &= \sigma \sum_s \psi_s \mu_t\left[(v_1 - \Delta_t - w_{os}, v_2 - \Delta_t - w_{os})\right] \\ &\quad + (1-\sigma)\,\sigma \sum_{s:\ w_{ys} \in (v_1, v_2)} \psi_s. \end{aligned} \tag{96}$$

From equations (94) and (96) it then follows that for every (v_1, v_2):

$$\mu\left[(v_1, v_2)\right] = \mu_t\left[(v_1 - \Delta_t, v_2 - \Delta_t)\right]. \tag{97}$$

[16] Even with no aggregate fluctuations (i.e. with $\Delta w_{o,t+1}$ identical to zero) promised values follow a random walk. However, contrary to Atkeson and Lucas (1992) an immizerizing result is not obtained because of the stochastic lifetimes. As people die and are replaced by young agents, there is enough "reversion to the mean" in promised values that an invariant distribution is obtained (see Phelan 1994). The immizerizeing result actually applies within each cohort of agents: Within each cohort the distribution of promised values keeps spreading out more and more over time.

That is, μ_t is just a Δ_t horizontal translation of the invariant distribution μ. In particular, since promised values increase during a boom, μ_t shifts to the right during such an episode.

Observe from equations (38), (74) and (77) that

$$u_{yst} = u_{ys} - \Delta \ln \lambda_t,$$

$$u_{ost}(v) = u_{os} + bv + \Delta f_t.$$

Then, the distribution of the utility of consumption ϕ_t satisfies that for every Borel set U:

$$\phi_t(U) = \sum_s \int_{\{v: u_{os} + bv + \Delta f_t \in U\}} \psi_s d\mu_t + \sum_{s: u_{ys} - \Delta \ln \lambda_t \in U} \psi_s.$$

It follows that for every (u_1, u_2),

$$\phi[(u_1, u_2)] = \sum_s \psi_s \mu \left[\left(\frac{u_1 - u_{os}}{b}, \frac{u_2 - u_{os}}{b} \right) \right] + \sum_{s: u_{ys} \in (u_1, u_2)} \psi_s$$

and

$$\begin{aligned} & \phi_t[(u_1 - \Delta \ln \lambda_t, u_2 - \Delta \ln \lambda_t)] \\ = & \sum_s \psi_s \mu_t \left[\left(\frac{u_1 - \Delta \ln \lambda_t - u_{os} - \Delta f_t}{b}, \frac{u_2 - \Delta \ln \lambda_t - u_{os} - \Delta f_t}{b} \right) \right] + \sum_{s: u_{ys} \in (u_1, u_2)} \psi_s. \end{aligned}$$

Using equations (95) and (97) we then have that

$$\phi_t[(u_1 - \Delta \ln \lambda_t, u_2 - \Delta \ln \lambda_t)] = \phi[(u_1, u_2)].$$

Thus, ϕ_t is just a $\Delta \ln \lambda_t$ horizontal translation of the steady state distribution ϕ. Since the utilities of consumption increase during a boom, ϕ_t shifts to the right during such an episode.

Observe that consumption levels are related to utilities of consumption according to $c = e^u$. Since this is a strictly increasing and strictly convex function it follows that when the distribution of utilities of consumption shifts to the right, that the dispersion of the distribution of consumption levels (measured, for example, as interdecile ranges) increases. Thus, the dispersion of the distribution of consumption levels increases during a boom.

From equations (39), (75) and (78) we have that

$$n_{yst} = n_{ys} - \Delta \ln \lambda_t - \Delta \ln q_t,$$

$$n_{ost}(v) = n_{os} + bv + \Delta f_t - \Delta \ln q_t.$$

Then, the distribution of utilities of leisure ζ_t satisfies that for every Borel set N:

$$\zeta_t(N) = \sum_s \int_{\{v: n_{os} + bv + \Delta f_t - \Delta \ln q_t \in N\}} \psi_s d\mu_t + \sum_{s: n_{ys} - \Delta \ln \lambda_t - \Delta \ln q_t \in N} \psi_s$$

It follows that for every (n_1, n_2),

$$\zeta[(n_1, n_2)] = \sum_s \psi_s \mu \left[\left(\frac{n_1 - n_{os}}{b}, \frac{n_2 - n_{os}}{b} \right) \right] + \sum_{s: n_{ys} \in (n_1, n_2)} \psi_s$$

and

$$\begin{aligned} &\zeta_t[(n_1 - \Delta \ln \lambda_t - \Delta \ln q_t, n_2 - \Delta \ln \lambda_t - \Delta \ln q_t)] \\ &= \sum_s \psi_s \mu_t \left[\left(\frac{n_1 - \Delta \ln \lambda_t - n_{os} - \Delta f_t}{b}, \frac{n_2 - \Delta \ln \lambda_t - n_{os} - \Delta f_t}{b} \right) \right] + \sum_{s: n_{ys} \in (n_1, n_2)} \psi_s \end{aligned}$$

Using equations (97) and (95) we then have that

$$\zeta_t[(n_1 - \Delta \ln \lambda_t - \Delta \ln q_t, n_2 - \Delta \ln \lambda_t - \Delta \ln q_t)] = \zeta[(n_1, n_2)].$$

Thus, ζ_t is just a $\Delta \ln \lambda_t + \Delta \ln q_t$ horizontal translation of the steady state distribution ζ. Since the utilities of leisure decrease during a boom, it follows that ζ_t shifts to the left during such an episode.

Observe that hours worked are related to utilities of leisure according to $h = 1 - e^n$. Since this is a strictly decreasing and strictly concave function it follows that when the distribution of utilities of leisure shifts to the left, that the dispersion of the distribution of hours (measured, for example, as interdecile ranges) decreases. Thus, the dispersion of the distribution of hours worked decreases during a boom.

5.4 Full information economy

In this section I consider a representative agent economy with full information. The social planning problem for this economy is the following:

$$V(z, K) = \max \{u + \bar{s}n + \theta E_z [V(z', K')]\} \tag{98}$$

subject to:

$$e^u + I \leq e^z K^\gamma H^{1-\gamma} \tag{99}$$

$$H \leq 1 - e^{n} \tag{100}$$

$$K' \leq (1-\delta) K + I \tag{101}$$

$$z' = \rho z + \varepsilon', \tag{102}$$

where $\bar{s} = s_H \psi_H + s_L \psi_L$. All parameters are assumed to have the same values as in the private information economy.

Using dated variables, the first order conditions to this problem are the following:

$$q_t = e^{z_t} K_{t-1}^{\gamma} (1-\gamma) H_t^{-\gamma} \tag{103}$$

$$\hat{\lambda}_t = \theta E_t \left\{ \hat{\lambda}_{t+1} \left[e^{z_{t+1}} \gamma K_t^{\gamma-1} H_{t+1}^{1-\gamma} + 1 - \delta \right] \right\} \tag{104}$$

$$K_t = (1-\delta) K_{t-1} + I_t \tag{105}$$

$$\frac{1}{\hat{\lambda}_t} + I_t = e^{z_t} K_{t-1}^{\gamma} H_t^{1-\gamma} \tag{106}$$

$$H_t = 1 - \frac{\bar{s}}{\hat{\lambda}_t q_t} \tag{107}$$

$$z_{t+1} = \rho z_t + \varepsilon_{t+1} \tag{108}$$

where $\hat{\lambda}_t$ and q_t are the Lagrange multipliers on equations (99) and (100), respectively.

Defining deviations from steady state values as in equation (66), we get that equations (103)-(108) can be rewritten as:

$$0 = e^{\Delta \ln q_t + \ln q} - e^{z_t} K_{t-1}^{\gamma} (1-\gamma) H_t^{-\gamma} \tag{109}$$

$$0 = -e^{\Delta \ln \hat{\lambda}_t} + \theta E_t \left\{ e^{\Delta \ln \hat{\lambda}_{t+1}} \left[e^{z_{t+1}} \gamma K_t^{\gamma-1} H_{t+1}^{1-\gamma} + 1 - \delta \right] \right\} \tag{110}$$

$$0 = K_t - (1-\delta) K_{t-1} - I_t \tag{111}$$

$$e^{-\Delta \ln \hat{\lambda}_t} \frac{1}{\hat{\lambda}} + I_t = e^{z_t} K_{t-1}^{\gamma} H_t^{1-\gamma} \tag{112}$$

$$H_t = 1 - e^{-\Delta \ln \hat{\lambda}_t - \Delta \ln q_t} \frac{\bar{s}}{\hat{\lambda} q} \tag{113}$$

$$z_{t+1} = \rho z_t + \varepsilon_{t+1} \tag{114}$$

I can now state the main result of this section.

Lemma 8 *Equations (109)-(114) are equivalent to equations (82)-(87).*

Proof: *The equivalence can be verified under the following relations:*

$$\lambda = (1-\sigma)\frac{\theta}{\theta-\sigma\beta}\hat{\lambda}, \tag{115}$$

$$\Delta \ln \lambda_t = \Delta \ln \hat{\lambda}_t. \blacksquare \tag{116}$$

The following corollary ties the lose end in the proof of Lemma 6.

Corollary 9 *A stationary solution to equations (82)-(87) exists.*

Proof: *It is standard to show that a recursive solution to equations (109)-(114) exists, with $\Delta \ln \hat{\lambda}_t$, $\Delta \ln q_t$, H_t and I_t being time invariant functions of the state variables (z_t, K_{t-1}), and that these decision rules generate a stationary stochastic process for all endogenous variables.*

Another direct consequence of Lemma 3 is the following:

Corollary 10 *All aggregate variables, i.e. K_{t-1}, H_t, I_t and $C_t = e^{z_t}K_{t-1}^{\gamma}H_t^{1-\gamma} - I_t$, follow identical stationary stochastic processes in the private information economy and in the representative agent economy.*

This result establishes that the information frictions introduced to the original model play no role whatsoever on aggregate business cycle fluctuations.

I would like to point out that the equivalence result presented here only holds for stationary allocations. The aggregate transitionary dynamics obtained from solving equations (109)-(114) for an arbitrary initial condition (z_o, K_0) in general will not coincide with the aggregate transitionary dynamics obtained from solving equations (43)-(64) for an identical initial (z_o, K_0), but arbitrary V_0. The reason, is that the restrictions imposed by equation (81), which is needed for the equivalence result, will generally be violated.[17] A consequence of this is that, contrary to the results obtained in Farhi and Werning (2012), the deterministic transitionary dynamics (obtained for the case of $z_t = 0$) will generally differ in the representative agent economy and the mechanism design problem with private information.

[17] Given the stochastic process $\left\{\Delta \ln \hat{\lambda}_t, \Delta \ln q_t, H_t, I_t, z_t, K_{t-1}\right\}_{t=0}^{\infty}$ that solves equations (109)-(114) and an initial condition (z_o, K_0), equations (115)-(116) can always be used to construct the stochastic process $\{\Delta u_{ot}, \Delta n_{ot}, \Delta w_{o,t+1}\}_{t=0}^{\infty}$ that solves equations (78)-(80). The equivalence result along the transitionary dynamics would then be obtained only for the value of V_0 that satisfies equation (81) at $t = 0$.

6 Computations

The previous section was able to provide a full characterization of the solution to the mechanism design problem because of the particular preferences considered. However, when preferences differ from the log-log case such characterization is no longer possible and the model must be solved numerically. This is a nontrivial task because of the high dimensionality of the state space. In this section I introduce a new method for computing equilibria of models with heterogeneous agents and aggregate shocks and apply it to the model considered in this paper. An important advantage of this computational method over existing alternatives in the literature is not only that it keeps track of an arbitrarily good approximation to the distribution of agents over individual states, but that the mapping from the current distribution of agents (and current individual decision rules) to the next-period distribution of agents is almost exact.[18] Thus, the method promises to be extremely useful for computing equilibria in cases where the distribution of individual states matters.

Before proceeding to describe its details it will be useful to sketch the main ingredients of the computational method. Instead of keeping track of the distribution of promised values μ as a state variable, what the computational method keeps track of is a long history of individual decision rules w_{os} and w_{ys}. Since the individual decision rules w_{os} are parametrized as spline approximations, the computational method only needs to keep track of a long but finite history of spline coefficients. The current distribution of promised values is then recovered by simulating the evolution of a large number of agents (and their descendants) over time using the history of individual decision rules kept as state variables.[19] The next period distribution of promised values is then obtained by simply updating by one period the history of individual decision rules using the decision rules chosen during the current period. All first order conditions and aggregate feasibility constraints are then linearized

[18] See Algan et al. (2014) for a survey of the alternatives.

[19] Because of the stochastic lifetimes, the truncation introduced by the finite history of decision rules generate arbitrarily small approximation errors as the lenght of the history becomes large. In fact, when this length becomes large the distribution used for drawing initial promised values for the simulations becomes irrelevant (although, in practice, I use the invariant distribution of the deterministic steady state).

with respect to the spline coefficients describing current and past individual decision rules.[20] This delivers a linear rational expectations model which, despite of its high dimensionality, can be solved using standard methods.

To streamline the presentation I will describe the computational method using the equations already derived for the log-log case. However, it is important to keep in mind that the method can be (and will be) applied to analogous equations derived under more general preferences.

6.1 Computing the deterministic steady state

While computing the deterministic steady state of the model is completely standard, this section describes the algorithm in detail since this will introduce objects and notation that will be needed later on.

Observe that the shadow value of labor q is known from the steady state versions of equations (31) and (32). In particular it is given by

$$q = (1-\gamma)\left\{\frac{1}{\gamma}\left[\frac{1}{\theta} - 1 + \delta\right]\right\}^{\frac{\gamma}{1-\gamma}}.$$

Given this value of q, the steady state planning problem for old agents can be solved. To this end, I find it convenient to use cubic spline approximations and iterate with the first order conditions to this problem, given by the steady state versions of equations (16)-(23).[21] In order to do this, I first restrict the promised values to lie on a closed interval $[v_{\min}, v_{\max}]$ and define an equidistant vector of grid points $(v_j)_{j=1}^{J}$, with $v_1 = v_{\min}$ and $v_J = v_{\max}$.[22] Given the function η from the previous iteration, which is used to value next period promised values in the steady state versions of equations (20) and (21), the values

[20] This is the computationally most intensive part of the method. The reason is that we need to take numerical derivatives with respect to each spline coefficient in the history, and each of these calculations requires simulating the evolution of a large panel of agents over the entire history of individual decision rules kept as state variables.

[21] Observe that the shadow value of consumption λ does not appear in the steady state version of these equations,

[22] When restricting promised values to lie in the interval $[v_{\min}, v_{\max}]$, the first order conditions (20)-(21) and (28)-(29) change by incorporating inequalities that check for corner solutions.

of $[u_{os}(v_j), n_{os}(v_j), w_{os}(v_j), \xi_o(v_j), \eta(v_j)]_{j=1}^{J}$ that satisfy the steady state versions of equations (16)-(23) are then solved for at the grid points $(v_j)_{j=1}^{J}$. Once these values are found, the functions are extended to the full domain $[v_{\min}, v_{\max}]$ using cubic splines.[23] The iterations continue until the values for $[u_{os}(v_j), n_{os}(v_j), w_{os}(v_j), \xi_o(v_j), \eta(v_j)]_{j=1}^{J}$ converge. Observe, that this solution does not depend on any other endogenous values, so it forms part of the steady state.

Given the steady state solution for η the steady state planning problem for young agents can be solved next. This problem is essentially static and has a finite number of decision variables. However, it has the complication is that it depends on the shadow price of consumption λ, which is an endogenous variable. Thus, conditional on a value for λ, the steady state versions of equations (24)-(30) can be solved for $(u_{ys}, n_{ys}, w_{ys}, \xi_y)$, but later on I will have to provide the side condition that λ must satisfy for this to form part of the steady state.

The steady state version of equation (36) describes the recursion that the invariant μ has to satisfy. This equation corresponds to the case of a continuum of agents. However, I will find it convenient to work with a large, but finite number of agents, and perform the recursion for this case. In particular, consider a large but finite number of agents I and endow them with promised values in the interval $[v_{\min}, v_{\max}]$. Using the functions w_{oL} and w_{oH} obtained from the steady state planning problem for old agents and the values w_{yL} and w_{yH} obtained from the steady state planning problem for young agents, simulate the evolution of the promised values of these I agents and their descendants for a large number of periods T. To be precise, if agent i was promised a value v at the beginning of the current period (conditional on being alive), then his promised value (or his descendant's, in case the agent dies) at the beginning of the following period will be given by:

$$v' = \begin{cases} w_{oL}(v), \text{ with probability } \sigma\psi_L, \\ w_{oH}(v), \text{ with probability } \sigma\psi_H, \\ w_{yL}, \text{ with probability } (1-\sigma)\psi_L, \\ w_{yH}, \text{ with probability } (1-\sigma)\psi_H. \end{cases} \tag{117}$$

Simulating the I agents for T periods using equation (117) we obtain a realized distri-

[23] In practice, I use the monotonicity preserving cubic splines described by Steffen (1990).

bution $(\bar{v}_i)_{i=1}^I$ of promised values (conditional on being alive) across the I agents. Observe that the last iteration of equation (117) also gives the corresponding realized values of leisure $(\bar{s}_i)_{i=1}^I$ across the I agents. The joint realized distribution of promised values and values of leisure $(\bar{v}_i, \bar{s}_i)_{i=1}^I$ can then be used to compute statistics under the invariant distribution. In particular, aggregate consumption can be obtained as:

$$C = \sigma \frac{1}{I} \sum_{i=1}^{I} e^{u_{o,\bar{s}_i}(\bar{v}_i)} + (1-\sigma) \sum_s e^{u_{ys}} \psi_s. \tag{118}$$

To understand this expression, suppose that we are at the beginning of period $T+1$. The joint realized distribution $(\bar{v}_i, \bar{s}_i)_{i=1}^I$ now corresponds to agents that were alive in the previous period, and thus a fraction σ of them will have survived and a fraction $(1-\sigma)$ of them will have died. The first term in equation (118) corresponds to those who have survived. It averages the consumption of these agents and multiplies the result by the probability of surviving σ. The second term corresponds to those who have died and thus have been replaced by young agents. It averages the consumption of young agents and multiplies the result by the probability of dying $(1-\sigma)$.

Aggregate hours worked can be similarly computed as

$$H = \sigma \frac{\sum_{i=1}^{I} \left[1 - e^{n_{o,\bar{s}_i}(\bar{v}_i)}\right]}{I} + (1-\sigma) \sum_s \left(1 - e^{n_{ys}}\right) \psi_s. \tag{119}$$

Observe that by a law of large numbers equations (118) and (119) will become arbitrarily good approximations to the steady state versions of equations (34) and (35) as I and T tend to infinity.

Given aggregate hours worked, aggregate capital can be then obtained from the fact that the social planner equates the marginal productivity of capital to its shadow price. In particular, from the steady state version of equation (32) we have that aggregate capital is given by

$$K = \left(\frac{\gamma}{\frac{1}{\theta} - 1 + \delta} \right)^{\frac{1}{1-\gamma}} H. \tag{120}$$

Then, aggregate investment is

$$I = \delta K. \tag{121}$$

The last equation that needs to be satisfied is the feasibility condition for consumption,

$$C + I = K^{\gamma} H^{1-\gamma}. \tag{122}$$

This is the side condition mentioned above for the shadow value of consumption λ. The shadow value of consumption determines the consumption, hours worked and promised values of young agents, and therefore each of the variables in equation (122). Therefore, it must be changed until equation (122) holds.[24]

6.2 Computing business cycle fluctuations

As has already been mentioned, computing business cycle fluctuations requires linearizing the first order conditions and aggregate feasibility constraints with respect to a convenient set of variables. The resulting linearized system can then be solved using standard methods.

6.2.1 Linearization

Linearizing equations (16)-(23), (24)-(30) and (31)-(37) present different types of issues. As a consequence, I classify them into different categories.

The first category is constituted by equations that only involve scalar variables. Equations (24)-(27), (30), and (31)-(33) fall into this category. For example, consider equation (25). This equation is a function of $\{\lambda_t, u_{yHt}, \xi_{yt}\}$, which are all scalars. Linearizing this equation around the deterministic steady state values $\left\{\bar{\lambda}, \bar{u}_{yHt}, \bar{\xi}_{yt}\right\}$ poses no difficulty.[25]

The second category is constituted by a continuum of equations that only involve scalar variables. Equations (16)-(19) and (22)-(23) fall into category. Consider, for example, equation (17). This equation depends on $\{u_{oHt}(v), \xi_{ot}(v), \eta_t(v)\}$ which are all scalars. The problem is that there is one of these equations for every value of v in the interval $[v_{\min}, v_{\max}]$. In this case the "curse of dimensionality" is solved by considering this equation only at the grid points $(v_j)_{j=1}^{J}$ that were used in the computation of the deterministic steady state. It is now straightforward to linearize each of these J equations with respect to $\{u_{oHt}(v_j), \xi_{ot}(v_j), \eta_t(v_j)\}$ at their deterministic steady state values $\left\{\bar{u}_{oH}(v_j), \bar{\xi}_o(v_j), \bar{\eta}(v_j)\right\}$. Extending $\{u_{oHt}(v), \xi_{ot}(v), \eta_t(v)\}$ to the full domain $[v_{\min}, v_{\max}]$ using cubic splines will

[24] In practice, this is done using a bisection root finding method.

[25] Although in this case derivatives can be taken analytically, throughout the section derivatives are assumed to be numerically obtained.

make equation (17) hold only approximately outside of the grid points $(v_j)_{j=1}^J$. The quality of this approximation will depend on how many grid points J we work with.

The third category is constituted by equations that involve both scalars and functions. Equations (28) and (29) fall in this category. For example, consider equation (29). This equation depends on λ_t, ξ_{yt}, λ_{t+1}, $w_{yH,t+1}$ and on the function η_{t+1}, which is a high dimensional object. In this case the "curse of dimensionality" is broken by considering that η_{t+1} is a spline approximation and, therefore, is completely determined by the finite set of values $\{\eta_{t+1}(v_j)\}_{j=1}^J$, i.e. the value of the function at the grid points. The equation can then be linearized with respect to $\left[\lambda_t, \xi_{yt}, \lambda_{t+1}, w_{yH,t+1}, \{\eta_{t+1}(v_j)\}_{j=1}^J\right]$ at the deterministic steady state values $\left[\bar{\lambda}, \bar{\xi}_y, \bar{\lambda}, \bar{w}_{yH}, \{\bar{\eta}(v_j)\}_{j=1}^J\right]$.

The fourth category is a combination of the previous two: it is constituted by a continuum of equations that involve both scalars and functions. Equations (20) and (21) fall in this category. For example, consider equation (21). Similarly to the third category, this equation depends on the scalars λ_t, $\xi_{ot}(v)$, λ_{t+1}, $w_{oH,t+1}(v)$ and on the function η_{t+1}. Similarly to the second category there is one of these equations for every value of v in the interval $[v_{\min}, v_{\max}]$. Given these similarities we can use the same strategy. In particular, we can consider this equation only at the grid points $(v_j)_{j=1}^J$ and linearize each of these J equations with respect to $\left[\lambda_t, \xi_{ot}(v_j), \lambda_{t+1}, w_{oH,t+1}(v_j), \{\eta_{t+1}(v_k)\}_{k=1}^J\right]$ at the deterministic steady state values $\left[\bar{\lambda}, \bar{\xi}_o(v_j), \bar{\lambda}, \bar{w}_{oH}(v_j), \{\bar{\eta}(v_k)\}_{k=1}^J\right]$.

The fifth category is much more involved. It is constituted by equations that involve scalars and integrals of variables with respect to the distribution μ_t. Equations (34) and (35) fall in this category. For example, consider equation (34). This equation depends on the real numbers $u_{yL,t}$, $u_{yH,t}$, I_t, z_t, K_{t-1}, and H_t, and on the integrals $\int e^{u_{oL,t}} d\mu_t$ and $\int e^{u_{oH,t}} d\mu_t$. To make progress it will be important to represent these integrals with a convenient finite set of variables. In order to do this, I will follow a strategy that is closely related to the one that was used in Section 6.1 for computing statistics under the invariant distribution. In particular, consider the same large but finite number of agents I that was used in that section and endow them with the same realized distribution of promised values $(\bar{v}_i)_{i=1}^I$ that was obtained when computing the steady state. Now, assume that these agents populated

the economy M time periods ago and consider the history

$$\{w_{oL,t-m}, w_{oH,t-m}, w_{yL,t-m}, w_{yH,t-m}\}_{m=0}^{M},$$

which describes the allocation rules for next-period promised values that were chosen during the last M periods (where t is considered to be the current period). Observe that since $w_{oL,t-m}$ and $w_{oH,t-m}$ are spline approximations, this history can be represented by the following finite list of values:

$$\left\{\left[w_{oL,t-m}\left(v_j\right)\right]_{j=1}^{J}, \left[w_{oH,t-m}\left(v_j\right)\right]_{j=1}^{J}, w_{yL,t-m}, w_{yH,t-m}\right\}_{m=0}^{M}. \tag{123}$$

Using the history of allocation rules for next-period promised values, we can simulate the evolution of promised values for the I agents and their descendants during the last M time periods to update the distribution of promised values from the initial $(\bar{v}_i)_{i=1}^{I}$ to a current distribution $(v_{i,t})_{i=1}^{I}$.

In particular, we can initialize the distribution of promised values at the beginning of period $t-M-1$ as follows:

$$v_{i,t-M-1} = \bar{v}_i,$$

for $i = 1, ..., I$. Given a distribution of promised values at the beginning of period $t-m-1$, the distribution of promised values at period $t-m$ is then obtained as follows:

$$v_{i,t-m} = \begin{cases} w_{oL,t-m}\left(v_{i,t-m-1}\right), \text{ with probability } \sigma\psi_L, \\ w_{oH,t-m}\left(v_{i,t-m-1}\right), \text{ with probability } \sigma\psi_H, \\ w_{yL,t-m}, \text{ with probability } (1-\sigma)\,\psi_L, \\ w_{yH,t-m}, \text{ with probability } (1-\sigma)\,\psi_H, \end{cases} \tag{124}$$

for $i = 1, ..., I$. Proceeding recursively for $m = M, M-1, ..., 0$, we obtain a realized distribution of promised values $(v_{i,t})_{i=1}^{I}$ at the beginning of period t.

Observe that the last iteration of equation (124) also gives the corresponding realized values of leisure $(s_{it})_{i=1}^{I}$ across the I agents. The joint realized distribution of promised values and values of leisure $(v_{it}, s_{it})_{i=1}^{I}$ can then be used to compute statistics under the distribution μ_t. In particular, equation (34) can be re-written as:

$$0 = (1-\sigma)\left[e^{u_{yL,t}}\psi_L + e^{u_{yH,t}}\psi_H\right] + \sigma\frac{1}{I}\sum_{i=1}^{I} e^{u_{os_{it}}(v_{it})} + I_t - e^{z_t}K_{t-1}^{\gamma}H_t^{1-\gamma}. \tag{125}$$

Since $u_{oL,t}$ and $u_{oH,t}$ are splines approximations, they can be summarized by their values at the grid points $(v_j)_{j=1}^J$. Therefore, equation (125) can be linearized with respect to

$$\begin{aligned} &I_t, z_t, K_{t-1}, H_t, u_{yL,t}, u_{yH,t}, [u_{oL,t}(v_j)]_{j=1}^J, [u_{oH,t}(v_j)]_{j=1}^J, \\ &\left\{[w_{oL,t-m}(v_j)]_{j=1}^J, [w_{oH,t-m}(v_j)]_{j=1}^J, w_{yL,t-m}, w_{yH,t-m}\right\}_{m=0}^M \end{aligned} \tag{126}$$

at their steady state values

$$\bar{I}, 0, \bar{K}, \bar{H}, \bar{u}_{yL}, \bar{u}_{yH}, [\bar{u}_{oL}(v_j)]_{j=1}^J, [\bar{u}_{oH}(v_j)]_{j=1}^J, \left\{[\bar{w}_{oL}(v_j)]_{j=1}^J, [\bar{w}_{oH}(v_j)]_{j=1}^J, \bar{w}_{yL}, \bar{w}_{yH}\right\}_{m=0}^M.$$

Observe that equation (126) provides a large but finite list of variables. In particular, there are $M(2J+2)$ variables in the second line of equation (126). Taking numerical derivatives with respect to each of these variables requires simulating I agents over M periods. As a consequence, linearizing equation (125) requires performing a massive number of Monte Carlo simulations. While this seems a daunting task it is easily parallelizable. Thus, using massively parallel computer systems can play an important role in reducing computing times and keeping the task manageable.[26]

The last category of equations has only one element: equation (36), which describes the law of motion for the distribution μ_t. While daunting at first sight, this equation is greatly simplified by our approach of representing the distribution μ_t using the history of values given by equation (123). In fact, updating the distribution μ_t is merely reduced to updating this history. In particular, the date-$(t+1)$ history can be obtained from the date-t history and the current values of $[w_{oL,t+1}(v_j)]_{j=1}^J$, $[w_{oH,t+1}(v_j)]_{j=1}^J$, $w_{yL,t+1}$ and $w_{yH,t+1}$ using the following equations:

$$\left[w_{oL,(t+1)-m}(v_j)\right]_{j=1}^J = \left[w_{oL,t-(m-1)}(v_j)\right]_{j=1}^J \tag{127}$$

$$\left[w_{oH,(t+1)-m}(v_j)\right]_{j=1}^J = \left[w_{oH,t-(m-1)}(v_j)\right]_{j=1}^J \tag{128}$$

$$w_{yL,(t+1)-m} = w_{yL,t-(m-1)} \tag{129}$$

$$w_{yH,(t+1)-m} = w_{yH,t-(m-1)} \tag{130}$$

for $m = 1, ..., M$. Observe that the law of motion described by equations (127)-(130) is already linear, so no further linearization is needed. Also observe that the variables that

[26] In practice, I heavily rely on GPU computing for performing the Monte Carlo simulations.

are M periods old in the date-t history are dropped from the date-$(t+1)$ history. Thus, the law of motion described by equations (127)-(130) introduces a truncation. However, the consequences of this truncation are expected to be negligible. The reason is that the truncation only affects the agents that had survived for M consecutive periods, and given a sufficiently small survival probability σ and/or a sufficiently large M there will be very few of these agents. Aside from this negligible truncation there are no further approximations errors in the representation of the law of motion given by equation (36).

6.2.2 Linearized system

Define the vector of endogenous state variables as follows:

$$x_{t-1} = \left(\triangle \ln K_{t-1}, \left\{ \triangle w_{yL,t-m}, \triangle w_{yH,t-m}, \left[\triangle w_{oL,t-m}\left(\bar{v}_j\right)\right]_{j=1}^{J}, \left[\triangle w_{oH,t-m}\left(\bar{v}_j\right)\right]_{j=1}^{J} \right\}_{m=0}^{M} \right),$$

and the vector of decision variables and Lagrange multipliers as follows:

$$\begin{aligned} y_t \;=\; & \Big(\triangle w_{yL,t+1}, \triangle w_{yH,t+1}, \left[\triangle w_{oL,t+1}\left(\bar{v}_j\right)\right]_{j=1}^{J}, \left[\triangle w_{oH,t+1}\left(\bar{v}_j\right)\right]_{j=1}^{J} \\ & \triangle u_{yL,t}, \triangle u_{yH,t} \triangle n_{yL,t}, \triangle n_{yH,t} \triangle \ln \xi_{yt}, \triangle \ln \lambda_t, \triangle \ln q_t, \\ & \left[\triangle \ln \eta_t\left(\bar{v}_j\right)\right]_{j=1}^{J}, \left[\triangle u_{oL,t}\left(\bar{v}_j\right)\right]_{j=1}^{J}, \left[\triangle u_{oH,t}\left(\bar{v}_j\right)\right]_{j=1}^{J}, \left[\triangle n_{oL,t}\left(\bar{v}_j\right)\right]_{j=1}^{J}, \\ & \left[\triangle n_{oH,t}\left(\bar{v}_j\right)\right]_{j=1}^{J}, \left[\triangle \ln \xi_{ot}\left(\bar{v}_j\right)\right]_{j=1}^{J}, \triangle \ln H_t, \triangle \ln I_t \Big). \end{aligned}$$

Then, using the approach described in the previous section, the linear approximation to equations (16)-(23), (24)-(30) and (31)-(37) can be written as

$$0 = Ax_t + Bx_{t-1} + Cy_t + Dz_t, \tag{131}$$

$$0 = Gx_t + Jy_{t+1} + Ky_t + Lz_{t+1}, \tag{132}$$

$$z_{t+1} = Nz_t. \tag{133}$$

where I have applied the certainty equivalence principle.

We seek a linear solution to equations (131)-(133) of the following form:

$$x_t = Px_{t-1} + Qz_t \tag{134}$$

$$y_t = Rx_{t-1} + Sz_t \tag{135}$$

Equations (131)-(135) have exactly the same structure as in Uhlig (1999), so his methods can be directly applied. Alternatively, one could iterate with equations (134)-(135) as follows. Suppose that at iteration j we have that

$$x_t = P^j x_{t-1} + Q^j z_t, \tag{136}$$

$$y_t = R^j x_{t-1} + S^j z_t. \tag{137}$$

and that we want to find

$$x_t = P^{j+1} x_{t-1} + Q^{j+1} z_t, \tag{138}$$

$$y_t = R^{j+1} x_{t-1} + S^{j+1} z_t, \tag{139}$$

for iteration $j+1$. Substituting equations (136)-(139) into equations (131)-(132), it is easy to show that P^{J+1}, Q^{j+1}, R^{j+1}, and S^{j+1} are the solution to the following system of linear equations:

$$\begin{bmatrix} A & C \\ (JR^j + G) & K \end{bmatrix} \begin{bmatrix} P^{j+1} & Q^{j+1} \\ R^{j+1} & S^{j+1} \end{bmatrix} = - \begin{bmatrix} B & D \\ 0 & (JS^j + L)\, N \end{bmatrix}, \tag{140}$$

which can be solved using a LU decomposition. Iterating with equation (140) until convergence is an alternative way of obtaining the solution P, Q, R and S that we seek.

The important thing is that whatever method one chooses to use, the linear rational expectations model given by equations (131)-(135) can be solved using standard methods. The only difficulty is its high dimensionality. Once equations (134)-(135) are obtained, they can be used to simulate the economy.

7 Numerical results

This section uses the computational method just described to explore the quantitative properties of different private information economies and compare them to those of their full information counterparts. In order to do this I first select parameter values for the benchmark economy with log-log preferences. Economies with more general preferences will be considered later on.

7.1 Parametrization

Except for the private information, the basic structure of the model corresponds to a standard real business cycle model. In fact, under log-log preferences the basic structure of the model is identical to the one in Cooley and Prescott (1995). For this reason, I calibrate all parameters associated with the neoclassical growth model to the same observations as theirs. In order to simplify computations, the model time period is selected to be one year.

Following Cooley and Prescott (1995) the labor share parameter $1-\gamma$ is set to 0.60, the depreciation rate δ is chosen to reproduce an investment-capital ratio I/K equal to 0.076, and the social discount factor θ is chosen to reproduce a capital-output ratio K/Y equal to 3.32. The values of leisure s_L and s_H are chosen to satisfy two criteria: that aggregate hours worked H equal 0.31 (another observation from Cooley and Prescott 1995) and that the hours worked by old agents with the high valuation of leisure and the highest possible promised value $n_{oH}(v_{\max})$ be a small but positive number. The rationale for this second criterion is that I want to maximize the relevance of the information frictions while keeping an internal solution for hours worked. The probability of drawing a high value of leisure ψ_H is chosen to maximize the standard deviation of the invariant distribution of promised values. It turns out that a value of $\psi_L = 0.50$ achieves this. The survival probability σ is chosen to generate an expected lifespan of 40 years. In turn, the individual discount factor β is chosen to be the same as the social discount factor θ. In terms of the parameters for the aggregate productivity stochastic process, ρ is chosen to be 0.95 and the variance of the innovations to aggregate productivity σ_ε^2 is chosen to be 4×0.007^2 (as in Cooley and Prescott 1995).

While the above parameters are structural, there are a number of computational parameters to be determined. The number of grid points in the spline approximations J, the total number of agents simulated I, the length of the simulations for computing the invariant distribution T, and the length of the histories kept as state variables when computing the business cycles M are all chosen to be as large as possible, while keeping the computational task manageable and results being robust to non-trivial changes in their values.[27] The lower

[27] Given the value selected for the survival probability σ, less than 0.1% of individuals survive more than M periods. Thus, the truncation imposed by keeping track of a finite history of decision rules introduces a

and upper bounds for the range of possible promised values $v_{\min}$ and $v_{\max}$ in turn were chosen so that the fraction of agents in the intervals $[v_1, v_2]$ and $[v_{J-1}, v_J]$ are each less than 0.1%. Thus, truncating the range of possible values at $v_{\min}$ and $v_{\max}$ should not play an important role in the results.

Table 1

Parameter values

Structural	Computational
$s_L = 1.513$	$v_{\min} = -28.5$
$s_H = 2.047$	$v_{\max} = -11$
$\psi_L = 0.50$	$T = 1,000$
$\theta = 0.9574$	$M = 273$
$\beta = 0.9574$	$I = 8,388,608$
$\sigma = 0.975$	$J = 20$
$\gamma = 0.40$	
$\delta = 0.076$	
$\rho = 0.95$	
$\sigma_\varepsilon = 0.014$	

Table 1 describes all parameter values. It turns out that under the computational parameters specified in this table the dimensionality of the linear system described by equations (131)-(135) is about $12,000 \times 12,000$, a large system indeed.

7.2 Results under log-log preferences

Before turning to business cycle dynamics I illustrate different features of the model at its deterministic steady state. Figure 1.A shows the invariant distribution of promised values across the $J-1$ intervals $[v_j, v_{j+1}]_{j=1}^{J-1}$ defined by the grid points of the spline approximations. While it is hard to see at this coarseness level, the distribution is approximately symmetrical. More importantly, we see that the invariant distribution puts very little mass at extreme

small approximation error.

values. As a consequence, in what follows I will report allocation rules only between the 7th and 15th ranges of the histogram. The reason is not only that there are very few agents at the tails of the distribution for them to matter, but being close to the artificial bounds $v_{\min}$ and $v_{\max}$ greatly distorts the shape of the allocation rules.

Figure 1: Deterministic steady state

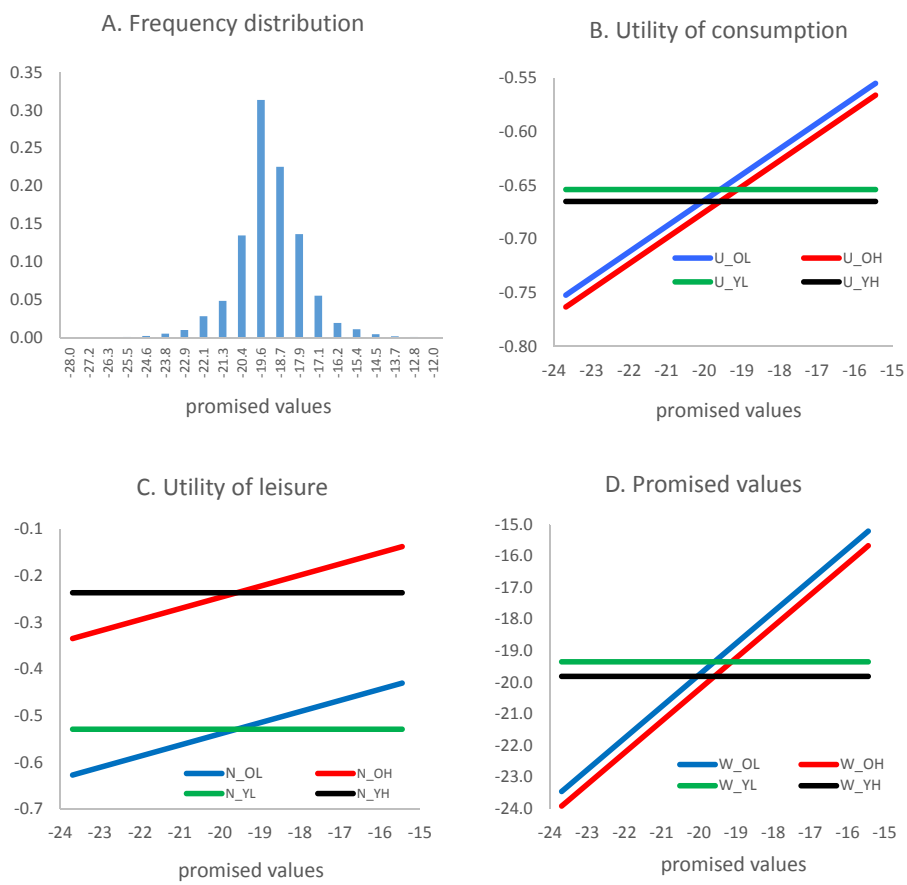
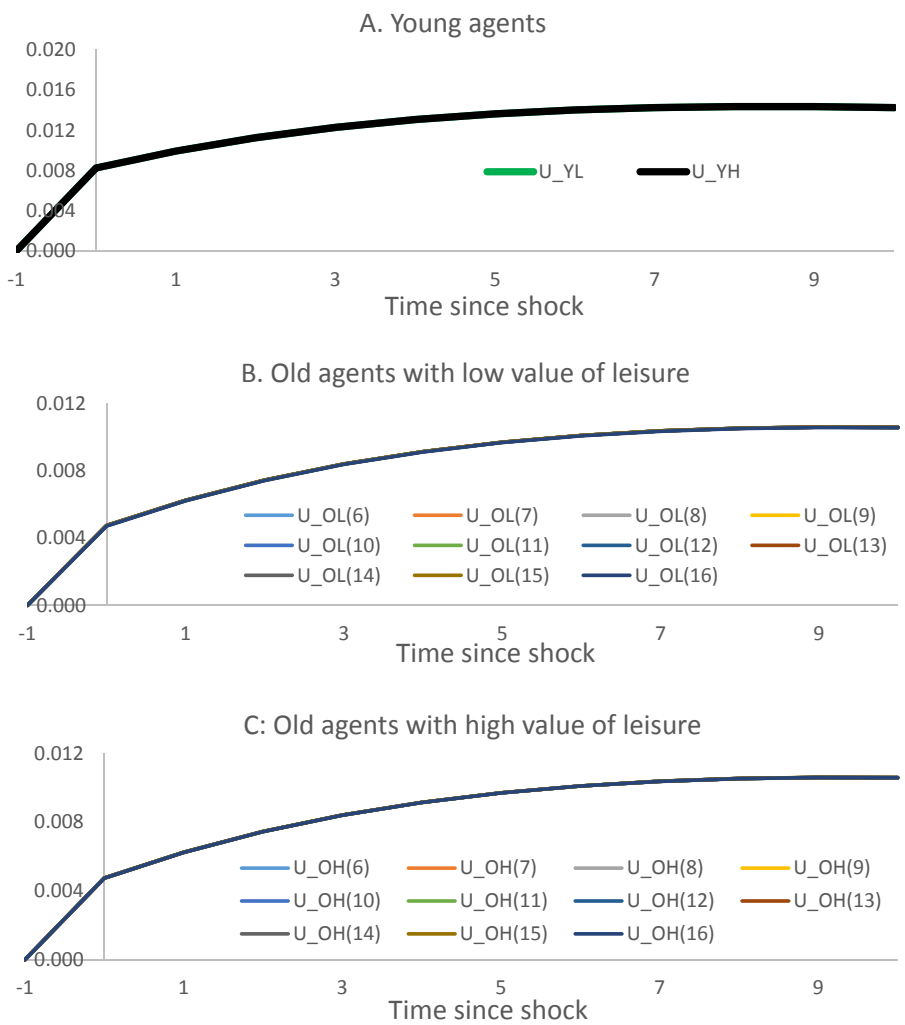


Figure 1.B reports the utility of consumption for old agents $u_{oL}(v)$ and $u_{oH}(v)$ across promised values v, as well as those of young agents u_{yL} and u_{yH} (which are independent of v). We see that, in all cases the utility of consumption is higher when the value of leisure is low. Both u_{oL} and u_{oH} are strictly increasing in the promised value v, are linear (with slope less than one) and parallel to each other. Moreover, the vertical difference between u_{oL} and u_{oH} is the same as between u_{yL} and u_{yH}. Figure 1.C reports the utility of leisure for old agents $n_{oL}(v)$ and $n_{oH}(v)$ across promised values v, as well as those of young agent n_{yL} and n_{yH}. We see that in all cases leisure is lower when the value of leisure is low. Both n_{oL} and n_{oH} are strictly increasing in the promised value v, are linear (with slope less than

one) and parallel to each other. Moreover, the vertical difference between n_{oL} and n_{oH} is the same as between n_{yL} and n_{yH}. In turn, Figure 1.D reports the next-period promised values for old agents $w_{oL}(v)$ and $w_{oH}(v)$ across promised values v, as well as those of young agent w_{yL} and w_{yH}. We see that in all cases next-period promised values are higher when the value of leisure is low. Both w_{oL} and w_{oL} are strictly increasing in the promised value v, are linear (with slope equal to one) and parallel to each other. We also see that the vertical difference between w_{oL} and w_{oH} is the same as between w_{yL} and w_{yH}. Thus, Figure 1 verifies the analytical steady state results of Lemmas 2, 3 and 5.

Figure 2: Impulse responses of utility of consumption



The discussion of business cycle dynamics that follows will be centered around the analysis of the impulse responses of different variables to a one standard deviation increase in

aggregate productivity. Figure 2.A shows the impulse responses of the utility of consumption of young agents u_{yL} and u_{yH}. We see that both impulse responses are identical and that their shape qualitatively resembles one for aggregate consumption in a standard RBC model. Figure 2.B shows the impulse response of the utility of consumption of old agents with a low value of leisure $u_{oL}(v)$, at each of the eleven grid points $(v_j)_{j=6}^{16}$. While the figure shows eleven impulse responses, only one of them is actually seen because they happen to overlap perfectly. This means that, in response to the aggregate productivity shock, the function u_{oL} depicted in Figure 1.B shifts vertically over time. Figure 2.C, which does the same for u_{oH}, is identical to Figure 2.B. Thus, u_{oH} also shifts vertically over time and its increments are the same as those of u_{oL}.

Figure 3: Impulse responses of utility of leisure

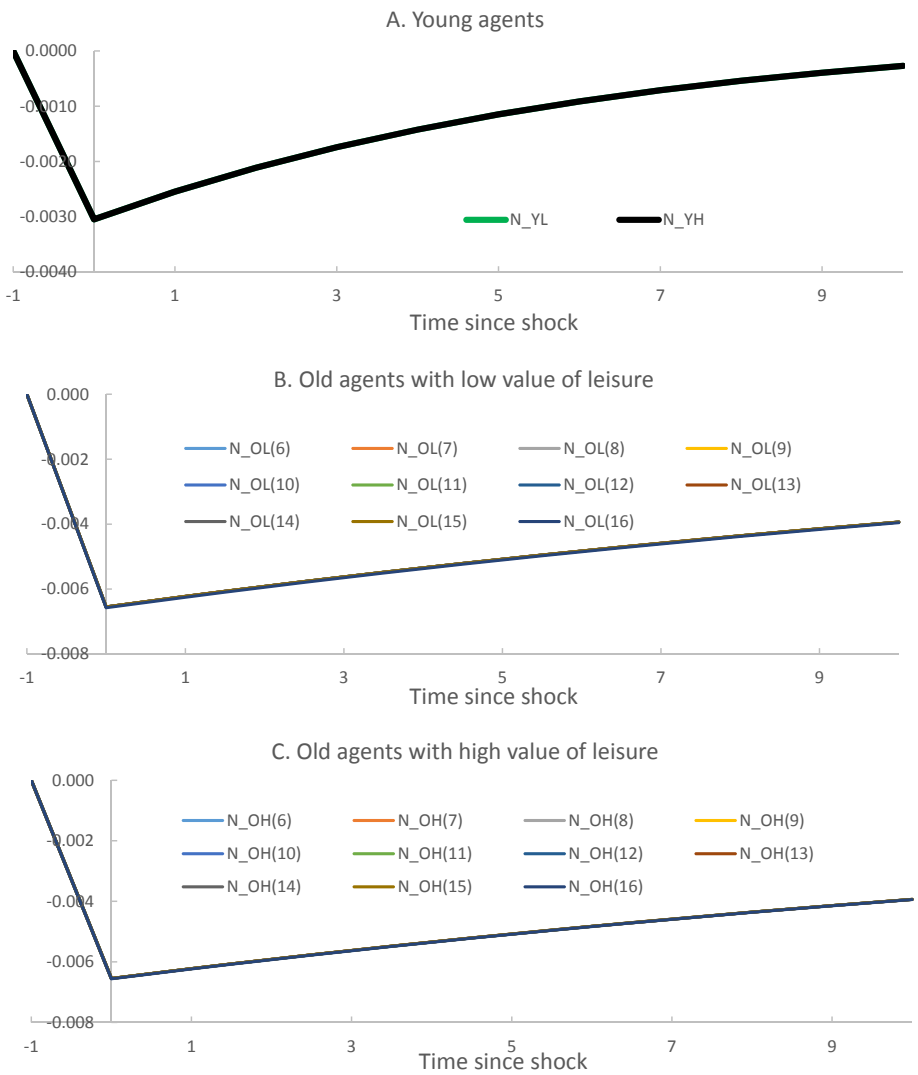


Figure 3 is analogous to Figure 2, except that they depict the behavior of the utility of leisure. Figure 3.A shows that the impulse responses of n_{yL} and n_{yH} are identical and that they resemble the response of leisure in a standard RBC model, while Figures 3.B and 3.C indicate identical vertical shifts of the functions n_{oL} and n_{oH} in response to the aggregate productivity shock.

Turning to promised values, Figure 4.A shows that the impulse responses of w_{yL} and w_{yH} coincide. In turn, Figures 4.B and 4.C show that w_{oL} and w_{oH} shift vertically by identical amounts in response to an aggregate productivity shock. Thus, taken together, we see that Figures 2 -4 reproduce the analytical results of equations (67)-(69) and (88)-(93).

Figure 4: Impulse responses of promised values

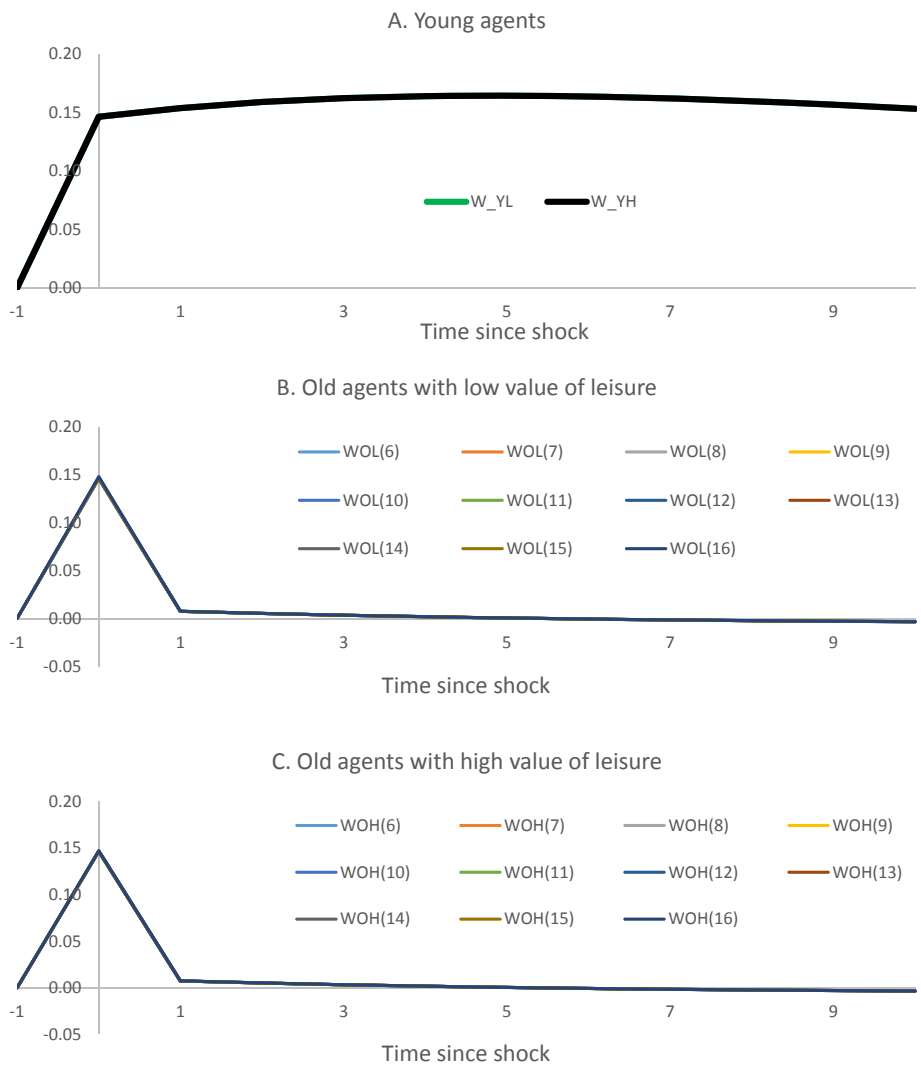
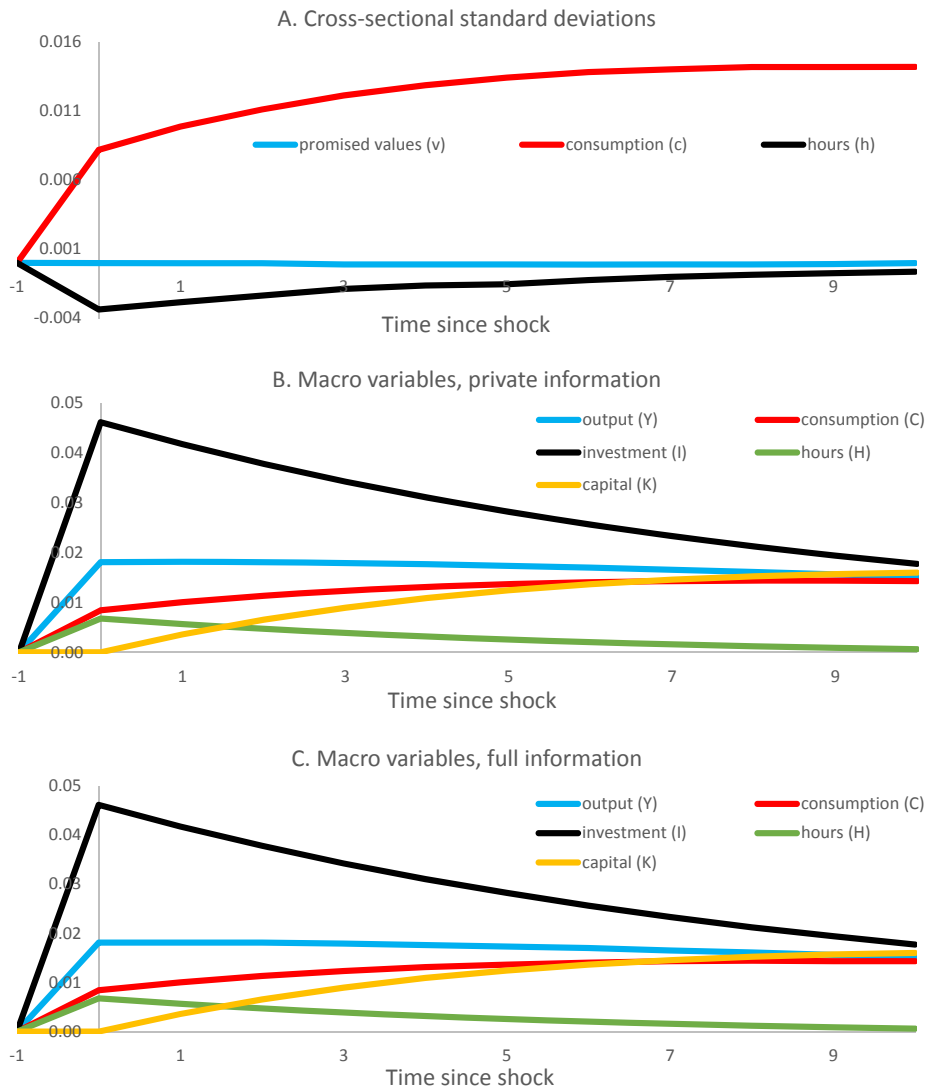


Figure 5.A shows the impulse responses of the cross sectional standard deviations of

promised values, consumption and hours worked. We see that in response to a positive aggregate productivity shock the standard deviation of promised values remains flat while the standard deviation of consumption increases and the standard deviation of hours worked decreases. Thus, Figure 5.A reproduces the analytical results of Section 5.3.

Figure 5: Cross-sectional distributions and macro variables



Finally, Figure 5.B shows the impulse responses of aggregate output Y, aggregate consumption C, aggregate investment I, aggregate hours worked H and aggregate capital K in the benchmark economy with private information. Figure 5.C reports the impulse responses for the same variables but for the full information economy given by equations (98)-(102). We see that both sets of impulse responses are identical. Thus, Figures 5.B and 5.C reproduce

the analytical result of Corollary 10.

We have verified that while the computational method was not designed to exploit any of the properties of the log-log case, it is able to exactly reproduce the analytical results derived for this case. This suggests that the computational method introduced in this paper could be quite useful not only for analyzing other functional forms, but as a general method for computing aggregate fluctuations of economies with heterogeneous agents.

7.3 Extension to other preferences

This section generalizes the preferences of equation (1) to the following form:

$$E_T \left\{ \sum_{t=T}^{\infty} \beta^{t-T} \sigma^{t-T} \left[\frac{c_t^{1-\pi} - 1}{1-\pi} + s_t \frac{(1-h_t)^{1-\alpha} - 1}{1-\alpha} \right] \right\},$$

where $\pi \neq 1$ and $\alpha \neq 1$. Since under this general functional form analytical results are no longer available the computational method becomes essential to evaluate these preferences.

Table 2

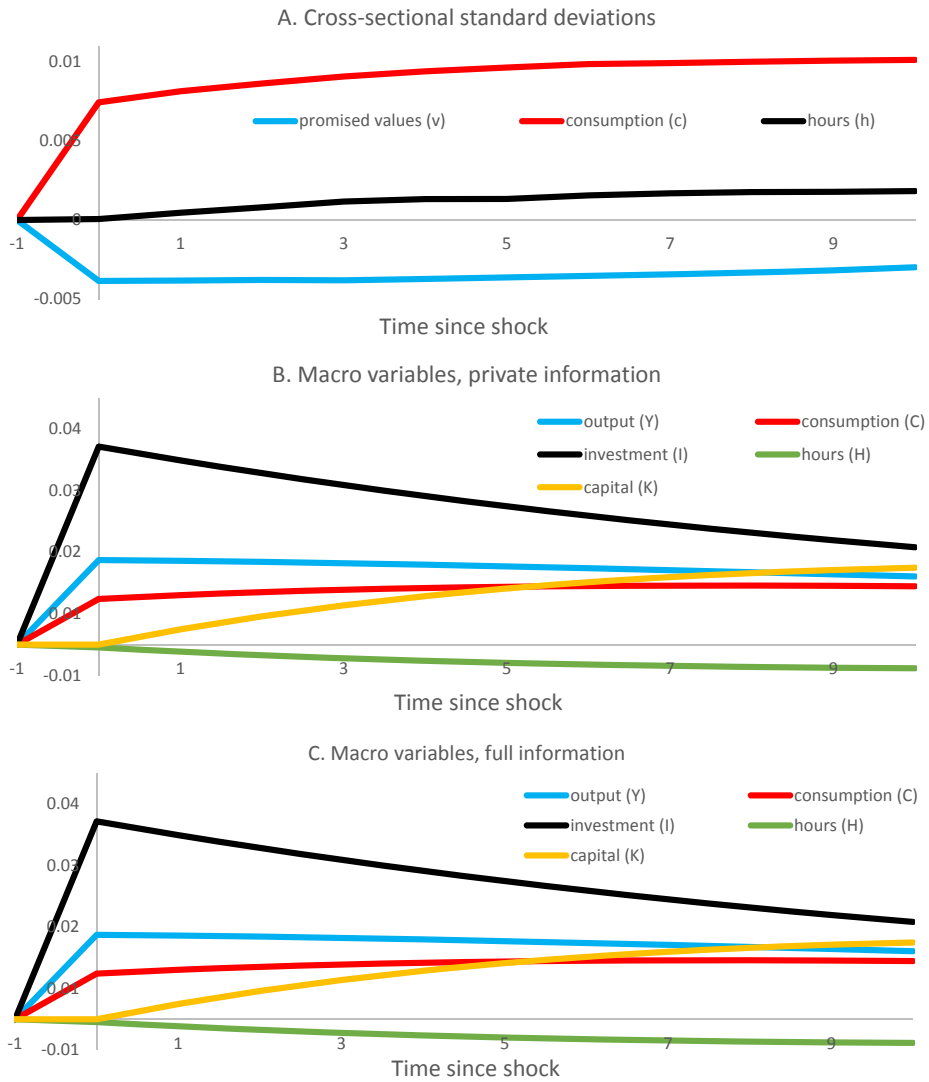
Steady state macroeconomic variables

(α, π)	Information	Y	C	I	H	K
$(1,1)$	Private	0.69155	0.51706	0.17449	0.31074	2.2959
	Full	0.69155	0.51706	0.17449	0.31074	2.2959
$(1,2)$	Private	0.56302	0.42096	0.14206	0.25299	1.8692
	Full	0.56305	0.42098	0.14207	0.25300	1.8693
$(2,1)$	Private	0.89539	0.66947	0.22592	0.40234	2.9727
	Full	0.89551	0.66956	0.22595	0.40239	2.9731
$(2,2)$	Private	0.76319	0.57062	0.19257	0.34293	2.5338
	Full	0.76327	0.57068	0.19259	0.34297	2.5341

Without recalibrating other parameters different values for π and α have been considered. However, in all cases similar results were obtained. For concreteness I here report results for unit deviations from the $\pi = 1$ and $\alpha = 1$ case. For each of these cases Table 2 reports the steady state values of all macroeconomic variables for the economies with private information

and full information. We see that in each parametrization all variables are nearly identical in both information scenarios.

Figure 6: Cross-sectional distributions and macro variables



In order to streamline the analysis of business cycle dynamics I consider the $\pi = 2$ and $\alpha = 2$ as a representative case. Figure 17 reports that, contrary to the log-log case, the cross sectional distribution of promised values now follows a non-trivial dynamics: Instead of being constant, the standard deviation of promised values decreases significantly in response to a positive aggregate productivity shock. Despite of this the information frictions still turn out to be irrelevant for aggregate dynamics. Figure 18 reports the impulse responses of all macroeconomic variables in the economy with private information while Figure 19 does the same for the economy with full information. We see that both sets of impulse responses

are identical. Thus, similarly to the log-log case, the stationary behavior of the aggregate variables of the economy is not affected by the presence of information frictions.

8 Conclusions

The paper considered a RBC model in which agents are subject to i.i.d. idiosyncratic shocks to their value of leisure. A key assumption of the model is that these shocks are private information of the agents. In this framework the paper analyzed the mechanism design problem of maximizing utility subject to incentive compatibility, promise keeping and aggregate feasibility constraints. For the case of log-log preferences the paper obtained sharp analytical characterizations. In particular, the utility of consumption, the utility of leisure and next-period promised values are all linear functions of current promised values. Over the business cycle these functions shift vertically in such a way that the distribution of promised values shifts horizontally while maintaining its shape. However, the cross-sectional dispersion of consumption levels turns out to be procyclical while the cross-sectional dispersion of hours worked is countercyclical. A key result of the paper is that the stationary business cycle fluctuations of all macroeconomic variables are exactly the same under private information as under full information.

For preferences other than the log-log case analytical results are no longer available. To analyze these other cases the paper developed a novel method for computing equilibria of economies with heterogeneous agents. Its basic strategy is to parametrize individual decision rules as spline approximations and to keep long histories of the spline coefficients as state variable. The model is then linearized with respect to these variables and solved. Two advantages of the computational method over alternatives is that it approximates the current distribution of promised values arbitrarily well and that the law of motion for this distribution is almost exact. Applying this method to other preference specifications produces a similar irrelevance result for aggregate dynamics. While the distribution of promised values may now change its shape over the business cycle, the business cycle fluctuations of all macroeconomic variables are still unaffected by the presence of private information.

The paper opens wide possibilities for future research. While the irrelevance result for the general CRRA preferences was obtained numerically, it is an open question if it could

be established analytically. While I ignore the answer to this question I speculate that if it could the proof would be much more involved than in this paper because of fluctuations in the shape of the distribution of promised values. Also, the irrelevance of private information for aggregate dynamics was obtained under a very particular framework (although a very interesting one, since the Mirlees structure considered constitutes a benchmark case in the dynamic public finance literature). It is an open question if information frictions could play an important role in aggregate dynamics in alternative settings, as in economies with moral hazard in unemployment insurance. In fact, the computational method developed here should prove extremely useful not only to evaluate these alternatives but to compute equilibria of more general models with aggregate fluctuations and heterogeneous agents.

A Appendix

Assuming that the aggregate productivity level z is identical to zero and imposing the condition that all variables are constant over time, equations (43)-(63) become the following:

$$0 = -e^{u_{oL}}\psi_L + e^{g_o} + e^f\psi_L, \tag{141}$$

$$0 = -e^{u_{oH}}\psi_H - e^{g_o} + e^f\psi_H, \tag{142}$$

$$0 = -qe^{n_{oL}}\psi_L + s_L e^{g_o} + e^f s_L\psi_L, \tag{143}$$

$$0 = -qe^{n_{oH}}\psi_H - s_L e^{g_o} + e^f s_H\psi_H, \tag{144}$$

$$0 = \beta\sigma e^{g_o} + e^f\beta\sigma\psi_L - \theta\sigma\psi_L e^{f+bw_{oL}}, \tag{145}$$

$$0 = -\beta\sigma e^{g_o} + e^f\beta\sigma\psi_H - \theta\sigma\psi_H e^{f+bw_{oH}}, \tag{146}$$

$$u_{oL} + s_L n_{oL} + \beta\sigma w_{oL} = u_{oH} + s_L n_{oH} + \beta\sigma w_{oH}, \tag{147}$$

$$0 = \{u_{oL} + s_L n_{oL} + \beta\sigma w_{oL}\}\psi_L + \{u_{oH} + s_H n_{oH} + \beta\sigma w_{oH}\}\psi_H, \tag{148}$$

$$0 = \psi_L - \lambda e^{u_{yL}}\psi_L + \lambda\xi_y, \tag{149}$$

$$0 = \psi_H - \lambda e^{u_{yH}}\psi_H - \lambda\xi_y, \tag{150}$$

$$0 = s_L\psi_L - \lambda qe^{n_{yL}}\psi_L + \lambda s_L\xi_y, \tag{151}$$

$$0 = s_H\psi_H - \lambda qe^{n_{yH}}\psi_H - \lambda s_L\xi_y, \tag{152}$$

$$0 = \beta\sigma\psi_L + \lambda\beta\sigma\xi_y - \theta\lambda\sigma\psi_L e^{f+bw_{yL}}, \tag{153}$$

$$0 = \beta\sigma\psi_H - \lambda\beta\sigma\xi_y - \theta\lambda\sigma\psi_H e^{f+bw_{yH}}, \tag{154}$$

$$u_{yL} + s_L n_{yL} + \beta\sigma w_{yL} = u_{yH} + s_L n_{yH} + \beta\sigma w_{yH}, \tag{155}$$

$$0 = q - K^{\gamma}(1-\gamma)H^{-\gamma} \tag{156}$$

$$0 = -1 + \theta\left[\gamma K^{\gamma-1}H^{1-\gamma} + 1 - \delta\right] \tag{157}$$

$$0 = \delta K - I \tag{158}$$

$$(1-\sigma)\sum_s e^{u_{ys}}\psi_s + V\sum_s e^{u_{os}}\psi_s + I = K^{\gamma}H^{1-\gamma}, \tag{159}$$

$$H = (1-\sigma)\sum_s (1 - e^{n_{ys}})\psi_s + \sigma - V\sum_s e^{n_{os}}\psi_s, \tag{160}$$

$$V = \sigma V\sum_s e^{bw_{os}}\psi_s + (1-\sigma)\sigma\sum_s e^{bw_{ys}}\psi_s. \tag{161}$$

It is straightforward to show that equations (159)-(161) actually reduce to the following:[28]

$$(1-\sigma)\frac{1}{\lambda}\frac{\theta}{\theta-\sigma\beta} + I = K^{\gamma}H^{1-\gamma}, \tag{162}$$

$$H = 1 - (1-\sigma)\frac{\bar{s}}{\lambda q}\frac{\theta}{\theta-\sigma\beta}, \tag{163}$$

$$V = (1-\sigma)\sigma\frac{\beta}{\lambda e^f}\frac{1}{(\theta-\sigma\beta)}. \tag{164}$$

References

Algan, Y., O. Allais, W. J. D. Haan and P. Rendhal, "Solving and Simulating Models with Heterogeneous Agents and Aggregate Uncertainty," in K. Schmedders and K. L. Judd, eds., *Handbook of Computational Economics*volume 3 (North-Holland, 2014), 277–324.

Atkeson, A. and R. E. Lucas, "On Efficient Distribution With Private Information," *Review of Economic Studies* 59 (1992), 427–453.

[28]See Section 3 in the Technical Appendix.

Cooley, T. F. and E. C. Prescott, "Economic growth and business cycles," in T. F. Cooley, ed., *Frontiers of business cycle research* (Princeton: Princeton University Press, 1995), 1–38.

DaCosta, C. and V. Luz, "The Private Memory of Aggregate Shocks," mimeo, Fundacao Getulio Vargas, 2013.

Farhi, E. and I. Werning, "Capital Taxation: Quantitative Explorations of the Inverse Euler Equation," *Journal of Political Economy* 120 (2012), 398 – 445.

Golosov, M., A. Tsyvinski and I. Werning, "New Dynamic Public Finance: A User's Guide," in *NBER Macroeconomics Annual 2006, Volume 21*NBER Chapters (National Bureau of Economic Research, Inc, 2007), 317–388.

Green, E. J., "Lending and the Smoothing of Uninsurable Income," in E. C. Prescott and N. Wallace, eds., *Contractual arrangements for intertemporal trade* (Minneapolis: University of Minnesota Press, 1987), 3–25.

Hopenhayn, H. A. and J. P. Nicolini, "Optimal Unemployment Insurance," *Journal of Political Economy* 105 (April 1997), 412–38.

Kocherlakota, N., "Figuring out the Impact of Hidden Savings on Optimal Unemployment Insurance," *Review of Economic Dynamics* 7 (July 2004), 541–554.

Krusell, P., A. A. Smith and Jr., "Income and Wealth Heterogeneity in the Macroeconomy," *Journal of Political Economy* 106 (October 1998), 867–896.

Phelan, C., "Incentives and Aggregate Shocks," *Review of Economic Studies* 61 (1994), 681–700.

Steffen, M., "A Simple Method for Monotonic Interpolation in One Dimension," *Astronomy and Astrophysics* 239 (1990), 443 – 450.

Uhlig, H., "A Toolkit for Analysing Nonlinear Dynamic Stochastic Models Easily," in R. Marimon and A. Scott, eds., *Computational Methods for the Study of Dynamics Economies* (Oxford: Oxford University Press, 1999), 30–61.

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